Advanced Scaling Techniques for the Modeling of Materials Processing

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OMS Methodology Flow Sheet:

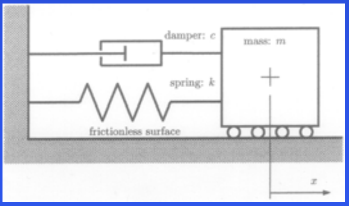
Determine the governing equation and boundary conditions that represent the physical process → Normalize the variables and boundary conditions and scale the governing equation. → Determine the vector of inputs and outputs. Build the Matrix of Balances. → Construct Matrix of Coefficients from the coefficients of each term of the scaled governing equation. Obtain the Normalized Matrix of Coefficients. → Perform matrix operation to obtain the Matrix of Estimations. Results in a scaling equation that captures the dominant driving forces involved in the process. The scaling equation is an estimate of unknown process variables. → Check for Self-Consistency.

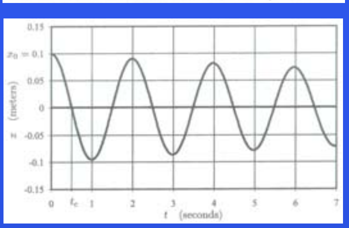
Order of Magnitude Scaling (OMS)

OMS deals with a system of coupled differential equations in their full complexity and provides an approximate solution to the set of governing equations. The solution is expressed in the form of power-laws, and it discriminates the dominant forces in the problem from the less relevant ones. OMS is an improvement over classical scaling techniques due to a more accurate scaling of the differential expressions. OMS lends itself to being implemented in a computer algorithm since the operations potentially involve a large number of matrices that could have a significant number of elements.

Example: Using OMS to obtain an expression for the unknown characteristic time, t_c. (Mechanical Oscillator)

OMS takes advantage of the following feature of governing equations in engineering problems: Each equation is a sum of terms, with each term representing a driving force. The competing driving forces in the Mechanical Oscillator example are: 1). Inertial force 2). Damping force 3). Spring force





Governing Equation:

$$m\ddot{x} + c\dot{x} + kx = 0 \quad x(0) = 0 \quad \dot{x}(0) = 0$$

Normalizing the variables:

$$x^* = \frac{x - x_{\min}}{x_{\max} - x_{\min}} = \frac{x-0}{x_0 - 0} = \frac{x}{x_0} \quad \therefore \quad x = x^* x_0 \qquad t^* = \frac{t - t_{\min}}{t_{\max} - t_{\min}} = \frac{t-0}{t_c - 0} = \frac{t}{t_c} \quad \therefore \quad t = t^* t_c$$

Scaled Governing Equation:

$$2\frac{mx_0}{t_c^2} f_1^*(t^*) + 2\frac{cx_0}{t_c} f_2^*(t^*) + kx_0 f_3^*(t^*) = 0$$

Vector of Inputs and Outputs:

$$\{\phi\} = \{-1 \quad -1 \quad 1\}$$

The Matrix of Balances:

$$[B]_1 = [1 \quad 1 \quad 3] \quad \text{and} \quad [B]_2 = [1 \quad 2 \quad 3]$$

Matrix of Coefficients (MOC):

$$\begin{array}{cc} & \begin{array}{cccccc} 2 & m & c & k & x_0 & t_c \end{array} \\ [C] = & \begin{bmatrix} 1 & 1 & 0 & 0 & 1 & -2 \\ 1 & 0 & 1 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

Matrix of Normalized Coefficients: Balance $[B]_1$

$$\begin{array}{cc} & \begin{array}{cccccc} 2 & m & c & k & x_0 & t_c \end{array} \\ [N] = & \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 1 \\ -1 & -1 & 0 & 1 & 0 & 2 \end{bmatrix} \end{array}$$

Matrix of Estimations (MOE):

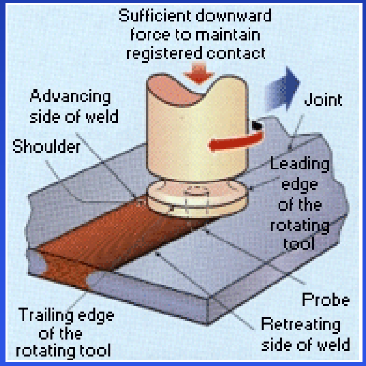
$$[S] = -[N_O]_S^{-1}[N_O]_{P'} = -\left[\frac{1}{2}\right][-1 \quad -1 \quad 0 \quad 1 \quad 0] = \left[\frac{1}{2} \quad \frac{1}{2} \quad 0 \quad -\frac{1}{2} \quad 0\right]$$

This leads to an estimation of the characteristic time: $\hat{t}_c = \sqrt{\frac{2m}{k}}$

CWJCR

Modeling Friction Stir Welding (FSW) with OMS

- The Welding Institute invented Friction Stir Welding in 1991
- Solid State Process that joins metal through mechanical deformation
- Can weld previously reported unweldable Al alloys (2xxx, 7xxx)
- Weld strength is 30-50% greater than with arc welding.



Objective of Research:

- Scale steady state FSW process parameters in high melting temperature alloys.
- No known scaling laws for FSW
- Test with Aluminum
- Scale up to high temperature melting alloys

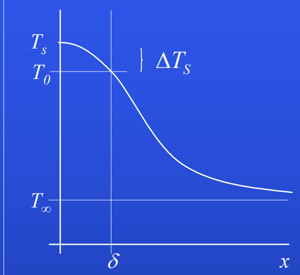
Analysis:

- Five coupled differential equations
- Shear Layer
  - I. Heat Transfer (1)
- I. Deformation (2)
  - I. Constitutive Law (1)
- Base Plate
  - I. Heat Transfer (1)

Governing Equation:

$$\frac{\partial^2 T}{\partial x^2} + \frac{q}{k} = 0 \quad T'(0) = 0, \quad T(\delta) = T_0$$

Heat Transfer in Shear Layer:



Normalizing the variables:

$$x^* = \frac{x - x_{\min}}{x_{\max} - x_{\min}} = \frac{x - 0}{\delta - 0} = \frac{x}{\delta} \quad \therefore \quad x = x^*\delta$$

$$T^* = \frac{T - T_{\min}}{T_{\max} - T_{\min}} = \frac{T - T_0}{\Delta T_S} \quad \therefore \quad T = T_0 + \Delta T_S T^*$$

$$q^* = \frac{q - q_{\min}}{q_{\max} - q_{\min}} = \frac{q - 0}{q_c - 0} = \frac{q}{q_c} \quad \therefore \quad q = q^* q_c$$

Scaled Governing Equation:

$$\underbrace{\frac{2\Delta T_S k}{\delta^2 q_c}\left(\frac{\partial^2 T}{\partial x^2}\right)^*}_{\text{OM(1)}} + \underbrace{q^*}_{\text{OM(1)}} = 0$$

Scaling Equations:

Heat Transfer in Shear Layer:

$$\boxed{2\frac{\Delta \hat{T}_S k}{\hat{\delta}^2 \hat{q}_c} = 1}$$

Estimate of Shear Layer thickness:

$$\frac{\delta}{a} = \left[2\overbrace{\left(\frac{\tau_1 \Delta T_m^{1/n} \omega^{1/n} f^{1/n}}{k v_1'^{1/n} \Delta T_0^{1+1/n}}\right)}^{\text{material properties}} \overbrace{\left(a^2 \omega f\right)}^{\text{process parameters}}\right]^{n/2}$$

The estimation of the shear layer thickness is only obtained by scaling the complete system of equations mentioned in the Analysis section above.

Conclusion:

Order of Magnitude Scaling (OMS) is a modeling technique that deals with a system of coupled differential equations in their full complexity. The solution of the system is expressed in the form of power-laws. The advantage of OMS over other modeling techniques is a resultant scaling equation that is based on system process parameters.