

# Strain energy release in ceramic-to-metal joints by ductile metal interlayers

Jin-Woo Park <sup>\*,1</sup>, Patricio F. Mendez <sup>2</sup>, Thomas W. Eagar

*Department of Materials Science and Engineering, Massachusetts Institute of Technology, Cambridge, MA 02139, USA*

Received 28 March 2005; accepted 2 June 2005

Available online 28 June 2005

## Abstract

We present analytical solutions for thermal strain energy in ceramic-to-metal joints where the coefficient of thermal expansion (CTE) of the interlayer is significantly larger than that of the metal and the ceramic. We consider plasticity and conclude that the relative CTE difference between the three materials and yield stress of the interlayer determine the thermal strain energy.

© 2005 Acta Materialia Inc. Published by Elsevier Ltd. All rights reserved.

**Keywords:** Joining; Residual stress; Finite element analysis

## 1. Introduction

Significant differences in the thermal and mechanical properties of ceramics and metals make it extremely difficult to obtain ceramic-to-metal joints with adequate mechanical integrity [1]. In cylindrical joints with a flat interface as analyzed in this work, large tensile and shear residual stresses are induced in the region around the edge and near the interface of the ceramic, respectively, on cooling from the joining temperature [2]. These stresses enhance the propensity for fracture of the ceramic [2]. Hence, various interlayers have been used to release the residual stresses in the ceramic [1–3].

To compare the effect of the interlayers, analytical models that can define the residual stress of a joint system based on a set of parameters have been proposed by several researchers [3,4]. However, no analytical model can account for both plastic deformation and the effects

of various geometries during bonding of two dissimilar materials involving temperature change. In our previous investigation [5,7], the elastic strain energy in the ceramic was a better strength metric than any alternative based on residual stress components. Using the finite element method (FEM) and order of magnitude scaling (OMS) [6], we described the dimensionless groups which are the important factors affecting the strain energy for those joint systems with a large coefficient of thermal expansion (CTE) mismatch between the ceramic and the metal. The following analytical solution provides a value of the elastic strain energy in the ceramic ( $U_{e,C}$ ) for the asymptotic case within 1% of the numerical calculation. This solution was verified experimentally [5,7].

$$U_{e,C} = \frac{\sigma_{YI}^2 r^3}{E_C} (0.03 \Pi_I + 0.11 \Phi + 0.50), \quad (1)$$

$$\Pi_I = \frac{(\alpha_M - \alpha_C) \Delta T E_I}{\sigma_{YI}}, \quad (2)$$

$$\Phi = 1 - \left( \frac{\alpha_M - \alpha_I}{\alpha_C - \alpha_I} \right)^m, \quad (3)$$

where  $\alpha_M$ ,  $\alpha_C$ ,  $\alpha_I$  are the CTE of the metal, the ceramic, and the interlayer, respectively.  $\sigma_{YI}$  is the yield stress of the interlayer and  $E_I$  and  $E_C$  are the elastic modulus of

\* Corresponding author. Tel.: +1 82 18 249 1656.

E-mail address: [jwpark@alum.mit.edu](mailto:jwpark@alum.mit.edu) (J.-W. Park).

<sup>1</sup> Present address: Central R&D Institute, Samsung Electro-Mechanics Co., Ltd, Suwon, Korea.

<sup>2</sup> Present address: Department of Metallurgical and Materials Engineering, Colorado School of Mines, CO, USA.

the interlayer and the ceramic, respectively.  $\Delta T$  is the temperature difference between the joining temperature and room temperature. In Eq. (3),  $m$  is 1 when  $\alpha_I$  is larger than  $(\alpha_M + \alpha_C)/2$  and  $m$  is  $-1$  for  $\alpha_I$  values smaller than  $(\alpha_M + \alpha_C)/2$  [5].  $r$  is the radial dimension.

The present work extends these analytical results to joint combinations with a smaller CTE mismatch between the ceramic and the metal, but a large CTE mismatch between the ductile interlayer and base materials. The physical meaning of  $\Phi$  in the joint systems has been highlighted in terms of its effect on plastic deformation in the interlayer.

## 2. Numerical analysis

Elastic strain energy has been calculated for various joint combinations in Table 1. To simplify the calculation as a two-dimensional problem, the specimen was modeled as a cylindrical, axisymmetric, rod-shaped specimen of 12 mm diameter. The height of each base material is 12 mm and the thickness of the interlayers is less than 2 mm [5]. Detailed descriptions of computational mesh can be found in our previous work [5,7].

Table 1  
Joint systems from Regime I in Fig. 1 and calculation results

Ceramic–interlayer–metal	$\alpha_M - \alpha_C$ ( $\times 10^{-6}$ , $^{\circ}\text{C}$ )	$\sigma_{YI}$ (MPa)	$\bar{\sigma}_{xx}$ $\sigma_{YI}$	$U_{e,C}$ (mJ)
AlN–Cu–Nb	2.8	75.8	0.71	1.32
SiC–Ti–Mo	2.7	172	0.55	5.76
SiC–Ti–W	2.1	172	0.65	4.03
Si <sub>3</sub> N <sub>4</sub> –Ni–Mo	2.1	148	0.76	4.19
Si <sub>3</sub> N <sub>4</sub> –Cu–Mo	2.1	75.8	0.82	0.76
Si <sub>3</sub> N <sub>4</sub> –Ni–W	1.5	148	0.83	2.53
Si <sub>3</sub> N <sub>4</sub> –Cu–W	1.5	75.8	0.86	0.52

The continuum models simulated cooling of a brazed specimen from the bonding temperature (800  $^{\circ}\text{C}$ ) to room temperature. Materials were assumed to be perfectly bonded at the interfaces. Uniform cooling and time-independent material properties were used. For the base metal and metallic interlayers, elastic–plastic responses were modeled. The temperature-dependency of the material properties was considered [8]. Numerical solutions were obtained using the ABAQUS computer program [9].

## 3. Results and discussion

Various joint combinations with a single interlayer can be classified into three categories by two previously determined dimensionless groups,  $\Pi_I$  and  $\Phi$  (Eqs. (2) and (3)), as shown in Fig. 1. Regime I is a group of joints that have been investigated in this study (Table 1). Our previous study [5] focused on the joint systems in Regime II. The interlayers of joint systems in Regime III have a CTE intermediate between the base materials, but a larger yield stress than the base metal. According to the FEM results, there is greater strain energy in the base materials in Regime III than when no interlayer is used. Therefore, Regime III has been excluded in this work.

$\Pi_I$  is the ratio of the thermal residual strain at the interface to the yield strain of the interlayer. The smaller  $\Pi_I$  is, the greater the portion of the interface which remains elastic. The second dimensionless group,  $\Phi$ , is the relative difference in CTEs between the three materials and quantifies the uniformity and symmetry of the residual stress distribution through the interlayer. As  $\Phi$  becomes closer to zero, the stress distribution in the interlayer has a smaller gradient and becomes more

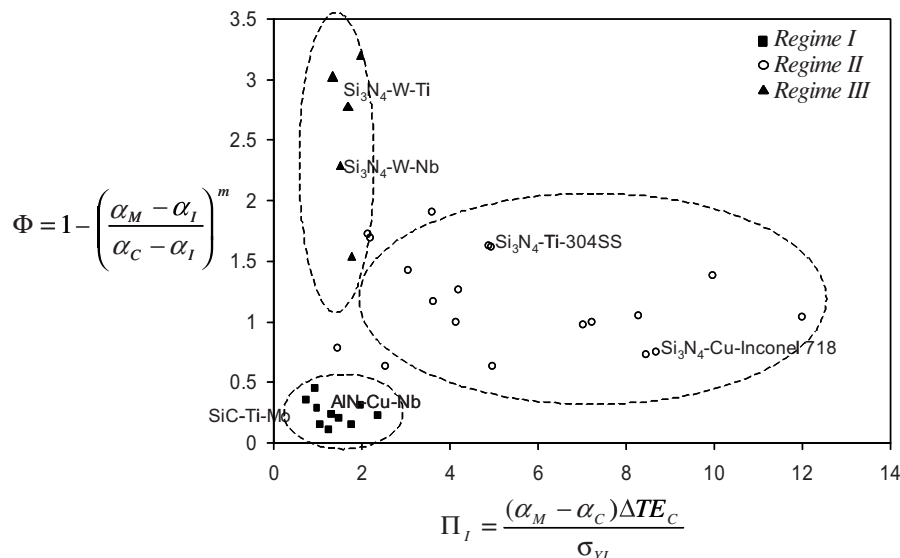


Fig. 1. Various joint systems (ceramic–interlayer–metal) classified by two dimensionless groups.

symmetric, which causes a larger volume of the interlayer to deform plastically and induces less strain energy in the ceramic [5].

The joints in *Regime II* have a relatively large CTE mismatch between the ceramic and the metal ( $\Pi_I \geq 3$ ) and less symmetric stress distributions ( $\Phi \geq 0.5$ ) in the interlayer. Eqs. (1)–(3) describe the residual strain energy in this type of joint [5]. The first term in Eq. (1) is the scaling factor that estimates the elastic strain energy in the ceramic ( $\hat{U}_{e,C}$ ) for the asymptotic case when the interlayer is entirely in the plastic regime. In this asymptotic case, the characteristic stress in the ceramic is given by the yield stress of the interlayer. Numerical simulations indicate that the volume of the ceramic that absorbs the strain energy is of the order of  $r^3$ . Two dimensionless groups capture the effect of partial plasticity and correct the deviation from the asymptotic case.  $\Pi_I$  and  $\Phi$  have much smaller coefficients than the constant term, which confirms that  $\Pi_I$  and  $\Phi$  are associated with secondary phenomena.

The joints in *Regime I* have a relatively small CTE mismatch between the ceramic and the metal ( $1 \leq \Pi_I \leq 3$ ) and a more symmetric stress distribution ( $\Phi \leq 0.5$ ). Calculation results of  $U_{e,C}$  are described in Table 1. With the same dimensionless groups, the following analytical solution has been obtained for the joint systems in *Regime I*:

$$U_{e,C} = \frac{\sigma_{YI}^2 r^3}{E_C} \cdot (0.06\Pi_I + 1.07\Phi - 0.1). \quad (4)$$

The coefficient of  $\Phi$  is much larger than the constant term. This indicates that  $\Phi$  is a part of the dominant phenomena, thus it should be included in the scaling factor. When including  $\Phi$  in the new scaling factor ( $\hat{U}_{e,C}$ ), the analytical solution is reformulated as follows:

$$\begin{aligned} U_{e,C} &= \frac{\sigma_{YI}^2 \Phi r^3}{E_C} f(\Pi_I) \\ &= \frac{r^3}{E_C} \left\{ \sigma_{YI} \sqrt{1 - \left( \frac{\alpha_M - \alpha_I}{\alpha_C - \alpha_I} \right)^2} \right\}^2 \cdot (0.26\Pi_I + 0.54). \end{aligned} \quad (5)$$

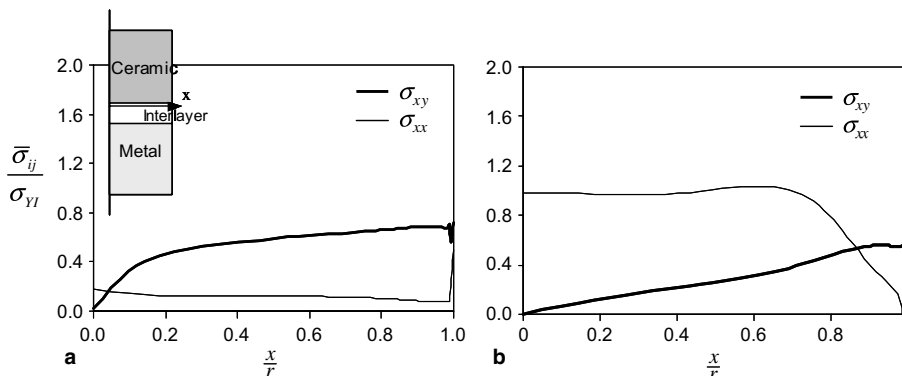


Fig. 2. Distributions of  $\sigma_{xy}$  and  $\sigma_{xx}$  along the  $x$ -direction near the interface in (a)  $\text{Si}_3\text{N}_4$ -Ni-Inconel 600 joint and (b)  $\text{AlN}$ -Cu-Nb joint.

The new analytical solution provides the value of strain energy in the ceramic within 1% of numerical calculations for joint combinations in *Regime I*. From the scaling factor, it is noted that the characteristic stress for the asymptotic case is smaller than the uniaxial yield stress of the interlayer.

The smaller characteristic stress for the joints in *Regime I* can be deduced from the von Mises yield criteria and stress distributions in the interlayer. In two-dimensional systems, the form of the von Mises yield criteria that is used in FEM calculations is the following:

$$\sigma_{YI} = \frac{1}{\sqrt{2}} \sqrt{(\sigma_{xx} - \sigma_{yy})^2 + \sigma_{xx}^2 + \sigma_{yy}^2 + 6\sigma_{xy}^2}. \quad (6)$$

The strain energy in the ceramic depends on the shear stress ( $\sigma_{xy}$ ) at the interface with an interlayer. In the analytical solution for *Regime II*, the magnitude of other stress components ( $\sigma_{xx}$ ,  $\sigma_{yy}$ ) near the interface is assumed to be negligible compared to  $\sigma_{xy}$ . Therefore, the asymptotic solution represents the case when the shear stress of the interface ( $\sigma_{xy}$ ) reaches  $\sigma_{YI}/\sqrt{3}$ . The schematic of stress distributions in the interlayer of joints in *Regime II* (Fig. 2(a)) supports the assumption that  $\sigma_{xy}$  is constant and is the largest stress component along the interface.

As  $\Phi$  becomes smaller, i.e., as the stress distribution in the interlayer becomes more symmetric,  $\sigma_{xy}$  is no longer the largest component of stress near the interface. As shown in Fig. 2(b),  $\sigma_{xx}$  increases rapidly and becomes largest near the interface of the joints in *Regime I*. The change in  $\sigma_{yy}$  with  $\Phi$  is negligible compared to the changes in the other two components. Therefore, from Eq. (6), the characteristic shear stress at the interface when yielding occurs,  $\bar{\sigma}_{xy,0}$ , will be as follows under a stress state similar to Fig. 2(b);

$$\begin{aligned} \bar{\sigma}_{xy,0} &= \frac{\sigma_{YI}}{\sqrt{3}} \cdot \sqrt{1 - \left( \frac{\bar{\sigma}_{xx}}{\sigma_{YI}} \right)^2 - \left( \frac{\bar{\sigma}_{yy}}{\sigma_{YI}} \right)^2 + \left( \frac{\bar{\sigma}_{xx}\bar{\sigma}_{yy}}{\sigma_{YI}^2} \right)} \\ &\approx \frac{\sigma_{YI}}{\sqrt{3}} \cdot \sqrt{1 - \left( \frac{\bar{\sigma}_{xx}}{\sigma_{YI}} \right)^2}, \end{aligned} \quad (7)$$

where  $\bar{\sigma}_{ij} = \frac{\sum_{k=1}^n \sigma_{ijk} V_k}{\sum_{k=1}^n V_k}$  ( $V_k$ : volume of each element  $k$ ).

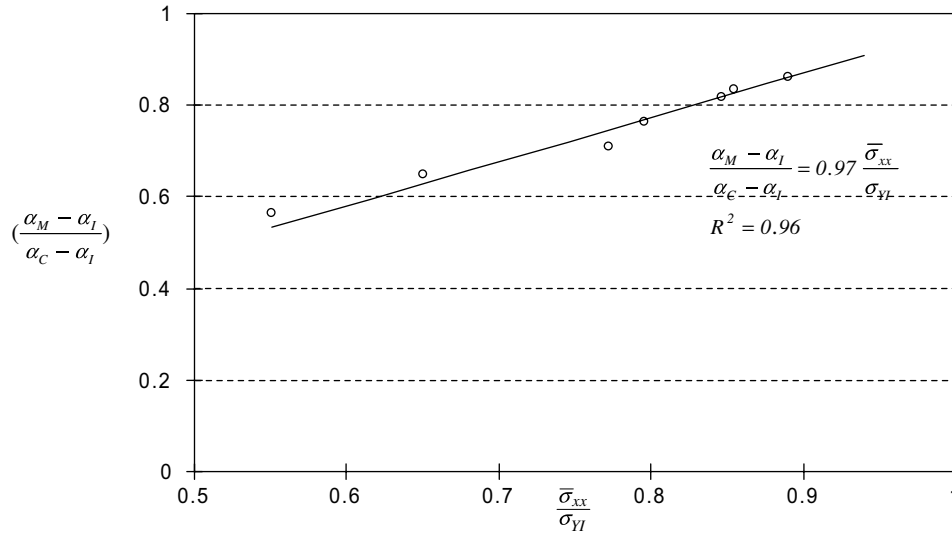


Fig. 3. Relative CTE differences between two base materials and the interlayer vs. normalized effective  $\sigma_{xx}$  near the interface.

As the characteristic stress for the asymptotic case of the joint depends on  $\bar{\sigma}_{xy,0}$ , the scaling factor in *Regime I* becomes

$$\hat{U}_{e,C} = \sigma_C \varepsilon_C V_C = \frac{\bar{\sigma}_{xy,0}^2 r^3}{E_C} = \frac{\sigma_{YI}^2}{3E_C} \left[ 1 - \left( \frac{\bar{\sigma}_{xx}}{\sigma_{YI}} \right)^2 \right] \cdot r^3. \quad (8)$$

According to FEM calculations, in regime I,  $\frac{\bar{\sigma}_{xx}}{\sigma_{YI}}$  is close to one (Table 1).

Therefore,  $\left( 1 - \left( \frac{\bar{\sigma}_{xx}}{\sigma_{YI}} \right)^2 \right) \approx 2 \left( 1 - \left( \frac{\bar{\sigma}_{xx}}{\sigma_{YI}} \right) \right)$ , and the Eq. (8) becomes

$$\begin{aligned} \hat{U}_{e,C} &= \frac{2\sigma_{YI}^2}{3E_C} \cdot \left[ 1 - \left( \frac{\bar{\sigma}_{xx}}{\sigma_{YI}} \right) \right] \cdot r^3 \\ &\approx \frac{r^3}{E_C} \left\{ \sigma_{YI} \sqrt{1 - \left( \frac{\bar{\sigma}_{xx}}{\sigma_{YI}} \right)} \right\}^2. \end{aligned} \quad (9)$$

Comparing Eq. (9) with (5), it is noted that the relative CTE difference between the three materials balances with the magnitude of  $\sigma_{xx}$  near the interface. Calculated  $\frac{\bar{\sigma}_{xx}}{\sigma_{YI}}$  is plotted with  $\left( \frac{\alpha_M - \alpha_I}{\alpha_C - \alpha_I} \right)$  in Fig. 3, which verifies this relationship. As  $\left( \frac{\alpha_M - \alpha_I}{\alpha_C - \alpha_I} \right)$  becomes closer to one, that is, as  $\Phi$  becomes closer to zero, the symmetric stress distribution promotes yielding at the interface at a smaller value than the yield stress of the interlayer. This induces less strain energy in the ceramic.

Eq. (5) considers the simultaneous effect of interlayer plasticity and CTE mismatch between the three materials. The equation shows that when the CTE mismatch of base materials is relatively small, the CTE of the interlayer is as important as its ductility in reducing the strain energy in the ceramic by inducing more effective plastic deformation in the interlayer. Previous analytical solutions [4,10] have been limited to elastically homogeneous

joint systems. These solutions make erroneous predictions when applied to real cases. For example, they predict that interlayers with an intermediate CTE between the ceramic and the metal will reduce residual stresses to the largest extent [3,4,10], which is not consistent with experiments [5,11].

#### 4. Conclusions

An analytical solution was formulated for joint systems with a relatively small CTE mismatch between the ceramic and the metal, but a large mismatch between the CTE of the ductile interlayer and the base materials. To the best of our knowledge, this equation is the first closed form analytical solution that considered plasticity in this particular type of ceramic-to-metal joint.

A smaller mismatch of CTE between the ceramic and the metal makes  $\Phi$  smaller for the same interlayers, which increases the normal stress in the radial direction within the interlayer. Due to the increased normal stress, the interlayer begins to yield at a lower stress near the interface, which induces less strain energy in the ceramic even though the CTE mismatch between the interlayer and the base materials has increased. The degree of increase in the normal stress depends on the relative CTE differences between the three materials, which makes the effect of  $\Phi$  more significant than in the joints having a larger CTE mismatch between the ceramic and the metal (*Regime II*).

#### References

- [1] Nicholas MG, editor. Joining of Ceramics. New York, NY: Chapman and Hall; 1990.

- [2] Williamson RL, Rabin BH, Byerly GE. *Compos Eng* 1995;5:851.
- [3] Kimura O. *J Am Ceram Soc* 1993;76:757.
- [4] Yu HY, Sanday SC, Rath BB. *J Am Ceram Soc* 1993;76:1661.
- [5] Park JW, Mendez PF, Eagar TW. *Acta Mater* 2002;50:883.
- [6] Mendez PF. Ph.D. Thesis, MIT, Cambridge, MA; 1999.
- [7] Park JW, Eagar TW. *Scripta Mater* 2004;50:555.
- [8] *Metals Handbook*, 2nd ed. ASM International, Materials Park, OH;1998.
- [9] ABAQUS, ver. 6.1-2, HKS; 2000.
- [10] Kimura O, Kawashima T. *J Appl Mech* 1989;56:707.
- [11] Xian AP, Si ZY. *J Am Ceram Soc* 1990;73:3462.