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Modeling of Materials Processing Using Dimensional Analysis and Asymptotic Considerations

P. F. Mendez and T. W. Eagar

Department of Materials Science and Engineering, Massachusetts Institute of Technology, Cambridge, Massachusetts, USA

Abstract

A new mathematical tool (Order of Magnitude Scaling, OMS) especially developed for the modeling of materials processes is presented. This tool combines elements of dimensional analysis and asymptotic considerations in order to provide useful insight into problems in which many driving forces act simultaneously. This is especially useful for the generalization of numerical or physical models, and for obtaining order of magnitude estimations of the unknowns of a problem before attempting to solve the governing equations. This technique has proved very valuable in the study of welding, where the geometries are relatively simple, but the physics are very complex due to the simultaneous and coupled interaction of many driving forces. Application OMS to weld pool dynamics shows quantitatively for the first time that aerodynamic drag of the arc is the driving force for flow in the weld pool in the high current regime (above 250 amperes). Our ongoing research of the welding arc using OMS combined with numerical analysis has generated universal maps for the flow of plasma in the region near the cathode.

Keywords: dimensional analysis; scaling; modeling; asymptotic

Introduction

Advances in computing capabilities currently allow one to model systems with a level of sophistication unattainable several years ago. This increase in modeling capabilities poses a new challenge: to interpret and generalize the results obtained. This task is essential for the transmission of knowledge all the way to final application on the shop floor. The reason that interpretation and generalization has become more complex is that newer models contain significantly more parameters and geometric details than before. After the application of dimensional analysis, a reduced number of parameters (m) will completely determine a particular problem. A sensitivity study based on ten experiments within the range of variation of each parameter requires on the order of 10^m experiments to characterize the whole space. Thus, the degree of difficulty in describing a process increases exponentially with the number of parameters.

The task of characterizing a problem might seem insurmountable; however, most material processes present a characteristic that makes them amenable to simplifications with little loss in accuracy. This characteristic is that they can be divided into different regimes usually. Within each regime, only one driving force is dominant and it is balanced mainly by one resistance. All the other forces are secondary and in many practical cases, the consideration of a few secondary forces (represented as dimensionless groups) can bring the simplified model within the accuracy of the measurements or numerical calculations.

The determination of the different regimes and the proper balance of forces are a difficult task and is beyond the capabilities of dimensional analysis [1] or inspectional analysis [2]. The technique of “dominant balance” or “ordering” [3, 4] is widely used by applied mathematicians to determine the balance for asymptotic cases in systems described by differential equations. In this technique, the terms that are expected to be small are removed from the differential equations, the system is solved, and finally the results are checked for consistency. This method is extremely powerful, but has the disadvantage that a complete set of differential equations must be solved at each iteration, which can be impractical in many cases. Engineers have tried alternative approaches based on dimensional analysis and an intuitive understanding of the governing equations [5, 6]. The downside of this approach is that much experience is necessary to acquire an intuitive understanding of a particular system and its governing equations.

The creation of the Order of Magnitude Scaling technique was motivated by the need for a tool that can interpret and generalize the information provided by numerical calculations or experiments. This technique is aimed at systems with simple geometry but many driving forces and resistances acting simultaneously. As an illustration of the application of OMS, the evolution of welding arc modeling is presented in Figure 1. In this chart, the vertical axis represents the maximum number of independent dimensionless groups that relate only to the geometry of the problem (m_g). Higher values in this axis correspond to models that capture more intricate geometries. The horizontal axis represents the number of groups related to the physics of the system (m_p; $m_p = m - m_g$). Higher values in this axis correspond to models including richer description of the physics (and consequently more simultaneous differential equations). The dashed line indicates the general trend in the evolution of arc modeling, which is also representative of the trends in modeling many other materials processes. It is observed that the increase in sophistication in the models has focused on the description of the physics.

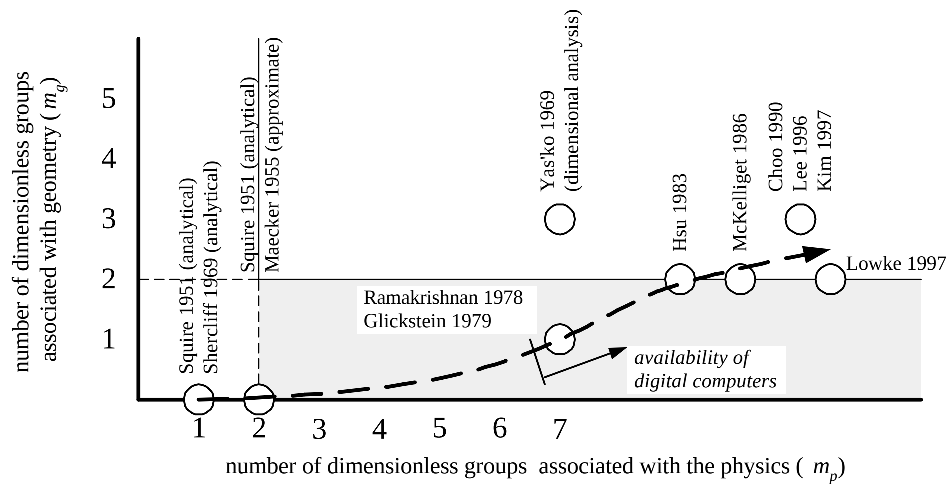

Figure 1

Evolution of the arc modeling. The horizontal and vertical axes indicate the difficulty of generalizing the physics and geometry of the system respectively.

Figure 1 is also useful to divide modeling problems into different categories relative to the difficulty of generalizing the results. A vertical line at m_p= 2 and a horizontal line at m_g = 2 divide the chart into four parts. The region on the lower left contains problems with simple geometries and simple physics. These problems are the easiest to generalize. This can be done by induction from the analysis of particular cases through numerical models or experiments. Generalization in this case can often be obtained also by deduction, through closed form solutions of the governing equations. The region on the upper left contains problems with relatively simple physics, but complex geometries. If geometric similarity is preserved, the physics of the problem can be generalized by induction from the analysis of particular cases through numerical models or experiments. The region at the bottom right represents the new generalizations that are possible by using OMS. The systems in this region have a relatively simple geometry and many driving forces and resistances acting simultaneously. The different possible balances between dominant and balancing forces determine distinct regimes into which different simplifications can be applied. The region at the top right represents systems with complex geometries and physics. The vast majority of these problems have solutions that are very difficult to generalize by induction (due to the large number of parameters involved) or by deduction, because of difficult geometry. The generalization of the results for these problems constitutes a challenge for future research in modeling of materials processes.

Order of Magnitude Scaling

In order to find the dominant and balancing force in a particular regime, the unknown functions that describe the system (velocity components, temperature, etc.) are normalized as shown below:

$$x_i = \frac{X_i - A_i}{B_i - A_i} \tag{1}$$

$$f_j(\mathbf{x}) = \pm \frac{F_j(\mathbf{X}) - F_j(\mathbf{A})}{\left|F_j(\mathbf{B})\right| - \left|F_j(\mathbf{A})\right|} \tag{2}$$

where F_j is a generic unknown function and X represents an independent argument. Upper case letters are used for magnitudes with units, and lower case for their dimensionless counterparts. **A** and **B** are the points in the domain where the function F_j reaches its minimum and maximum absolute value respectively. The magnitude $|F_j(\mathbf{B})|$ is the characteristic value of F_j , and $|F_j(\mathbf{B})|$-$|F_j(\mathbf{A})|$ is the characteristic value of the variation of F_j. One special feature of this normalization is that even if the characteristic values are unknown, the normalized function still has an absolute value lesser or equal to one. After normalizing the unknown functions, each equation is normalized by the coefficients of the term expected to be dominant, and the unknown characteristic values act as degrees of freedom for the balancing terms. These degrees of freedom are fixed in an iterative process. This iterative process stops when the dimensionless coefficient of the dominant and balancing terms is equal to one, and the coefficient of all other terms is less than one. It is not necessary to solve the differential equations as in the dominant balance technique. This allows for a formulation of the problems in terms of matrices, which is very convenient in terms of speed and efficiency.

The reason why it is possible to obtain estimations without solving the equations is that most materials processes have a particular characteristic. This characteristic is that the domain in which the equations must be solved can be split into a small number of subdomains (of the order of one to three), in which the solutions behave in an approximately linear fashion (i.e. the dimensionless second derivative of the functions is of the order of one, or less). This property establishes an upper bound of the order of one for the functions involved in the dimensionless governing equations. When a lower bound of the order of magnitude of one can also be established, the governing equations can be characterized by their coefficients alone, as if they were algebraic equations.

Application to Welding at High Current

The analysis of the weld pool at high current and velocity [7-9] is well suited for the OMS technique, because it has a relatively simple geometry and many forces acting on it. Figure 2 shows a photograph and a schematic of the weld pool at high current and velocity. It can be seen that the molten metal turns into a thin film under the electrode.

The driving forces acting on this system are gas shear on the free surface, arc pressure, electromagnetic forces, hydrostatic pressure, capillary forces, Marangoni forces, and buoyancy forces. Inertial and viscous forces in the molten metal oppose to these forces. The governing equations for this system include the Navier-Stokes equations, the equations of conservation of mass and energy, Maxwell's equations, and boundary conditions considering Marangoni shear stresses and capillary effects. The unknown functions that describe the problem are the velocities in the X and Y components, $U(X;Y)$ and $V(X; Y)$ respectively, the pressure $P(X; Y)$, temperature $T(X; Y)$, current density in the X and Y directions $J_X(X; Y)$ and $J_Y(X; Y)$ respectively, induction $B(X;Y)$, electric potential $\Phi(X; Y)$, and position of the free surface $d(X)$.

The OMS technique used herein determined quantitatively for the first time that the dominant driving force in a thin weld pool is the aerodynamic shear of the arc. The balancing resistance is the viscous force. Most secondary dimensionless groups are very small for a typical case, as indicated in Figure 2, the only exceptions are the inertial forces which are small but not negligible. A simplified model could safely neglect all secondary effects but these, which is represented by only one dimensionless group. The expressions of some important estimations are the following:

$$\hat{d}_C = \left(2\, m U_\infty D / t_{\max}\right)^{1/2} \tag{3}$$

$$\hat{T}_C = Q_{\max} \hat{d}_C / k \tag{4}$$

$$\hat{U}_C = 2 U_\infty D / \hat{d}_C \tag{5}$$

where D is indicated in Figure 2; m, k are the viscosity, and thermal conductivity of the molten metal; Q_{max} and t_{max} are the maximum power density and aerodynamic shear of the arc over the free surface, and U_∞ is the welding velocity.

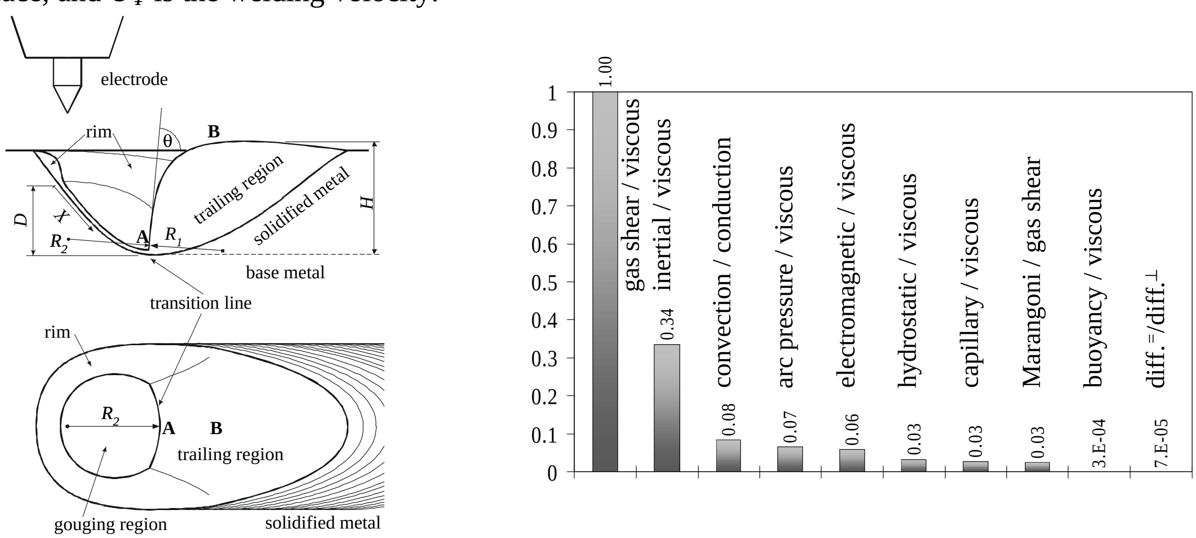

Figure 2

On the left: weld pool at high currents and speeds. The photograph on the upper left corner is a longitudinal section; the one on the upper right corner is a cross section; and the photograph at the bottom, a top view of a weld in which the welding current was suddenly cut off. On the right: typical values of the governing dimensional groups for a thin weld pool. Gas shear on the free surface is the dominant driving force, and viscosity is the dominant resistance.

Application to a Transferred Plasma Arc

The electric arc is a problem with much relevance to industrial applications such as welding, steel-making and plasma-spraying. As part of our current research, the cathode region of a transferred plasma arc has been scaled [10]. In this region, indicated in Figure 3, the arc can be considered essentially isothermal and the electromagnetic forces are transformed into momentum of the plasma. The driving forces acting on this system are electromagnetic forces in the radial and axial direction, which are opposed by inertial and viscous forces. The governing equations for this system include the Navier-Stokes equations, the equations of conservation of mass and Maxwell's equations. The unknown functions that describe the problem are the radial and axial velocities (V_R (R,Z) and $V_Z(R,Z)$ respectively), the pressure $P(R,Z)$, current density in the radial and axial directions ($J_R(R,Z)$ and $J_Z(R,Z)$ respectively) and induction $B(R,Z)$. The OMS technique provides an estimation of the length of the cathode region ($\hat{Z}_S$), and of the characteristic values of the unknown functions. These expressions are presented below.

$$\hat{Z}_S = \sqrt{I/(4pJ_C)} \quad (8)$$

$$\hat{V}_{RS} = \sqrt{m_0 I J_C/(4pr)} \quad (9)$$

$$\hat{V}_{ZS} = \sqrt{m_0 I J_C/(4pr)} \quad (10)$$

$$\hat{P}_S = m_0 I J_C/(2p) \quad (11)$$

where I is the welding current, J_C is the critical current density for thermionic emission at the cathode, r is the density of the plasma at the temperature of the cathode region, and m_0 is the magnetic permeability of vacuum. Estimations for axial velocity and pressure [11, 12] were found to be comparable to those obtained through OMS. OMS also determined that for a flat electrode the radial electromagnetic forces are dominant, and that for most practical cases the viscous forces are negligible. The range for which this balance holds was also determined. The dimensionless groups that are relevant for this problem were determined (the Reynolds number and the dimensionless arc length). Correction functions can be built based on the estimations obtained, the dimensionless groups most relevant in this system, and numerical results.

$$f_Z = 0.88\,\mathrm{Re}^{0.058}(h/R_C)^{0.34} \quad (12)$$

$$f_{VR} = 0.22\,\mathrm{Re}^{-0.026}(h/R_C)^{0.086} \quad (13)$$

$$f_{VZ} = 0.55\,\mathrm{Re}^{0.073}(h/R_C)^{0.0068} \quad (14)$$

$$f_P = 0.13\,\mathrm{Re}^{0.17}(h/R_C)^{-0.057} \quad (15)$$

where Re is the Reynolds number based on $\hat{Z}_S$ and $\hat{V}_{ZS}$, h is the arc length, and R_C is the radius of the cathode spot. With these correction functions it is possible to construct dimensionless maps, such as the one shown in Figure 3, that capture the behavior of the system in general terms, independent of the gas, current or other particular characteristic. Maps like this permits one to compare in a single figure various experiments or simulations of a system under a wide variety of conditions. Once a dimensionless map is built and the scaling relationships are determined, the map can be considered a canonical reference. One can envision, for example, storing the maps for velocity fields, pressure fields, etc. as computer files in a server accessible through the Internet, so when an engineer or a researcher needs a particular field of properties for a system, it is not necessary to build a numerical model for it. Instead, the appropriate file can be downloaded and scaled for the particular problem in significantly less time that it would take to build a numerical simulation.

Conclusions

The new technique presented herein is a useful tool for the generalization of problems with relatively simple geometry and many forces acting simultaneously. No previous techniques have this capability. OMS permits one to divide a problem into clearly delimited regimes, obtain estimations of the characteristic values of the solutions of the problem for each regime. Based on these estimations, sensitivity studies between input and output of the system become almost trivial. OMS also permits one to determine the most important dimensionless groups for practical cases, and based on these groups and accurate information about the problem (from experiments or calculations) build correction functions with simple expressions (e.g. power laws). The estimations corrected in this way have accuracy comparable to that of experimental or numerical error. Based on the corrected estimations it is possible to generalize the experiments or calculations to a wide variety of situations, and in the case of maps of a function over a domain, the map can be generalized in a dimensionless form. It is hoped that this technique will help experimenters and numerical modelers alike in their effort to extract the maximum value from their results, and transmit these results in a simple way for a wide range of systems.

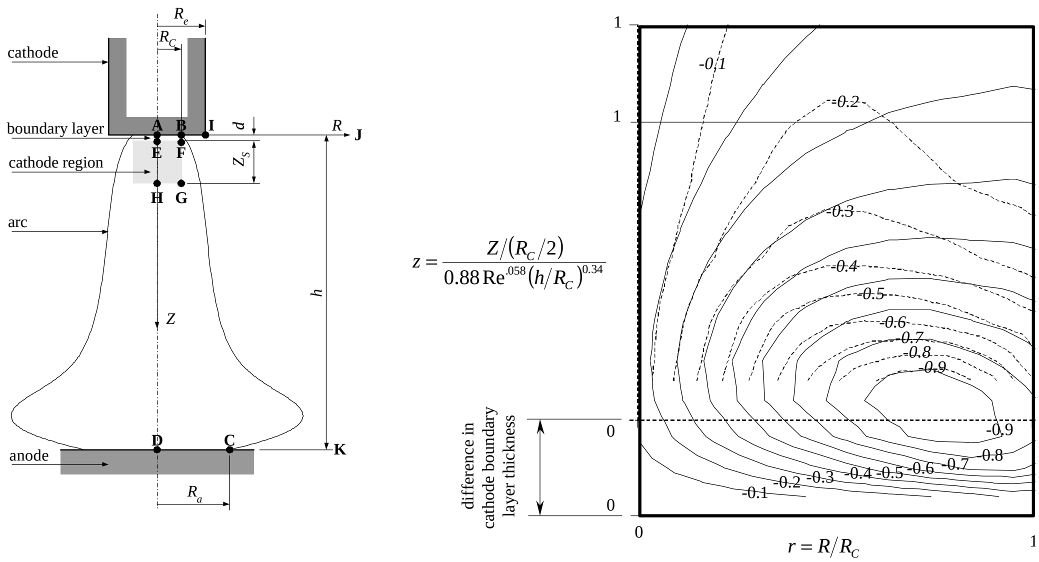

Figure 3

On the left: schematic of a transferred plasma arc. The cathode region is contained in the EFGH domain. On the right: normalized contour plot of $V_R(R, Z)=V_{RS}$ for the numerical calculations of an welding argon arc of 200 A and 10 mm length (solid lines), and for a melting furnace air arc of 2160 A and 7 cm length (dashed lines, italicized numbers). These two very different arcs converge to a similar normalized representation by using OMS.

Acknowledgements

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