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Modeling of Welding Processes through Order of Magnitude Scaling

Patricio F. Mendez, Thomas W. Eagar

Abstract

A new technique for mathematical modeling has been developed: Order of Magnitude Scaling (OMS). It combines elements of dimensional analysis and asymptotic considerations in order to provide useful insight into problems in which many driving forces act simultaneously. It is especially useful for the generalization of numerical results, because it permits one to reduce all numerical results within a range to a single dimensionless map. This technique is also useful to generalize results from physical experiments, and to obtain order of magnitude estimations of the unknowns of a problem before attempting to solve the governing equations. This technique has proved very valuable in the study of welding, where the geometries are relatively simple, but the physics are very complex due to the simultaneous and coupled interaction of many driving forces. Application of OMS to weld pool dynamics shows quantitatively for the first time that the aerodynamic drag of the arc is the driving force for flow in the weld pool in the high current regime (above 250 amperes). Our ongoing research of the welding arc using OMS combined with numerical analysis has generated universal maps for the flow of plasma in the region near the cathode.

Introduction

Mathematical models are extremely valuable for the understanding of materials processing. There are two main reasons for this: 1) They bring insight into the physics of the process and, 2) They enable one to explore new processes or process modifications in an inexpensive way before a significant economic effort is committed into experiments, prototypes or pilot plants.

There are many different kinds of mathematical models [1, 2, 3]; however, they all share a common characteristic: that a desired aspect of a process can be predicted through mathematical expressions.

Frequently the mathematical models of materials processes are based on equations of general validity that describe the physics and chemistry of the process. The first step of modeling in this case is the selection of what phenomena are going to be taken into account and what will be disregarded. This choice determines the range of validity of the model. The equations corresponding to the selected physics will be called here "governing equations", and they might be, for example, the Navier-Stokes equations, Maxwell equations, Fourier equation, Fick's law or any other mathematical expression of a law of nature. Success in the selection of what phenomena to include or disregard can only be tested through experiments or by contrasting the results with more general models.

Once a decision is made about the relevant physics of the system and the corresponding mathematical expression (almost always in the form of differential equations), the second step of modeling consists of solving these equations. This can be done by closed form expressions, mathematical approximations, numerical methods, or with the help of physical models.

The rapid progress in computing power has enabled researchers to apply numerical methods to problems of increasing difficulty, in both geometry and underlying physics. Many problems are still beyond the reach of current computers; however, all trends suggest that it is only a matter of time before this obstacle is overcome. Progress in software is not as fast as that in hardware, and mathematical problems (e.g. convergence) which are unrelated to computing power are still frequently an obstacle. The most complex multicoupled problems, such as modeling of heat and fluid flow in systems with free surfaces (e.g. the weld pool in arc welding), fall into this category. Due to limitations in the solution of the equations, it frequently happens that the physics considered must be simplified in order to eliminate particularly troubling equations. In the case of welding, a common simplification of the physics for some time consisted of neglecting the free surface deformation of the weld pool [4, 5, 6]. Even today, only relatively small free surface deformations can be included in numerical calculations of arc welding [7, 8].

The ability of modeling very complex systems in a computer also creates new challenges: the vast amount of information obtained must be interpreted and generalized in order to be used by other engineers and researchers. In a complex system, many driving forces and resistances act simultaneously, and a large set of parameters is necessary to characterize the system (geometric parameters, thermophysical properties, mechanical properties, etc.). Tools such as dimensional analysis and theory of similarity are of limited help in these cases, because a large number of dimensionless groups are generated, and currently no tool exists that can determine the most meaningful choice for dimensionless groups and their relative importance. Without a proper choice of dimensionless groups, it is not possible to determine in a general way the different regimes that characterize the behavior of a process.

The creation of an Order of Magnitude Scaling technique was motivated by the need for a tool that can help to interpret and generalize the information provided by numerical calculations or experiments. This technique is aimed at systems with simple geometry but many driving forces and resistances acting simultaneously, as indicated in Figure 1. The axes in this figure represent sets of dimensionless groups obtained through dimensional analysis of the governing equations. The total number of independent dimensionless groups (m) is determined using Buckingham's theorem. This set of dimensionless groups can be divided into two subsets: one subset, Π_g, is made up by the maximum number of independent dimensionless groups that relate only to the geometry of the problem (m_g groups, represented as the vertical axis of Figure 1). The other subset, Π_p, involves groups related to the physics of the system, and consists of the remainder of the dimensionless groups ($m_p = m - m_g$ groups, represented as the horizontal axis of Figure 1) tools. The horizontal line indicates the limit for the generalization of complex geometries of the new technique introduced here (OMS). The region on the lower left contains problems with simple geometries and simple physics. These problems are the easiest to generalize. This can be done by induction from the analysis of particular cases through numerical models or experiments. Generalization in this case can often be obtained also by deduction, through closed form solutions of the governing equations. The region on the upper left contains problems with relatively simple physics, but complex geometries. If geometric similarity is preserved, (i.e. the many parameters of subset Π_g are held constant) the physics of the problem can be generalized by induction from the analysis of particular cases through numerical models or experiments. The generalization of the problem for different geometries is much more difficult. Because there are many geometric parameters, exploration of the space they describe through particular cases becomes prohibitive. If 10 points are used to characterize the effect of each geometric dimensionless group, the number of particular cases to be studied is of the order of 10^{mg}. This type of analysis becomes impractical very soon for even a few geometric parameters. In some cases closed form solutions are possible; for example when solving the Laplace equation in a polygonal domain by conformal mapping [9]. These cases, however, are very uncommon.

The region at the bottom right of Figure 1 represents the new generalizations that are possible by using OMS. The systems in this region have a relatively simple geometry and many driving forces and resistances acting simultaneously. The different possible balances between dominant and balancing forces determine distinct regimes into which different simplifications can be applied. The region at the top right represents systems with complex geometries and physics. The vast majority of these problems have solutions that are very difficult to generalize by induction (due to the large number of parameters involved) or by deduction, because of difficult geometry. The generalization of the results for these problems constitutes a challenge for future research in modeling of materials processes.

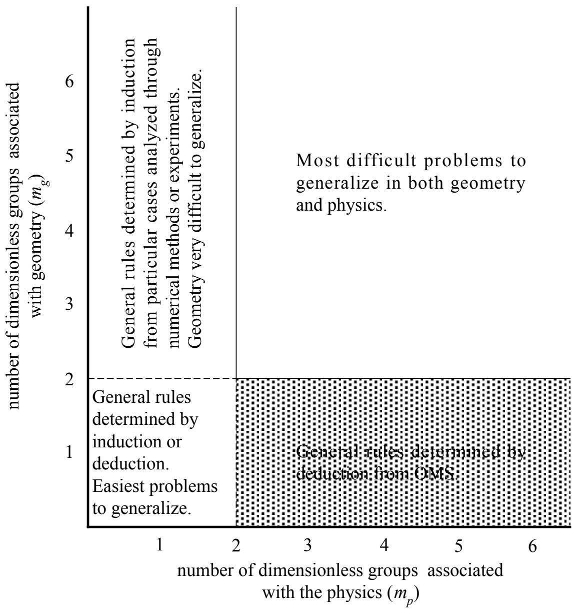

Figure 1: Degrees of difficulty for generalizing models of materials processes measured as the number of dimensionless groups necessary to describe a system. The horizontal and vertical axes indicate difficulty for generalizing the physics and geometry of the system respectively

Figure 2 shows the location of particular problems in the frame of Figure 1. Point **A** corresponds to fluid flow through a cylinder [10, 11]. In this case there is at most one dimensionless group associated only with the geometry (for example the ratio of length to diameter). The remaining dimensionless group could be arbitrarily defined as the Reynolds number. Point **B** represents a viscous boundary layer problem [10, 11, 12, 13]. There is only one dimensionless group associated with this problem; this group can be chosen to be the Reynolds number again. Point **C** corresponds to a thermal boundary layer problem [10, 11, 13]. In this case, in addition to the Reynolds number, the Prandtl number is considered. Point **D** represents the model for thermocapillary flows by Rivas and Ostrach [13, 14]. In this case, the dimensionless groups considered are the ratio of width to depth for the geometry, and the Reynolds and Prandtl numbers for the physics. Point **E** represents the modeling of the cathode region of a transferred plasma arc [15], which will be discussed later in this paper. The geometric parameters are the ratio of arc length to cathode spot radius and ratio of anode spot radius to cathode spot radius. A dimensionless group with physical meaning similar to the Reynolds number is chosen to represent the physics of the system. Point **F** represents the model of a weld pool under high currents and velocities [13, 16, 17], which will also be discussed later in this paper. The OMS technique is especially appropriate for this system, because its geometry is relatively simple (characterized by the ratio of welding penetration to heat source size), but the physics are very complex involving many driving forces and resistances. The dimensionless groups associated with the physics of this system are the ratio of viscous forces and other forces: inertial, arc pressure, electromagnetic, hydrostatic, capillary and buoyancy. The ratio of convection to conduction and Marangoni forces to aerodynamic drag from the arc complete the set Π_p for this system. Point **G** represents the analysis of incompressible fluid flow in complex geometries, for example the analysis of an airfoil in the subsonic, laminar regime. A characteristic Reynolds number is the only parameter that contains the physics of the problem, while a large number of groups are necessary to describe the geometry. Point **H** represents a problem such as integral model of the arc welding process, including deformation of the free surface. In this case, many dimensionless groups are necessary to describe the geometry and the physics of the problem.

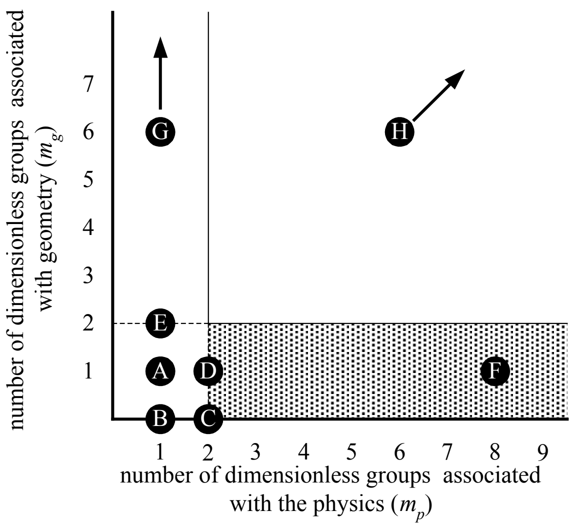

Figure 2: Degrees of difficulty for the generalization of selected systems. **A**: fluid flow through a cylinder. **B**: viscous boundary layer. **C**: thermal boundary layer. **D**: thermocapillary flows. **E**: cathode region of a transferred plasma arc. **F**: weld pool under high currents an velocities. **G**: incompressible fluid flow in complex geometries. **H**: integral model of the arc welding process.

Order of Magnitude Scaling

The OMS technique provides order of magnitude estimations of the characteristic values of the descriptive functions. These functions are the solution of the governing equations chosen to model a system. The set $\{P\}$ contains the parameters of the model. When the governing equations are normalized, each of them has the form of a sum of terms, with each term being the multiplication of a dimensionless coefficient C_i by a dimensionless function g_i. Dimensional analysis of the set $\{P\}$ determines that it is possible to make the normalization in such a way that the dimensionless coefficient is a function only of the set of governing dimensionless groups $\{\Pi\}$, thus $C_i = C_\mathrm{i}(\Pi)$, and the dimensionless function is a function only of the set of normalized independent arguments $\{\mathbf{x}\}$ and the set of governing dimensionless groups $\{\Pi\}$, thus $g_i = g_i(\mathbf{x}, \Pi)$. Each dimensionless coefficient C_i has a different dependence on the elements of $\{\Pi\}$; for this reason some terms in the dimensional equation might be grouped together and there is not always a one to one correspondence between the terms of the original governing equations and its normalized counterpart. A normalized equation will have the following form:

$$\sum C_i(\mathbf{\Pi})g_i(\mathbf{x},\mathbf{\Pi}) = 0 \tag{1}$$

Normalization of the governing equations can be carried out by normalizing first the descriptive functions and its arguments, and then by normalizing each equation by the dimensional coefficient of one of its terms (this term will then have a coefficient equal to 1 in the normalized equation). A convenient normalization of the independent arguments and the descriptive functions is the following:

$$x_i = \frac{X_i - A_i}{B_i - A_i} \tag{2}$$

$$f_j(\mathbf{x}) = \pm\frac{F_j(\mathbf{X}) - F_j(\mathbf{A})}{|F_j(\mathbf{B})| - |F_j(\mathbf{A})|} \tag{3}$$

where **A** and **B** are the points in the domain where the function F_j reaches its minimum and maximum absolute value respectively. The magnitude $|F_j(\mathbf{B})|$ is the characteristic value of F_j , and $|F_j(\mathbf{B})|$-$|F_j(\mathbf{A})|$ is the characteristic value of the variation of F_j. The normalization proposed above has the property that for many practical situations the dimensionless functions g_I are of the order of magnitude of one, i.e. their maximum value has an upper and lower bound close to one (within an order of magnitude). Not all of the characteristic values are known a priori. Those that are not known will be used as redundant parameters for dimensional analysis. This set of redundant parameters, $\{S\}$, contains extra degrees of freedom. These degrees of freedom allow the arbitrary assignment of values to as many independent dimensionless groups as redundant parameters were generated. If this arbitrary assignment of values is done with the appropriate criteria, an estimation of the unknown characteristic values can be obtained.

It is also possible to chose a particular set of governing dimensionless groups such that when all of these groups tend to zero, the normalized equation can be reduced to basically three terms: a dominant term (containing f_d), a balancing term (containing f_b) and a term of secondary importance (containing f_s). The term of secondary importance is the sum of all the terms of the equation that tend to zero. The dominant term is the one with a coefficient of one, and the balancing term contains the unknown characteristic values.

$$f_d(\mathbf{x},\mathbf{\Pi}) + C_b(\mathbf{P},\mathbf{S}) f_b(\mathbf{x},\mathbf{\Pi}) + C_s(\mathbf{\Pi}) f_s(\mathbf{x},\mathbf{\Pi}) = 0 \quad (4)$$

When the governing equations are properly normalized, the functions f_d and f_b are of the order of magnitude of one and the secondary term tends to zero. If C_b contains unknown characteristic values (elements of $\{S\}$), it can be estimated as having the value of one. This estimation generates a linear equation that will permit the estimation of one unknown characteristic value. The proper normalization is found through an iterative process, in which different terms in the equations are assumed dominant and balancing. When a coefficient in any equation is larger than one, that choice of dominant and balancing terms is incorrect, and another iteration needs to be made.

Application to Welding at High Current

The analysis of the weld pool at high current and velocity [13, 16, 17] is well suited for the OMS technique, because it has a relatively simple geometry and many forces acting on it. Figure 3 shows a photograph and a schematic of the weld pool at high current and velocity. It can be seen that the molten metal turns into a thin film under the electrode.

The driving forces acting on this system are gas shear on the free surface, arc pressure, electromagnetic forces, hydrostatic pressure, capillary forces, Marangoni forces, and buoyancy forces. Inertial and viscous forces in the molten metal oppose to these forces. The governing equations for this system include the Navier-Stokes equations, the equations of conservation of mass and energy, Maxwell's equations, and boundary conditions considering Marangoni shear stresses and capillary effects. The unknown functions that describe the problem are the velocities in the X and Y components, $U(X;Y)$ and $V(X; Y)$ respectively, the pressure $P(X; Y)$, temperature $T(X; Y)$, current density in the X and Y directions $J_X(X; Y)$ and $J_Y(X; Y)$ respectively, induction $B(X;Y)$, electric potential $\Phi(X; Y)$, and position of the free surface $\delta(X)$.

The OMS technique used herein determined quantitatively for the first time that the dominant driving force in a thin weld pool is the aerodynamic shear of the arc. The balancing resistance is the viscous forces. Most secondary dimensionless groups are very small for a typical case, as indicated in Figure 4, the only exception are the inertial forces which are small but not negligible. A simplified model could safely neglect all secondary effects but this, which is represented by only one dimensionless group. Estimations of the characteristic values for each of these functions are indicated in Figure 5 where L is indicated in Figure 3; ρ, k, σ_e, ν are the density, thermal conductivity, electric conductivity and kinematic viscosity of the molten metal; Q_{max}, J_{max}, τ_{max} are the maximum power density, current density and aerodynamic shear of the arc over the free surface, U_∞ is the welding velocity and μ_0 is the magnetic permeability of vacuum. The magnitude C'_1 relates to the geometry of the problem $C'_1 = L/(2D)$ (D is indicated in Figure 3). The elements of the matrix of Figure 5 are the exponents of the parameters (indicated above top row) in the estimations indicated at left. These estimations were critical for understanding of defect formation in arc welding at high currents [13, 18]. The expanded expression of some important estimations is the following:

$$\hat{\delta}_C = \left(2\,\mu U_\infty D/\tau_{\max}\right)^{1/2} \quad (5)$$

$$\hat{T}_C = Q_{\max}\hat{\delta}_C/k \quad (6)$$

$$\hat{U}_C = 2U_\infty D/\hat{\delta}_C \quad (7)$$

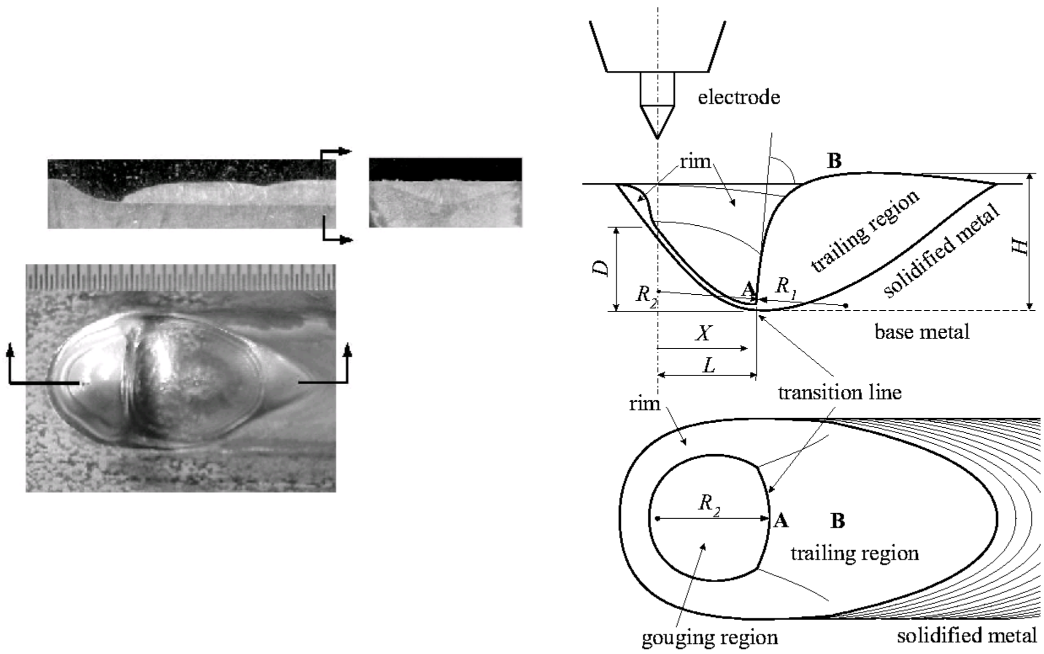

Figure 3: Weld pool at high currents and speeds. The photograph on the upper left corner is a longitudinal section; the one on the upper right corner is a cross section; and the photograph at the bottom, a top view of a weld in which the welding current was suddenly cut off. The schematic shows that the free surface is very depressed, turning into a thin liquid film under the arc. A thicker rim of liquid runs around the edge of the weld pool carrying molten metal to the bulk of liquid at the rear of the weld pool. The transition line marks the abrupt change from the thin liquid film into the bulk of liquid at the rear.

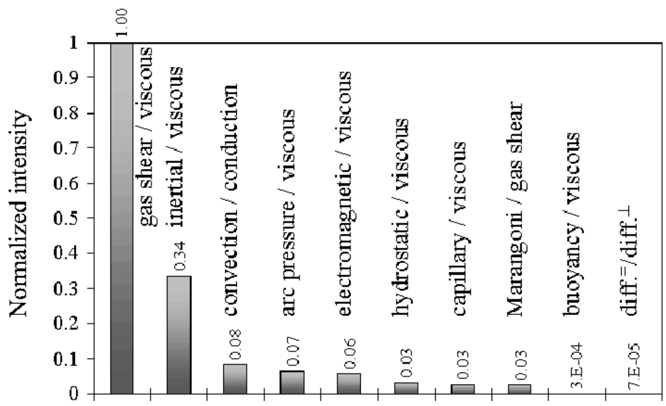

Figure 4: Typical value of the governing dimensional groups for a thin weld pool. The gas shear on the free surface is the dominant driving force, and viscosity is the dominant resistance. All other forces and effects are normalized by them. In the asymptotic case when all other forces and effects are negligible, the ratio of gas shear/viscous effect is one. For this typical case, gas shear is an order of magnitude stronger than any other force. The viscous forces are the dominant resistance, but inertial effects are not negligible.

	L	ρ	k	Q_{max}	J_{max}	σ_e	g	ν	C_1'	τ_{max}	U_∞	μ_0
$\widehat{\delta}$	1/2	1/2					1/2	-1/2		-1/2	1/2	
$\widehat{U}$	1/2	-1/2					-1/2	-1/2		1/2	1/2	
$\widehat{W}$								-1			1	
$\widehat{P}$	1/2	-1/2					-1/2	1/2		3/2	-1/2	
$\widehat{T}$	1/2	1/2	-1	1			1/2	-1/2		-1/2	1/2	
$\widehat{\Phi}$	1/2	1/2			1	-1	1/2	-1/2		-1/2	1/2	
$\widehat{J}$	-1/2	1/2			1		1/2	-1/2		-1/2	1/2	
$\widehat{B}$	1				1							1

Figure 5: Matrix of estimations for a thin weld pool.

Application to a Transferred Plasma Arc

The electric arc is a problem with much relevance to industrial applications such as welding, steel-making and plasma-spraying. As part of our current research, the cathode region of an transferred plasma arc has been scaled [15]. In this region, indicated in Figure 6, the arc can be considered essentially isothermal and the electromagnetic forces are transformed into momentum of the plasma. The driving forces acting on this system are electromagnetic forces in the radial and axial direction, which are opposed by inertial and viscous forces. The governing equations for this system include the Navier-Stokes equations, the equations of conservation of mass and Maxwell's equations. The unknown functions that describe the problem are the radial and axial velocities (V_R (R,Z) and $V_Z(R,Z)$ respectively), the pressure $P(R,Z)$, current density in the radial and axial directions ($J_R(R,Z)$ and $J_Z(R,Z)$ respectively) and induction $B(R,Z)$. The OMS technique provided an estimation of the length of the cathode region ($\hat{Z}_S$), and of the characteristic values of the unknown functions. Their expressions are presented below.

$$\hat{Z}_S = \left(\frac{I}{4\pi J_C} \right)^{1/2} \tag{8}$$

$$\hat{V}_{RS} = \left(\frac{\mu_0 I J_C}{4\pi\rho} \right)^{1/2} \tag{9}$$

$$\hat{V}_{ZS} = \left(\frac{\mu_0 I J_C}{4\pi\rho} \right)^{1/2} \qquad (10)$$

$$\hat{P}_S = \frac{\mu_0 I J_C}{2\pi} \qquad (11)$$

where I is the welding current, J_C is the critical current density for thermionic emission at the cathode, and ae is the density of the plasma at the temperature of the cathode region. Estimations for axial velocity and pressure found in the literature [19, 20, 21] were found to be comparable to those obtained through OMS. OMS also determined that for a flat electrode the radial electromagnetic forces are dominant, and that for most practical cases the viscous forces are negligible. The range for which this balance holds was also determined. The dimensionless groups that are relevant for this problem were determined (the Reynolds number and the dimensionless arc length). Correction functions can be built based on the estimations obtained, the dimensionless groups most relevant in this system and numerical results.

$$f_Z = 0.88\,\mathrm{Re}^{0.058}\left(h/R_C\right)^{0.34} \qquad (12)$$

$$f_{VR} = 0.22\,\mathrm{Re}^{-0.026}\left(h/R_C\right)^{0.086} \qquad (13)$$

$$f_{VZ} = 0.55\,\mathrm{Re}^{0.073}\left(h/R_C\right)^{0.0068} \qquad (14)$$

$$f_P = 0.13\,\mathrm{Re}^{0.17}\left(h/R_C\right)^{-0.057} \qquad (15)$$

where Re is the Reynolds number based on $\hat{Z}_S$ and $\hat{V}_{ZS}$, h is the arc length, and R_C is the radius of the cathode spot.

With the correction functions shown above it is possible to construct dimensionless maps, such as the one shown in Figure 7, that capture the behavior of the system in general terms, independent of the gas, current or other particular characteristic. Maps like this permits one to compare in a single figure various experiments or simulations of a system under a wide variety of conditions. Once a dimensionless map is built and the scaling relationships are determined, the map can be considered a canonical reference. One can envision, for example, storing the maps for velocity fields, pressure fields, etc. as computer files in a server accessible through the Internet, so when an engineer or a researcher needs a particular field of properties for a system, it is not necessary to build a numerical model for it. Instead, the appropriate file can be downloaded and scaled for the particular problem in significantly less time that it would take to build a numerical simulation.

Conclusions

The new technique presented herein is a useful tool for the generalization of problems with relatively simple geometry and many forces acting simultaneously. No previous techniques have this capability. OMS permits one to divide a problem into clearly delimited regimes, obtain estimations of the characteristic values of the solutions of the problem for each regime. Based on these estimations, sensitivity studies between input and output of the system become almost trivial. OMS also permits one to determine the most important dimensionless groups for practical cases, and based on these groups and accurate information about the problem (from experiments or calculations) build correction functions with simple expressions (e.g. power law). The estimations corrected this way have accuracy comparable to that of experimental or numerical error. Based on the corrected estimations it is possible to generalize the experiments or calculations to a wide variety of situations, and in the case of maps of a function over a domain, the map can be generalized in a dimensionless form. It is hoped that this technique will be of help to experimenters and numerical modelers alike in their effort to extract the maximum value from their results, and transmit it in a simple way to people in great need of results for systems that have similar physics but different conditions than the particular cases analyzed.

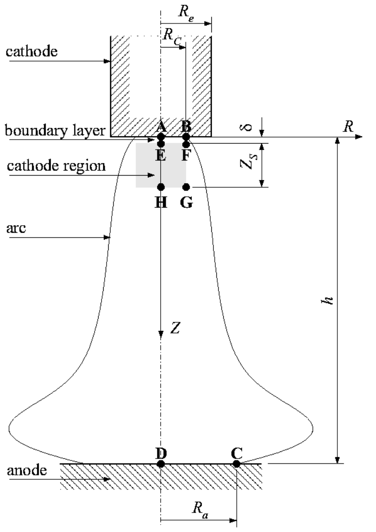

Figure 6: Schematic of a transferred plasma arc. The cathode region is contained in the EFGH domain

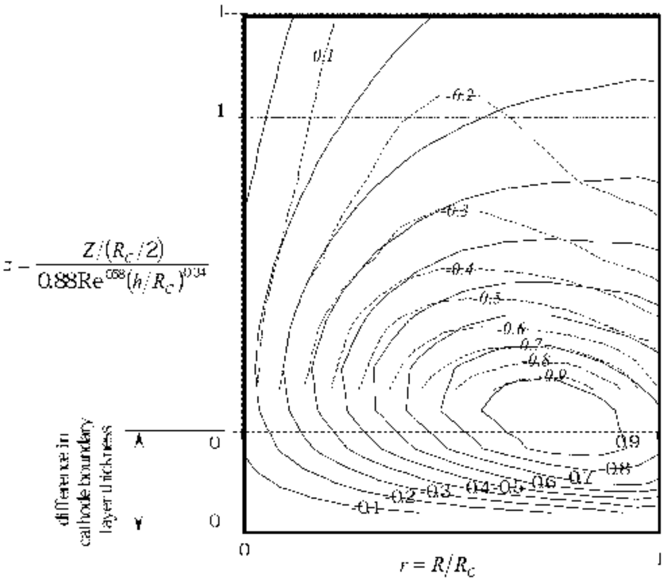

Figure 7: Normalized contour plot of $V_R(R, Z)=V_{RS}$ for the numerical calculations of an welding argon arc of 200 A and 10 mm length (solid lines), and for a melting furnace air arc of 2160 A and 7 cm length (dashed lines, italicized numbers). These two very different arcs converge to a similar normalized representation by using OMS.

Acknowledgment

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