Scaling Laws in Welding Modeling

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Abstract

There are few simple formulas to predict the results of a welding process. Such formulas would be of enormous help in the design of welding processes, and they are ubiquitous in other engineering disciplines such as strength of materials, fluid mechanics, and heat transfer, typically in the form of scaling laws. Scaling laws provide accurate approximations and display clearly the trends in a problem. In this paper I review the reasons why scaling laws are so scarce in welding, highlight previous and current efforts to develop scaling laws for welding, and present two complimentary approaches of great potential to develop scaling laws specific for welding.

Introduction

The design of a new welding process involves so many parameters that it can seldom be predicted reliably; therefore, extensive experimentation must take place in order to determine an ideal process set-up. A set of simple and intuitive design laws based only on the most relevant parameters would be of enormous help in this case. Scaling laws are particularly well suited for this purpose.

Scaling laws appear in several disciplines such as physics, biology[1, 2], geophysical [3, 4], Internet traffic[5], and even economic systems[6]. A broad sample of problems that can be described with such scaling laws is presented in [7]. Scaling laws are ubiquitous in engineering; some of the reasons for this are: 1) the combination of units has the form of a power law, 2) the expressions of many physical phenomena have the form of power laws as noted above, and 3) many empirical regressions of engineering data in log-log plots tend to give a straight line, which corresponds to a power law.

In engineering in particular, scaling laws constitute the backbone of handbooks, together with the tables of values for the parameters involved. For example, the maximum deflection of a cantilever beam is universally presented as a simple power law involving the properties of the cross section of the beam and the modulus of the beam material. This simple formula is accompanied by tables of properties of cross sections and modulus of different materials. This power law is valid for steel beams of a bridge, wooden beams of a house, or silicon beams in MEMS.

This "handbook formulas" approach would be very useful for welding, but there are very few universally accepted such formulas in this field. Very relevant welding characteristics such as penetration, arc size, current in GMAW, voltage in GTAW lack such a general expression. For welders in the field, who must have an approximate estimation of the right welding parameters before they weld, there are rough aids such as the "Miller Calculators" [8], which provide ballpark values for some particular welding activities. Sophisticated software is often difficult to operate, time consuming, and not widely available to welders or welding engineers.

Scaling laws are of enormous utility during the early stages of welding design, when the configuration of a system is still uncertain. In this case, scaling laws could provide quick estimations of the feasibility of a proposed process, help determine costs, and contribute to decisions about configuration and materials. Scaling laws can be calculated in negligible time; therefore, they can also be useful for robotic welding systems predicting the behavior of a system in real time, much faster than computationally intensive models such as finite element analysis, or computational fluid mechanics.

When experimental databases or numerical models exist, scaling laws can be used to generalize and extrapolate the results obtained. For existing processes, scaling laws are useful for set-up and tuning operations and to compare different welding alternatives.

When modeling a welding process, it is convenient to divide it into submodels such as the weld pool, arc, electrode, etc. Power laws are useful to link the submodels together. For example weld pool depression depends on arc pressure. Power law solutions for the arc can relate the arc pressure to the controllable parameters of the arc, thus embedding the arc into the weld pool model. Power laws do not present convergence problems of the type of numerical solutions.

Scaling Laws, Dimensional Analysis, and Similarity

Scaling laws give us an accurate estimation a magnitude (e.g. penetration) as a function of the welding parameters. Welding parameters are not only the obvious current, voltage, wire feed speed, etc. They also include thermophysical properties of materials involved such as heat conductivity, density, and heat capacity for heat transfer in the solid, and the same plus viscosity and surface tension for heat transfer in the weld pool. Even more parameters are necessary to include a characterization of the arc, electrode, laser beam or electron beam. Some of these parameters can vary within the domain of the problem, for example the surface tension depends on temperature, which varies spatially.

Scaling laws have the functional form of a power law of the relevant problem parameters such as the following proposed scaling law for penetration in SMAW [9]

$P = G\ I^{4/3}V^{-1/3}E^{-2/3}$..(1)

Where G is a constant, I is the current, V is the travel speed, and E is the voltage. We observe in this power law that the parameters are raised to a constant power. Expressions where a constant is raised to a variable power are not power laws in this context; thus, if a is a constant, and P, is a parameter, P^a is a power law, but a^P is not. As power laws are aimed to compare different welding processes, they are based on the parameters, and not on the problem variables (space, time). Thus if L is a length in the x direction, L^a is a useful power law to compare welding alternatives, while x^a is not.

Since power laws do not depend on the space and time coordinates of the problem, a representative value must be chosen for those parameters that vary with the coordinates. For example, a representative value of the density of the molten metal is chosen for scaling laws of the weld-pool. Occasionally, spatial variations are relevant and they generate driving forces, for example variations in density cause buoyancy forces, and variations in surface tension cause Marangoni forces. In these cases, the varying magnitude can be expressed as two parameters, one capturing a characteristic value, and another capturing the variation. For density we thus have the Boussinesq approximation: $\rho=\rho_0+\beta(T-T_0)$, where β captures the variation of density with temperature. For Marangoni flows we use $\sigma=\sigma_0+\sigma_T(T-T0)$.

Power laws are a natural consequence of dimensional homogeneity[10], which states that all terms in an equation must have the same dimensions, and dimensions can only be formed as power laws of basic units such as m, kg, s. Power laws often appear naturally as the exact solution of asymptotic cases. As a welding example, temperature distribution in Rosenthal's solution[11], is expressed as a power law in units of temperature, multiplied by a dimensionless function:

$T-T_0 = q/(4\pi kR)\exp[-\lambda\nu(x+R)]$...(2)

The fundamental nature of power laws is also evident in the engineering wisdom that "everything is a straight line when plotted in a log-log graph." In mathematical terms, the straight line y'=ax'+b in a log-log plot corresponds to the power law $y=10^b x^a$, where y'=log y, and x'=log x.

Power laws are indeed very powerful. They clearly indicate trends and can yield accurate predictions over several orders of magnitude. They also convey much intuitive meaning: the sensitivity of a power law to a given welding parameter is directly proportional to the exponent of the parameter. Using Equation (1) as an example, we see that penetration will increase approximately 13% if we increase the current by 10%.

Dimensionless Groups

Dimensionless groups are a particular type of power law. These groups have no units; therefore, their value is independent of the unit system used, conferring them generality. Well known dimensionless groups include the Reynolds number and the Peclet number.

Dimensionless groups are at the core of the technique of Dimensional Analysis, in which a problem is represented in a simpler way without any loss of generality. A cornerstone of Dimensional Analysis is Buckingham's Π theorem[12], which roughly states that a problem involving n parameters and k units can be represented in a simpler form by m dimensionless groups, where m=n-k. The technique of Dimensional Analysis can greatly simplify a problem, and has been of substantial help to the fields of Aerodynamics and Fluid Mechanics.

A fundamental requirement for proper dimensional analysis is that all relevant parameters must be included. For simpler problems this is not too difficult; for example, in Aerodynamics all geometrically similar problems can be completely described by the properties of the fluid (density ρ and viscosity μ), a characteristic length L, and a velocity V. Such a complete description is practically impossible in welding because of the large number of parameters required. The obvious current, voltage, wire feed speed, etc., are only a small fraction of all the parameters needed. In fact, relevant parameters have been overlooked at times; for example, before the early 1980's[13], the effect of surface tension variation on Marangoni forces had not been considered yet[14], and welding engineers could not explain large penetration variations in apparently identical welding conditions.

The large amount of parameters to be considered makes welding modeling especially challenging. While not all parameters are simultaneously critical, we often do not know which ones to neglect before modeling. And models considering all imaginable parameters can become intractable.

Dimensional Analysis is less powerful for problems with several parameters such as welding; while the number of parameters easily exceeds 10, the number of units is fixed to approximately 5. Buckingham's theorem indicates that the problem still requires 5 or more dimensionless groups. In comparison, in Aerodynamics, the four parameters mentioned above (ρ, μ, L, V) involve three units (m, kg, s), and a single dimensionless parameter (typically the Reynolds number) is enough to characterize all problems. In this case, tabulation and understanding of the problem are relatively simple, while the same is virtually impossible for welding.

Complete and Incomplete Similarity

When two problems can be described using the same set of dimensionless groups, the two problems will be considered “similar” when the corresponding groups have the same value for both problems. For similar problems, the behavior of one can be deduced from the behavior of the other through the use of scaling laws. For example, a model airplane in a wind tunnel and a real airplane in flight can be considered similar if both have the same Reynolds number, and measurements made in the wind tunnel can translate directly to the actual plane with the use of the appropriate scaling laws.

When only some dimensionless groups have the same value on two systems, these models have “incomplete similarity.” Incomplete similarity is especially important in welding, since it is not possible to control all the dimensionless groups associated with the large number of parameters. If the dimensionless groups with the same value capture the dominant parameters, and the groups with different values represent secondary phenomena, we can still obtain useful and representative results. Using weld pool flows as an example, buoyancy is a secondary force when Marangoni forces are dominant. In this case, a model and experiment with a different Grashoff number (which captures buoyancy) will still show essentially the same behavior.

The choice of which dimensionless groups to preserve and which ones can be discarded is critical, and far from trivial. Dimensional Analysis is useful to generate dimensionless groups, but does not provide guidelines for which ones to keep. In addition, there are several sets of dimensionless groups that can represent a given problem; thus, there is the additional challenge of selecting the best set of dimensionless groups that would provide for the minimum influence of the discarded groups.

The problem of selecting the most representative dimensionless groups in partially similar problems has become increasingly relevant since the introduction of powerful personal computers in the 1970’s. At that time there was a surge of mathematical models of increasing complexity, as illustrated in Figure 1.

Figure 1 uses the modeling of a welding arc as an example. In it, we appreciate the significant jump in the number of dimensionless groups involved in the models after 1970. Similar trends are also evident for other aspects of welding modeling such as the weld pool.

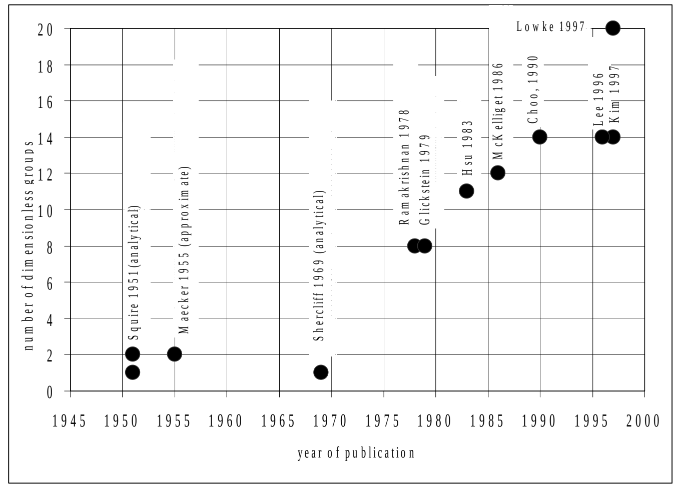

Figure 1: Evolution of arc modeling complexity [15-26]

I propose that the reason why there are relatively few general scaling laws for welding is because useful models involve an excessive number of dimensionless groups, and there are no established techniques that can help the engineer to form and select the most useful dimensionless groups on which to establish similarity between model and reality, and upon which to develop scaling laws. Especial tools are necessary to accomplish this, and will be reviewed in a later section. In the next section we will discuss past and current applications of scaling laws to welding.

Scaling and Dimensional Analysis in Welding

Possibly the oldest scaling law developed for welding is the estimation of penetration presented in Equation (1) and its successors[27]. This law was developed empirically for manual SMAW of ½” steel plate. We see that the constant G needs units for this equation to be homogeneous, thus it captures some physics such as the thermophysical properties of the material being welded, not considered by the explicit parameters. This formula is seldom seen today, possibly because of its lack of generality.

The next improvement for scaling laws in welding was the scaling of analytical solutions for heat transfer in the solid, neglecting the effects of the fluid flow in the weld pool.

Scaling of analytical solutions for heat transfer in the solid

Rosenthal’s analytic solution for a point heat source in a semi-infinite solid[11] was generalized with great success by Christensen, and it is still widely used today. This problem is formulated in terms of the thermophysical properties of the solid: heat conductivity k and heat diffusivity α, process parameters: heat input q and traveling velocity V, and a characteristic temperature jump ΔT. These parameters involve the units m, kg, s, K, and thus the whole formulation of this problem could be characterized by a single dimensionless group. This is actually the case, and Christensen used the "operating parameter" n

$$n = qV/(4\pi k\alpha\Delta T) \quad (3)$$

All unknowns of this problem can be obtained from scaling laws involving only the four parameters listed above and a dimensionless function of the operating parameter.

This solution is restricted to thick plates and temperatures significantly lower than the melting temperature of the base metal, thus it does not provide accurate information about weld pool shape and HAZ thermal history. One possible reason for the success of Christensen's generalization despite its limitations is that it is derived from first principles; therefore, its generality is assured as long as the starting hypotheses are valid.

There is a family of solutions following a similar approach to Rosenthal and Christensen for thin plates, line heat sources and more. Comprehensive lists of solutions can be found in the compilations by Grong [28, 29]. These last references also list solutions for point heat sources on plates of intermediate thickness. In this case, the plate thickness becomes an additional parameter, and the problem needs two dimensionless parameters to be completely described.

There are also solutions for multiple point heat sources, and for distributed heat sources. Eagar and Tsai presented solutions for a circular gaussian heat source on a thick plate.[30, 31] In this case, in addition to the operating parameter n, there is an additional dimensionless number u, the "distribution parameter." Further generalization to a gaussian heat source on a plate of intermediate thickness were presented by Manca *et al.[32]*. As expected, their solutions depend on three dimensionless groups.

Similarly to the earliest work on arc illustrated in Figure 1, these analytical solutions trace their roots to the times in which computers were not readily available. The welding problems were then just as complex as today, but their physics needed to be simplified until a tractable problem was obtained.

Scaling laws and dimensional analysis of the welding arc

For the arc, two of the earliest scaling laws are those for pressure and velocity by Maecker[21]. These laws involve two dimensionless groups at the most, and they have little influence on the resulting scaling laws. Thorough dimensional analysis of an electric arc was performed by Yas'ko and Shaskov[33, 34]. In their work they identified seven or more dimensionless groups, proposed no particular solution employing them, and indicated which groups were more relevant for similarity of 10 different types of arc. Scaling laws for pressure, velocity and temperatures in a laminar welding arc are presented in[35, 36]. Most current work on arc modeling will include a standard set of dimensionless groups such as the Reynolds number, Peclet number, and magnetic Reynolds number.

The increase in the number of dimensionless groups observed in Figure 1 is due to the addition of thermal effects of the arc and variations in geometry of the electrode or weld pool surface.

Dimensional analysis of the whole welding process

There have been attempts to capture the whole welding process with dimensionless groups. All of these efforts are based on heat transfer in the solid, and are aimed at generalizing numerical solutions or experiments. Krivosheya[37] presented an analysis of SAW butt joints based on seven parameters, thus obtaining three dimensionless groups. Kou[38, 39] developed a dimensionless formulation that considers the Stefan problem of heat of melting. A relevant aspect of his normalizations is the use of the same characteristic length for all three spatial directions. Fuerschbach proposed a dimensionless parameter model for arc welding[40], and Fuerschbach and Knorovsky proposed a single dimensionless group to characterize the melting efficiency of melting in PAW and GTAW[41]. A scaling analysis for the butt welding of thermoplastics is developed in[42].

Scaling laws and dimensional analysis of the weld pool

In the early 1980's, a simultaneous development of weld pool understanding and the availability of computer power prompted the introduction of fluid flow and heat transfer in the models of the weld pool. Dimensional Analysis is a standard tool of fluid mechanics, and soon models incorporated the traditional Peclet number, Reynolds number, and Marangoni number.

Wang and Kou[43] extended Kou's dimensionless formulation of the welding problem to include convection in the weld pool. Landmark studies of convection in the weld pool during spot welding were developed by Oreper, Szekely, *et al.*[44-47]. In them, the governing equations are normalized, generating the Reynolds, Prandtl, Grashoff, Stefan, Rayleigh, surface tension, and Marangoni numbers. An analysis of the relevance of the driving forces is performed using these numbers, and scaling laws are proposed for the characteristic velocity of the weld pool when the driving forces are Marangoni, electromagnetic, or buoyancy. In [48], Szekely also proposes scaling laws for time constants in melting and solidification during the transient regime of a spot weld pool. This approach of determining the dominant forces using known dimensionless groups, then using the appropriate scaling laws is being used today by DebRoy *et al*. for weld pools in GMAW[49], and spot[50] and traveling GTAW[51]. A well explained scaling analysis of fluid flow is presented by Rivas and Ostrach[52]. In this work they considered three different regimes of thermocapillary driven flows: I) when there is no surface boundary layer, II) when there is a viscous surface boundary layer, and III) when there is a thermal surface boundary layer.

Another systematic scaling of fuid flow in the weld pool is presented by[53]. These last two references differ from the analysis by Kou in that they use intrinsic scales for lengths in different directions. This enables the estimation of differential expressions without the need to solve the corresponding differential equations. For weld pools under high currents, dimensional analysis and scaling laws are presented in[54].

Dimensional analysis of LBW

For weld penetration in LBW, early correlations based on three dimensionless groups were presented by Lubin[55]. An empirical study of energy transfer efficiency based on dimensionless groups has been presented by Fuerschbach[56], and dimensionless maps for laser processing of materials based on dimensionless groups have been developed by Ion, Shercliff and Ashby[57]. An extension to LBW of the GTAW weld pool scaling laws and dimensional analysis is presented in [58, 59].

Dimensional analysis of GMAW

Several correlations for metal transfer in GMAW based on dimensionless groups are presented by Murray in[60]. In[61], he presents dimensionless correlations for penetration, and in [62] there is a dimensionless analysis of droplet detachment. An scaling analysis for flows in the weld pool during fillet welding is presented in [49], and a thorough dimensional analysis of weld pool phenomena involving eight parameters is presented in[63].

Tools for Generation of Scaling Laws

Welding modeling has a difficulty that other disciplines do not have: the number and parameters and dimensionless groups is relatively large. Partial similarity is key to simplifying the models and generating useful scaling laws, and to accomplish it effectively, two tasks must be completed; first, an appropriate set of dimensionless groups must be generated; second, these dimensionless groups must be ranked by relevance.

Dimensional Analysis provides no guidance to accomplish these tasks. Applied mathematics tools such as dominant balance[64] are limited to single equations, and are not effective for the systems of equations often found in welding modeling.

Most scaling laws for welding were generated either from exact solutions of simplified problems, or from intuitive normalization of governing equations. The intuitive approach is not necessarily systematic, and requires a great deal of experience to determine the best normalization scheme. A systematic approach would involve trying all possible normalization schemes, and checking for consistency of the assumptions made. Such an approach was followed by Rivas and Ostrach for a simplified weld pool with no electromagnetic or buoyancy forces[52]. This approach has been generalized using linear algebra, and improved normalization of the differential expressions. It is called Order of Magnitude Scaling, and it is described in[65-70]. A similar approach to modeling was developed by Yip[71] using concepts of Artificial Intelligence.

Another alternative for generating scaling laws is to analyze sensitivity data from experiments or numerical simulations. In this case scaling laws can be generated by minimizing fitting error. The field of artificial intelligence has been active in this task, although the models generated are difficult to extend to welding. An algorithm called SLAW[72, 73] and associated software were developed to generate scaling laws and dimensionless groups ranked by relevance. This algorithm correctly reproduced the scaling laws previously developed for ceramic to metal joining[74].

Summary

Summarizing, scaling laws are desirable in welding. They would help integrate models, speed-up weld process design, and would facilitate costing estimations. However, scaling laws are much less developed in the welding field than in other disciplines such as fluid mechanics. The reason for this disparity is that welding involves a much larger number of parameters than other disciplines. It is difficult to decide before modeling which parameters to neglect, and it is difficult to model without neglecting some parameters. Traditional tools such as Dimensional Analysis or dominant balance are not powerful enough for the complexity of welding. Current scaling laws were generated by solving analytically very simplified systems, or by inspectional analysis of the normalized governing equations. Two methods are proposed to systematically generate scaling laws and associated dimensionless groups. The first, Order of Magnitude Scaling, is proposed for analyzing systems of coupled differential equations. The second, SLAW is proposed to analyze experimental or numerical sensitivity studies.

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