Two-loop electroweak corrections for four-fermion processes at LHC and ILC

Alexander Penin

TTP Karlsruhe & INR Moscow

Loops and Legs in Quantum Field Theory, Eisenach, Germany, April 23-28 2006

Based on

B. Feucht/Jantzen, J.H. Kühn, A.A. Penin, V.A. Smirnov,

Phys. Rev. Lett. **93** *(2004) 101802*

Phys. Rev. **D72** *(2005) 051301 (R)*

Nucl. Phys. **B731** *(2005) 188*

Why two-loop electroweak corrections?

- Massive W, Z bosons ⇨ exclusive reactions

Why two-loop electroweak corrections?

- Massive W, Z bosons $\Rightarrow$ exclusive reactions
- Sudakov double logs: $\ln^2(s/M_{Z,W}^2)$ per loop

Why two-loop electroweak corrections?

- Massive W, Z bosons $\Rightarrow$ exclusive reactions
- Sudakov double logs: $\ln^2(s/M_{Z,W}^2)$ per loop
- LHC, ILC: $\ln^2(s/M_{Z,W}^2) \sim 25$

Why two-loop electroweak corrections?

- Massive W, Z bosons ⇨ exclusive reactions
- Sudakov double logs: $\ln^2(s/M^2_{Z,W})$ per loop
- LHC, ILC: $\ln^2(s/M^2_{Z,W}) \sim 25$
  - *30% in one loop*

Why two-loop electroweak corrections?

- Massive W, Z bosons ⇨ exclusive reactions
- Sudakov double logs: $\ln^2(s/M^2_{Z,W})$ per loop
- LHC, ILC: $\ln^2(s/M^2_{Z,W}) \sim 25$
  - *30% in one loop*
  - *5% in two loops*

Why two-loop electroweak corrections?

- Massive W, Z bosons ⇨ exclusive reactions
- Sudakov double logs: $\ln^2(s/M^2_{Z,W})$ per loop
- LHC, ILC: $\ln^2(s/M^2_{Z,W}) \sim 25$
  - *30% in one loop*
  - *5% in two loops*
- Luminosity monitoring:

Why two-loop electroweak corrections?

- Massive W, Z bosons $\Rightarrow$ exclusive reactions
- Sudakov double logs: $\ln^2(s/M^2_{Z,W})$ per loop
- LHC, ILC: $\ln^2(s/M^2_{Z,W}) \sim 25$
  - *30% in one loop*
  - *5% in two loops*
- Luminosity monitoring:
  - *Drell-Yan (LHC)*

Why two-loop electroweak corrections?

- Massive W, Z bosons ⇨ exclusive reactions
- Sudakov double logs: $\ln^2(s/M_{Z,W}^2)$ per loop
- LHC, ILC: $\ln^2(s/M_{Z,W}^2) \sim 25$
  - *30% in one loop*
  - *5% in two loops*
- Luminosity monitoring:
  - *Drell-Yan (LHC)*
  - *Bhabha scattering (ILC)*

Why two-loop electroweak corrections?

$\boxed{\text{Drell-Yan}}$

$\mathcal{O}(\alpha_s^2)$ *corrections* ⇨ $\delta^{NNLO}\sigma/\sigma^{NLO} \sim 10\%$

(Matsuura, Hamberg, van Neerven; Anastasiou, Dixon, Melnikov, Petriello)

Why two-loop electroweak corrections?

$\boxed{\text{Drell-Yan}}$

$\mathcal{O}(\alpha_s^2)$ *corrections* ➪ $\delta^{NNLO}\sigma/\sigma^{NLO} \sim 10\%$

(Matsuura, Hamberg, van Neerven; Anastasiou, Dixon, Melnikov, Petriello)

$\boxed{\text{Bhabha}}$

$\mathcal{O}(\alpha_e^2)$ *corrections* ➪ *only low energy or small angle scattering*

(Bonciani, Ferroglia, Mastrolia, Remiddi, van der Bij; Penin)

Luminosity spectrum at ILC ➪ *wide angle scattering*

Four-fermion processes $f\bar{f} \rightarrow f'\bar{f}'$

- Current status of two-loop calculation:

Four-fermion processes $f\bar{f} \to f'\bar{f}'$

- Current status of two-loop calculation:
  - LL: $\alpha_{ew}^2 \ln^4(s/M_{Z,W}^2)$ (Fadin, Lipatov, Martin, Melles)

Four-fermion processes $f\bar{f} \to f'\bar{f}'$

- Current status of two-loop calculation:
  - LL: $\alpha_{ew}^2 \ln^4(s/M_{Z,W}^2)$ (Fadin, Lipatov, Martin, Melles)
  - NLL: $\alpha_{ew}^2 \ln^3(s/M_{Z,W}^2)$ (Kühn, Penin, Smirnov)

Four-fermion processes $f\bar{f} \to f'\bar{f}'$

- Current status of two-loop calculation:
  - LL: $\alpha_{ew}^2 \ln^4(s/M_{Z,W}^2)$ (Fadin, Lipatov, Martin, Melles)
  - NLL: $\alpha_{ew}^2 \ln^3(s/M_{Z,W}^2)$ (Kühn, Penin, Smirnov)
  - NNLL: $\alpha_{ew}^2 \ln^2(s/M_{Z,W}^2)$ (Kühn, Moch, Penin, Smirnov)

Four-fermion processes $f\bar{f} \to f'\bar{f}'$

- Current status of two-loop calculation:
  - LL: $\alpha_{ew}^2 \ln^4(s/M_{Z,W}^2)$ (Fadin, Lipatov, Martin, Melles)
  - NLL: $\alpha_{ew}^2 \ln^3(s/M_{Z,W}^2)$ (Kühn, Penin, Smirnov)
  - NNLL: $\alpha_{ew}^2 \ln^2(s/M_{Z,W}^2)$ (Kühn, Moch, Penin, Smirnov)
  - $\boxed{\text{N}^3\text{LL:}}$ $\alpha_{ew}^2 \ln(s/M_{Z,W}^2)$ (Feucht/Jantzen, Kühn, Penin, Smirnov)

Four-fermion processes $f\bar{f} \to f'\bar{f}'$

- Current status of two-loop calculation:
  - LL: $\alpha_{ew}^2 \ln^4(s/M_{Z,W}^2)$ (Fadin, Lipatov, Martin, Melles)
  - NLL: $\alpha_{ew}^2 \ln^3(s/M_{Z,W}^2)$ (Kühn, Penin, Smirnov)
  - NNLL: $\alpha_{ew}^2 \ln^2(s/M_{Z,W}^2)$ (Kühn, Moch, Penin, Smirnov)
  - $\boxed{\text{N}^3\text{LL:}}$ $\alpha_{ew}^2 \ln(s/M_{Z,W}^2)$ (Feucht/Jantzen, Kühn, Penin, Smirnov)
- Topics discussed in this talk:

Four-fermion processes $f\bar{f} \to f'\bar{f}'$

- Current status of two-loop calculation:
  - LL: $\alpha_{ew}^2 \ln^4(s/M_{Z,W}^2)$ (Fadin, Lipatov, Martin, Melles)
  - NLL: $\alpha_{ew}^2 \ln^3(s/M_{Z,W}^2)$ (Kühn, Penin, Smirnov)
  - NNLL: $\alpha_{ew}^2 \ln^2(s/M_{Z,W}^2)$ (Kühn, Moch, Penin, Smirnov)
  - $\boxed{\text{N}^3\text{LL:}}$ $\alpha_{ew}^2 \ln(s/M_{Z,W}^2)$ (Feucht/Jantzen, Kühn, Penin, Smirnov)
- Topics discussed in this talk:
  - *General approach*

Four-fermion processes $f\bar{f} \to f'\bar{f}'$

- Current status of two-loop calculation:
  - LL: $\alpha_{ew}^2 \ln^4(s/M_{Z,W}^2)$ (Fadin, Lipatov, Martin, Melles)
  - NLL: $\alpha_{ew}^2 \ln^3(s/M_{Z,W}^2)$ (Kühn, Penin, Smirnov)
  - NNLL: $\alpha_{ew}^2 \ln^2(s/M_{Z,W}^2)$ (Kühn, Moch, Penin, Smirnov)
  - $\boxed{\text{N}^3\text{LL:}}$ $\alpha_{ew}^2 \ln(s/M_{Z,W}^2)$ (Feucht/Jantzen, Kühn, Penin, Smirnov)
- Topics discussed in this talk:
  - *General approach*
  - *Approximation*

Four-fermion processes $f\bar{f} \to f'\bar{f}'$

- Current status of two-loop calculation:
  - LL: $\alpha_{ew}^2 \ln^4(s/M_{Z,W}^2)$ (Fadin, Lipatov, Martin, Melles)
  - NLL: $\alpha_{ew}^2 \ln^3(s/M_{Z,W}^2)$ (Kühn, Penin, Smirnov)
  - NNLL: $\alpha_{ew}^2 \ln^2(s/M_{Z,W}^2)$ (Kühn, Moch, Penin, Smirnov)
  - $\boxed{\text{N}^3\text{LL:}}$ $\alpha_{ew}^2 \ln(s/M_{Z,W}^2)$ (Feucht/Jantzen, Kühn, Penin, Smirnov)
- Topics discussed in this talk:
  - *General approach*
  - *Approximation*
  - *Results and phenomenology*

How to compute?

- Many scales: $M_Z,\ M_W,\ M_H,\ \lambda,\ m_f, m_{f'}$

How to compute?

- Many scales: $M_Z,\ M_W,\ M_H,\ \lambda,\ m_f, m_{f'}$
- Scales hierarchy: $s,\ t,\ u \gg \underbrace{M_Z \approx M_W \sim M_H}_{M} \gg \lambda,\ \underbrace{m_f,\ m_{f'}}_{0}$

How to compute?

- Many scales: $M_Z,\ M_W,\ M_H,\ \lambda,\ m_f, m_{f'}$
- Scales hierarchy: $s,\ t,\ u \gg \underbrace{M_Z \approx M_W \sim M_H}_{M} \gg \lambda,\ \underbrace{m_f,\ m_{f'}}_{0}$
- Leading asymptotics in: $M^2/s, \quad \lambda/M$

How to compute?

- Many scales: $M_Z,\ M_W,\ M_H,\ \lambda,\ m_f, m_{f'}$
- Scales hierarchy: $s,\ t,\ u \gg \underbrace{M_Z \approx M_W \sim M_H}_{M} \gg \lambda,\ \underbrace{m_f,\ m_{f'}}_{0}$
- Leading asymptotics in: $M^2/s, \quad \lambda/M$

Two types of large logs

Electroweak	*QED*
$\ln(s/M^2)$	$\ln(s/\lambda^2)$

How to compute?

- Many scales: $M_Z,\ M_W,\ M_H,\ \lambda,\ m_f, m_{f'}$
- Scales hierarchy: $s,\ t,\ u \gg \underbrace{M_Z \approx M_W \sim M_H}_{M} \gg \lambda,\ \underbrace{m_f,\ m_{f'}}_{0}$
- Leading asymptotics in: $M^2/s, \quad \lambda/M$

Two types of large logs

Electroweak	*QED*
$\ln(s/M^2)$	$\ln(s/\lambda^2)$

hard evolution equation

How to compute?

- Many scales: $M_Z,\ M_W,\ M_H,\ \lambda,\ m_f, m_{f'}$
- Scales hierarchy: $s,\ t,\ u \gg \underbrace{M_Z \approx M_W \sim M_H}_{M} \gg \lambda,\ \underbrace{m_f,\ m_{f'}}_{0}$
- Leading asymptotics in: $M^2/s, \quad \lambda/M$

Two types of large logs

Electroweak	*QED*
$\ln(s/M^2)$	$\ln(s/\lambda^2)$

hard evolution equation *infrared evolution equation*

Hard evolution

(Mueller; Collins; Sen; Sterman,...)

Form factor

$$\frac{\partial}{\partial \ln Q^2}\mathcal{F} = \left[\int_{M^2}^{Q^2} \frac{\mathrm{d}x}{x}\gamma(\alpha(x)) + \zeta(\alpha(Q^2)) + \xi(\alpha(M^2))\right]\mathcal{F}$$

Reduced amplitude

$$\frac{\partial}{\partial \ln Q^2}\tilde{\mathcal{A}} = \chi(\alpha(Q^2))\tilde{\mathcal{A}}$$

Amplitude decomposition ⇨ $\mathcal{A}(f\bar{f} \to f'\bar{f}') = -\frac{ig^2(Q^2)}{Q^2}\mathcal{F}^2\tilde{\mathcal{A}}$

Hard evolution

$\boxed{\text{Solution}}$

$$\mathcal{F} \quad = \quad F_0(\alpha(M^2))\,\exp\left\{\int_{M^2}^{Q^2}\frac{\mathrm{d}x}{x}\left[\int_{M^2}^{x}\frac{\mathrm{d}x'}{x'}\gamma(\alpha(x')) + \zeta(\alpha(x)) + \xi(\alpha(M^2))\right]\right\}$$

$$\tilde{\mathcal{A}} \quad = \quad \tilde{\mathcal{A}}_0(\alpha(M^2))\mathrm{Pexp}\left[\int_{M^2}^{Q^2}\frac{\mathrm{d}x}{x}\chi(\alpha(x))\right]$$

Anomalous dimensions ⇨ $\gamma, \quad \zeta, \quad \chi$

Initial conditions ⇨ $\xi, \quad F_0, \quad \mathcal{A}_0$

Cusp anomalous dimension

$$\gamma(\alpha) = \sum_n \left(\frac{\alpha}{4\pi}\right)^n \gamma^{(n)}, \ldots$$

$$\gamma^{(1)} = 2C_F$$

$$\gamma^{(2)} = C_F \left[\left(-\frac{134}{9} + \frac{2}{3}\pi^2\right) C_A + \frac{8}{9}\left(5n_f + 2n_s\right) T_F\right]$$

(Kodaira, Trentedue)

$$\gamma^{(3)} = C_F \left[\left(-\frac{245}{3} + \frac{268}{27}\pi^2 - \frac{44}{3}\zeta(3) - \frac{22}{45}\pi^4\right) C_A^2 + \ldots\right]$$

(Moch, Vermaseren, Vogt)

Two-loop corrections to $\sigma(f\bar{f} \to f'\bar{f}')$

$\boxed{\text{Two-loop log}^{4,3,2}}$

$$\gamma^{(2)}, \quad \zeta^{(1)}, \quad \chi^{(1)}, \quad \xi^{(1)}, \quad F_0^{(1)}, \quad \mathcal{A}_{0i}^{(1)}$$

Two-loop corrections to $\sigma(f\bar{f} \to f'\bar{f}')$

$\boxed{\text{Two-loop log}^{4,3,2}}$

$$\gamma^{(2)}, \quad \zeta^{(1)}, \quad \chi^{(1)}, \quad \xi^{(1)}, \quad F_0^{(1)}, \quad \mathcal{A}_{0i}^{(1)}$$

$\boxed{\text{Two-loop linear log}}$

Two-loop corrections to $\sigma(f\bar{f} \to f'\bar{f}')$

Two-loop $\log^{4,3,2}$

$$\gamma^{(2)}, \quad \zeta^{(1)}, \quad \chi^{(1)}, \quad \xi^{(1)}, \quad F_0^{(1)}, \quad \mathcal{A}_{0i}^{(1)}$$

Two-loop linear log

$\chi^{(2)}$ ⇨ *two-loop massless amplitudes*

(Anastasiou, Glover, Oleari, Tejeda-Yeomans; Glover)

Two-loop corrections to $\sigma(f\bar{f} \to f'\bar{f}')$

$\boxed{\text{Two-loop } \log^{4,3,2}}$

$$\gamma^{(2)}, \quad \zeta^{(1)}, \quad \chi^{(1)}, \quad \xi^{(1)}, \quad F_0^{(1)}, \quad \mathcal{A}_{0i}^{(1)}$$

$\boxed{\text{Two-loop linear log}}$

$\chi^{(2)}$ ⇨ *two-loop massless amplitudes*

(Anastasiou, Glover, Oleari, Tejeda-Yeomans; Glover)

$\zeta^{(2)} + \xi^{(2)}$ ⇨ *two-loop massive form factor*

(Feucht/Jantzen, Kühn, Penin, Smirnov)

Two-loop corrections to $\sigma(f\bar{f} \to f'\bar{f}')$

Massive $SU(2)$ *model,* $M_H = M$*, six left-handed massless doublets*

Two-loop corrections to $\sigma(f\bar{f} \to f'\bar{f}')$

Massive $SU(2)$ *model,* $M_H = M$*, six left-handed massless doublets*

$$\left[\frac{\delta\sigma}{\sigma}\right]_{\uparrow\downarrow} = \left(\frac{\alpha_{ew}}{4\pi}\right)^2 \left[\frac{9}{2}\ln^4\left(\frac{s}{M^2}\right) - \frac{125}{6}\ln^3\left(\frac{s}{M^2}\right) + \left(-\frac{799}{9} + \frac{37\pi^2}{3}\right)\ln^2\left(\frac{s}{M^2}\right) + \left(\frac{51613}{216} - \frac{815}{18}\pi^2 - 122\zeta(3) + 15\sqrt{3}\pi + 26\sqrt{3}\mathrm{Cl}_2\left(\frac{\pi}{3}\right)\right)\ln\left(\frac{s}{M^2}\right)\right]$$

Two-loop corrections to $\sigma(f\bar{f} \to f'\bar{f}')$

Massive $SU(2)$ *model,* $M_H = M$*, six left-handed massless doublets*

$$
\begin{aligned}
\left[\frac{\delta\sigma}{\sigma}\right]_{\uparrow\downarrow} \quad = \quad & \left(\frac{\alpha_{ew}}{4\pi}\right)^2 \left[\frac{9}{2}\ln^4\left(\frac{s}{M^2}\right) - \frac{125}{6}\ln^3\left(\frac{s}{M^2}\right) + \left(-\frac{799}{9} + \frac{37\pi^2}{3}\right)\ln^2\left(\frac{s}{M^2}\right)\right. \\
& \left. + \left(\frac{51613}{216} - \frac{815}{18}\pi^2 - 122\zeta(3) + 15\sqrt{3}\pi + 26\sqrt{3}\mathrm{Cl}_2\left(\frac{\pi}{3}\right)\right)\ln\left(\frac{s}{M^2}\right)\right] \\
\approx & \left(\frac{\alpha_{ew}}{4\pi}\right)^2 \left[4.50\ \ln^4\left(\frac{s}{M^2}\right) - 20.83\ln^3\left(\frac{s}{M^2}\right) + 32.95\ \ln^2\left(\frac{s}{M^2}\right) - 227.25\ \ln\left(\frac{s}{M^2}\right)\right]
\end{aligned}
$$

Two-loop corrections to $\sigma(f\bar{f} \to f'\bar{f}')$

Massive $SU(2)$ *model,* $M_H = M$, *six left-handed massless doublets*

$$\left[\frac{\delta\sigma}{\sigma}\right]_{\uparrow\downarrow} = \left(\frac{\alpha_{ew}}{4\pi}\right)^2 \left[\frac{9}{2}\ln^4\left(\frac{s}{M^2}\right) - \frac{125}{6}\ln^3\left(\frac{s}{M^2}\right) + \left(-\frac{799}{9} + \frac{37\pi^2}{3}\right)\ln^2\left(\frac{s}{M^2}\right)\right.$$
$$\left. + \left(\frac{51613}{216} - \frac{815}{18}\pi^2 - 122\zeta(3) + 15\sqrt{3}\pi + 26\sqrt{3}\mathrm{Cl}_2\left(\frac{\pi}{3}\right)\right)\ln\left(\frac{s}{M^2}\right)\right]$$
$$\approx \left(\frac{\alpha_{ew}}{4\pi}\right)^2 \left[4.50\ \ln^4\left(\frac{s}{M^2}\right) - 20.83\ln^3\left(\frac{s}{M^2}\right) + 32.95\ \ln^2\left(\frac{s}{M^2}\right) - 227.25\ \ln\left(\frac{s}{M^2}\right)\right]$$

$$\left[\frac{\delta\sigma}{\sigma}\right]_{\uparrow\uparrow} = \left(\frac{\alpha_{ew}}{4\pi}\right)^2 \left[\frac{9}{2}\ln^4\left(\frac{s}{M^2}\right) - \frac{449}{6}\ln^3\left(\frac{s}{M^2}\right) + \left(\frac{4855}{18} + \frac{37\pi^2}{3}\right)\ln^2\left(\frac{s}{M^2}\right)\right.$$
$$\left. + \left(\frac{34441}{216} - \frac{1247}{18}\pi^2 - 122\zeta(3) + 15\sqrt{3}\pi + 26\sqrt{3}\mathrm{Cl}_2\left(\frac{\pi}{3}\right)\right)\ln\left(\frac{s}{M^2}\right)\right]$$
$$\approx \left(\frac{\alpha_{ew}}{4\pi}\right)^2 \left[4.50\ \ln^4\left(\frac{s}{M^2}\right) - 74.83\ln^3\left(\frac{s}{M^2}\right) + 391.45\ \ln^2\left(\frac{s}{M^2}\right) - 543.62\ \ln\left(\frac{s}{M^2}\right)\right]$$

$SU(2) \times U(1)$ model

$SU(2) \times U(1)$ **model**

Factorization of infrared singularities

$$\mathcal{A}_{f\bar{f}\to f'\bar{f}'} = \exp\left[-\frac{\alpha_e}{4\pi}(Q_f^2 + Q_{f'}^2)\ln^2\left(\frac{s}{\lambda^2}\right) + \ldots\right]\bar{\mathcal{A}}(M^2/s) + \mathcal{O}(\lambda/M)$$

$SU(2) \times U(1)$ model

Factorization of infrared singularities

$$\mathcal{A}_{f\bar{f}\to f'\bar{f}'} = \exp\left[-\frac{\alpha_e}{4\pi}(Q_f^2 + Q_{f'}^2)\ln^2\left(\frac{s}{\lambda^2}\right) + \ldots\right]\bar{\mathcal{A}}(M^2/s) + \mathcal{O}(\lambda/M)$$

NNLL approximation ➪ no nontrivial dependence on λ/M

$SU(2) \times U(1)$ model

Factorization of infrared singularities

$$\mathcal{A}_{f\bar{f}\to f'\bar{f}'} = \exp\left[-\frac{\alpha_e}{4\pi}(Q_f^2 + Q_{f'}^2)\ln^2\left(\frac{s}{\lambda^2}\right) + \ldots\right] \bar{\mathcal{A}}(M^2/s) + \mathcal{O}(\lambda/M)$$

NNLL approximation ➪ no nontrivial dependence on λ/M

- *Compute in symmetric phase with* $\lambda = M$

$SU(2)\times U(1)$ **model**

Factorization of infrared singularities

$$\mathcal{A}_{f\bar{f}\to f'\bar{f}'} = \exp\left[-\frac{\alpha_e}{4\pi}(Q_f^2+Q_{f'}^2)\ln^2\left(\frac{s}{\lambda^2}\right)+\ldots\right]\bar{\mathcal{A}}(M^2/s)+\mathcal{O}(\lambda/M)$$

NNLL approximation ➪ no nontrivial dependence on λ/M

- *Compute in symmetric phase with* $\lambda = M$
- *Factorize* $\exp\left[-\frac{\alpha_e}{4\pi}(Q_f^2+Q_{f'}^2)\ln^2\left(\frac{s}{M^2}\right)+\ldots\right]$

$SU(2)\times U(1)$ model

Factorization of infrared singularities

$$\mathcal{A}_{f\bar{f}\to f'\bar{f}'} = \exp\left[-\frac{\alpha_e}{4\pi}(Q_f^2+Q_{f'}^2)\ln^2\left(\frac{s}{\lambda^2}\right)+\ldots\right]\bar{\mathcal{A}}(M^2/s)+\mathcal{O}(\lambda/M)$$

NNLL approximation ➪ no nontrivial dependence on λ/M

- *Compute in symmetric phase with* $\lambda = M$
- *Factorize* $\exp\left[-\frac{\alpha_e}{4\pi}(Q_f^2+Q_{f'}^2)\ln^2\left(\frac{s}{M^2}\right)+\ldots\right]$

N^{3}LL approximation ➪ $\xi^{(2)}(\lambda/M)$

$SU(2) \times U(1)$ model

$\boxed{\text{No-mixing } SU(2)_M \times U(1)_\lambda \text{ model}}$

- *Higgs boson of zero hypercharge, no mixing*

$SU(2) \times U(1)$ **model**

$\boxed{\text{No-mixing } SU(2)_M \times U(1)_\lambda \text{ model}}$

- *Higgs boson of zero hypercharge, no mixing*
- *Two-loop heavy-light interference term is identical to* $U(1)_M \times U(1)_\lambda$*model*

$SU(2) \times U(1)$ model

$\boxed{\text{No-mixing } SU(2)_M \times U(1)_\lambda \text{ model}}$

- *Higgs boson of zero hypercharge, no mixing*
- *Two-loop heavy-light interference term is identical to* $U(1)_M \times U(1)_\lambda$ *model*
- *For* $U(1)_M \times U(1)_\lambda$ *model* $\xi^{(2)} = 0$ (Feucht/Jantzen, Kühn, Penin, Smirnov)

  In fact $\xi^{(n)} = 0, \; n > 1$ ⇨ $\boxed{\textit{Bhabha}}$ (Penin)

$SU(2) \times U(1)$ model

$\boxed{\text{No-mixing } SU(2)_M \times U(1)_\lambda \text{ model}}$

- *Higgs boson of zero hypercharge, no mixing*
- *Two-loop heavy-light interference term is identical to* $U(1)_M \times U(1)_\lambda$*model*
- *For* $U(1)_M \times U(1)_\lambda$ *model* $\xi^{(2)} = 0$ (Feucht/Jantzen, Kühn, Penin, Smirnov)

  In fact $\xi^{(n)} = 0, \; n > 1$ ⇨ $\boxed{\textit{Bhabha}}$ (Penin)
- *For* $SU(2)_M \times U(1)_\lambda$ *model* $\xi^{(2)}_{\lambda=0} = \xi^{(2)}_{\lambda=M}$

naïve factorization of infrared logs holds to N ^{3}*LL*

$SU(2) \times U(1)$ **model**

$\boxed{\text{Two-loop log}^{4,3,2}}$

$SU(2) \times U(1)$ **model**

$\boxed{\text{Two-loop log}^{4,3,2}}$

- *Symmetric phase calculation,* $M_W = M_Z$

$SU(2) \times U(1)$ **model**

$\boxed{\text{Two-loop log}^{4,3,2}}$

- *Symmetric phase calculation,* $M_W = M_Z$
- *Naïve factorization of QED logs*

$SU(2) \times U(1)$ **model**

$\boxed{\text{Two-loop log}^{4,3,2}}$

- *Symmetric phase calculation,* $M_W = M_Z$
- *Naïve factorization of QED logs*
- $M_W \neq M_Z$ *through one-loop result* ⇨ *5% effect*

$SU(2) \times U(1)$ **model**

Two-loop $\log^{4,3,2}$

- *Symmetric phase calculation,* $M_W = M_Z$
- *Naïve factorization of QED logs*
- $M_W \neq M_Z$ *through one-loop result* ⇨ *5% effect*

Two-loop linear log

$SU(2) \times U(1)$ **model**

Two-loop $\log^{4,3,2}$

- *Symmetric phase calculation,* $M_W = M_Z$
- *Naïve factorization of QED logs*
- $M_W \neq M_Z$ *through one-loop result* ⇨ *5% effect*

Two-loop linear log

- *Approximation: Higgs boson of zero hypercharge,* $M_H = M_W = M_Z$

$SU(2) \times U(1)$ **model**

Two-loop log^{4,3,2}

- *Symmetric phase calculation,* $M_W = M_Z$
- *Naïve factorization of QED logs*
- $M_W \neq M_Z$ *through one-loop result* ⇨ *5% effect*

Two-loop linear log

- *Approximation: Higgs boson of zero hypercharge,* $M_H = M_W = M_Z$
- *No mixing* ⇨ *naïve factorization of infrared logs*

$SU(2) \times U(1)$ **model**

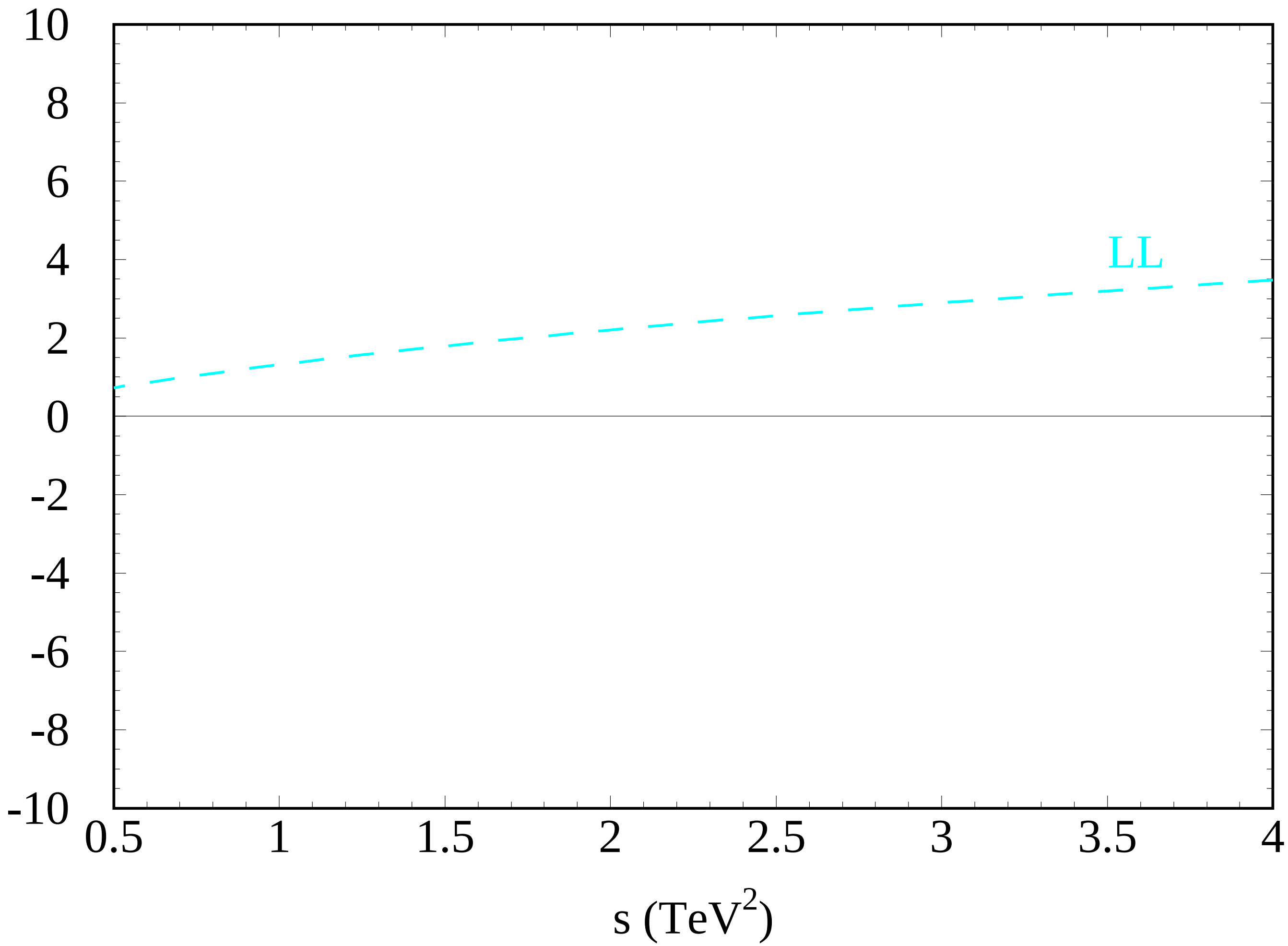
Two-loop $\log^{4,3,2}$

- *Symmetric phase calculation,* $M_W = M_Z$
- *Naïve factorization of QED logs*
- $M_W \neq M_Z$ *through one-loop result* ⇨ *5% effect*

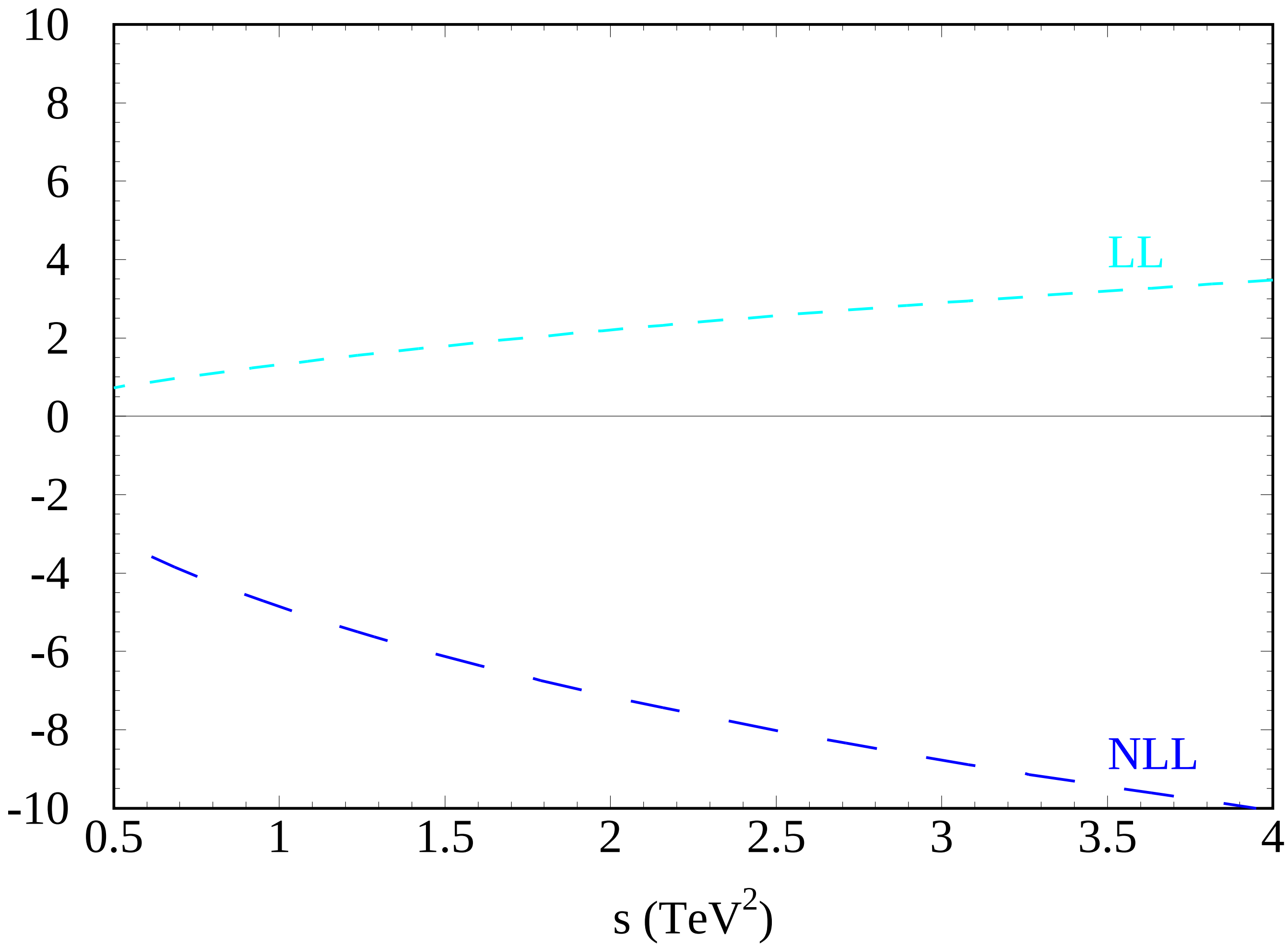
Two-loop linear log

- *Approximation: Higgs boson of zero hypercharge,* $M_H = M_W = M_Z$
- *No mixing* ⇨ *naïve factorization of infrared logs*
- *Mixing effects are suppressed by* $\sin^2\theta_W$ ⇨ *20% error,* $M_H \neq M_W$ *effect is negligible*

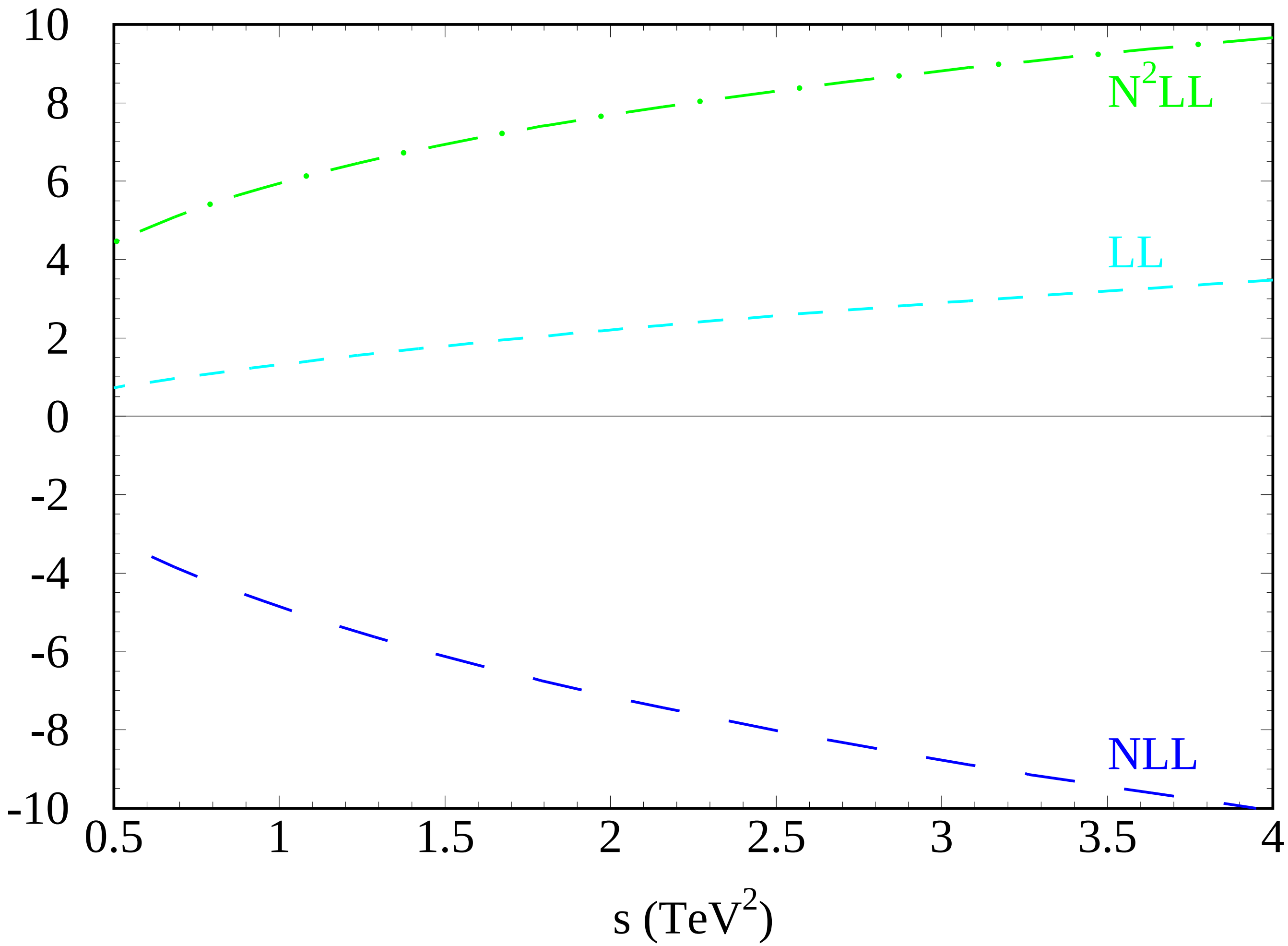
Two-loop corrections to $\sigma/\sigma_{\rm Born}(d\bar{d} \to l\bar{l})$



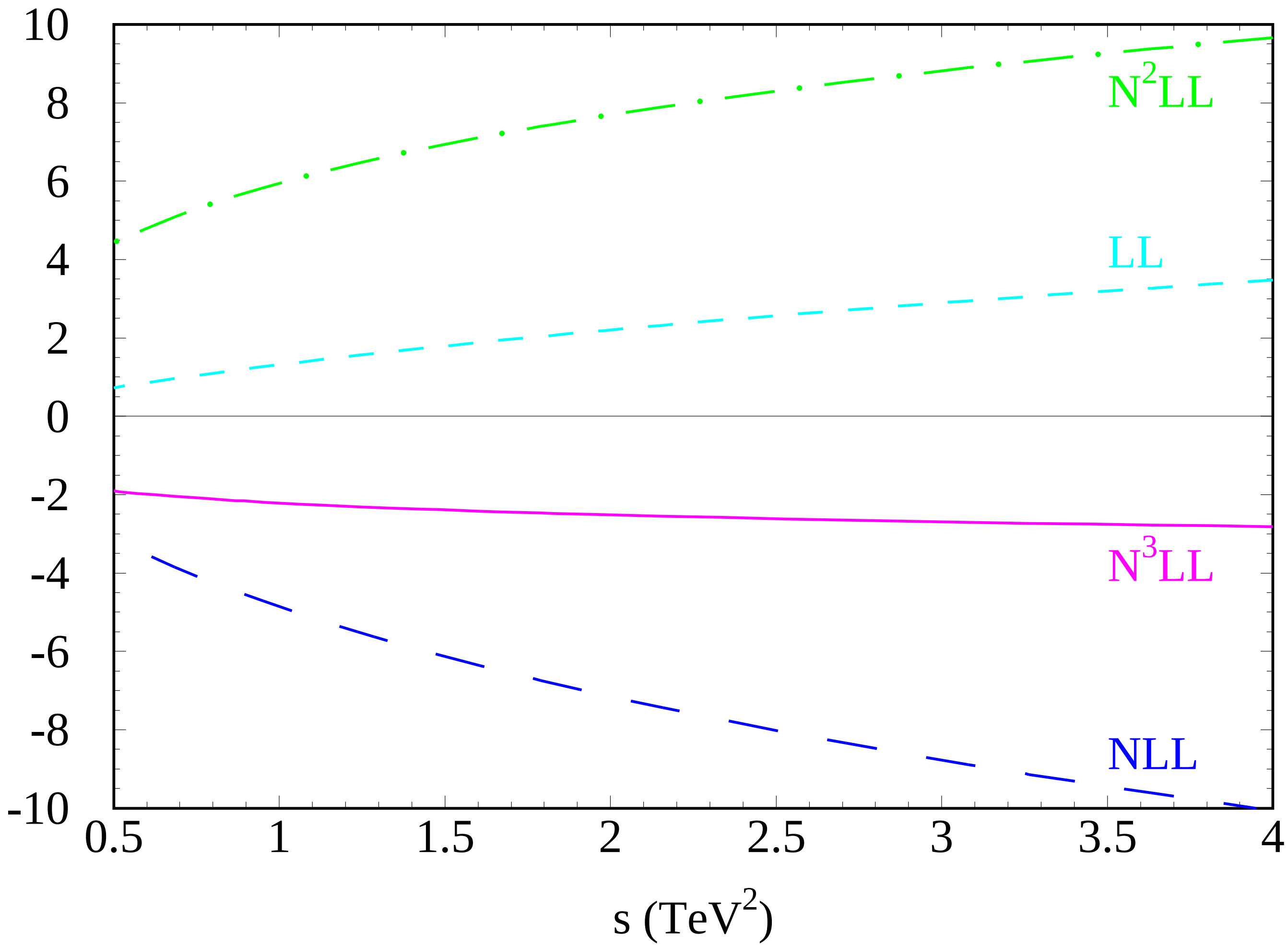
Two-loop corrections to $\sigma/\sigma_{\rm Born}(d\bar{d} \to l\bar{l})$



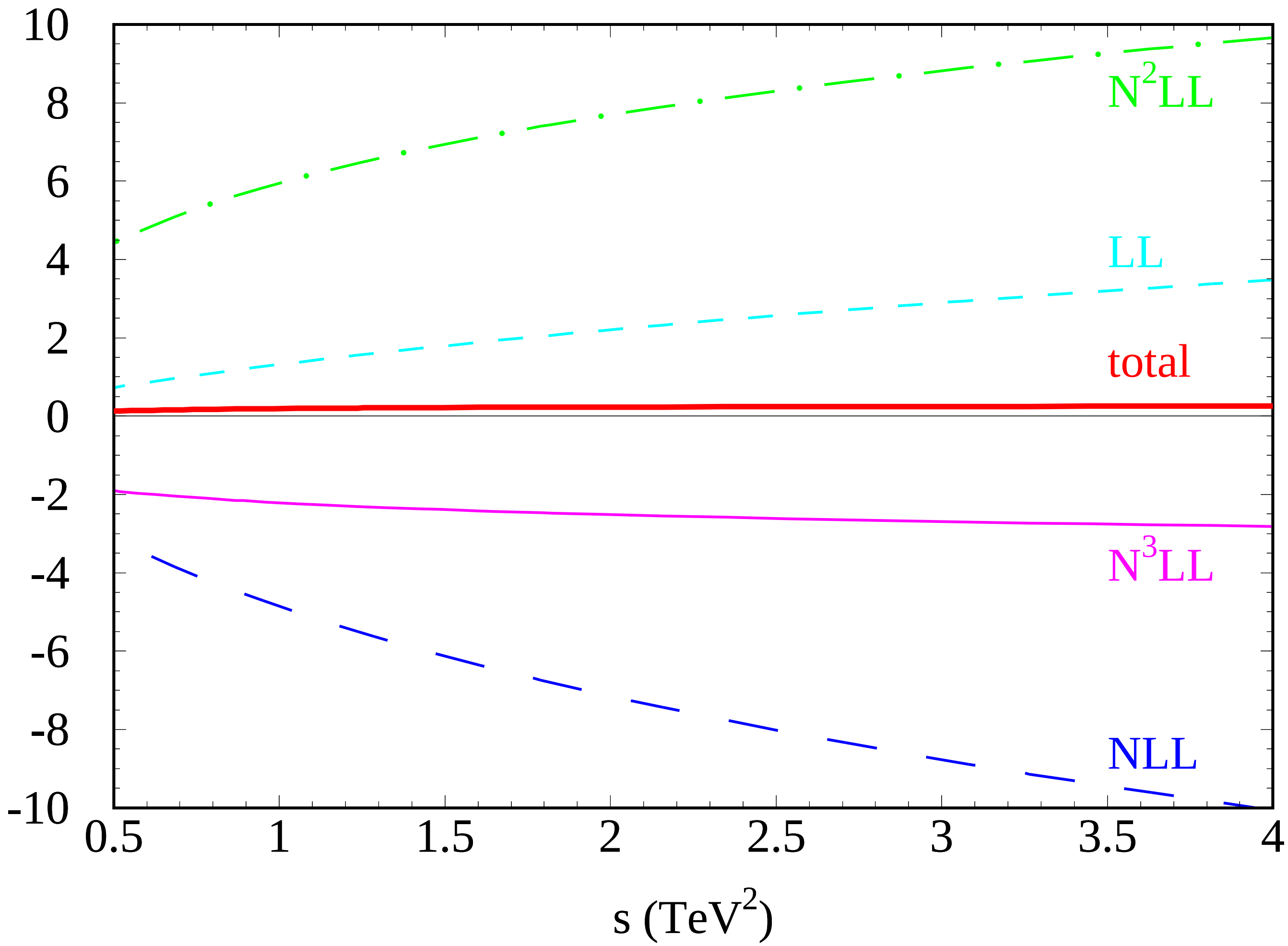
Two-loop corrections to $\sigma/\sigma_{\rm Born}(d\bar{d} \to l\bar{l})$



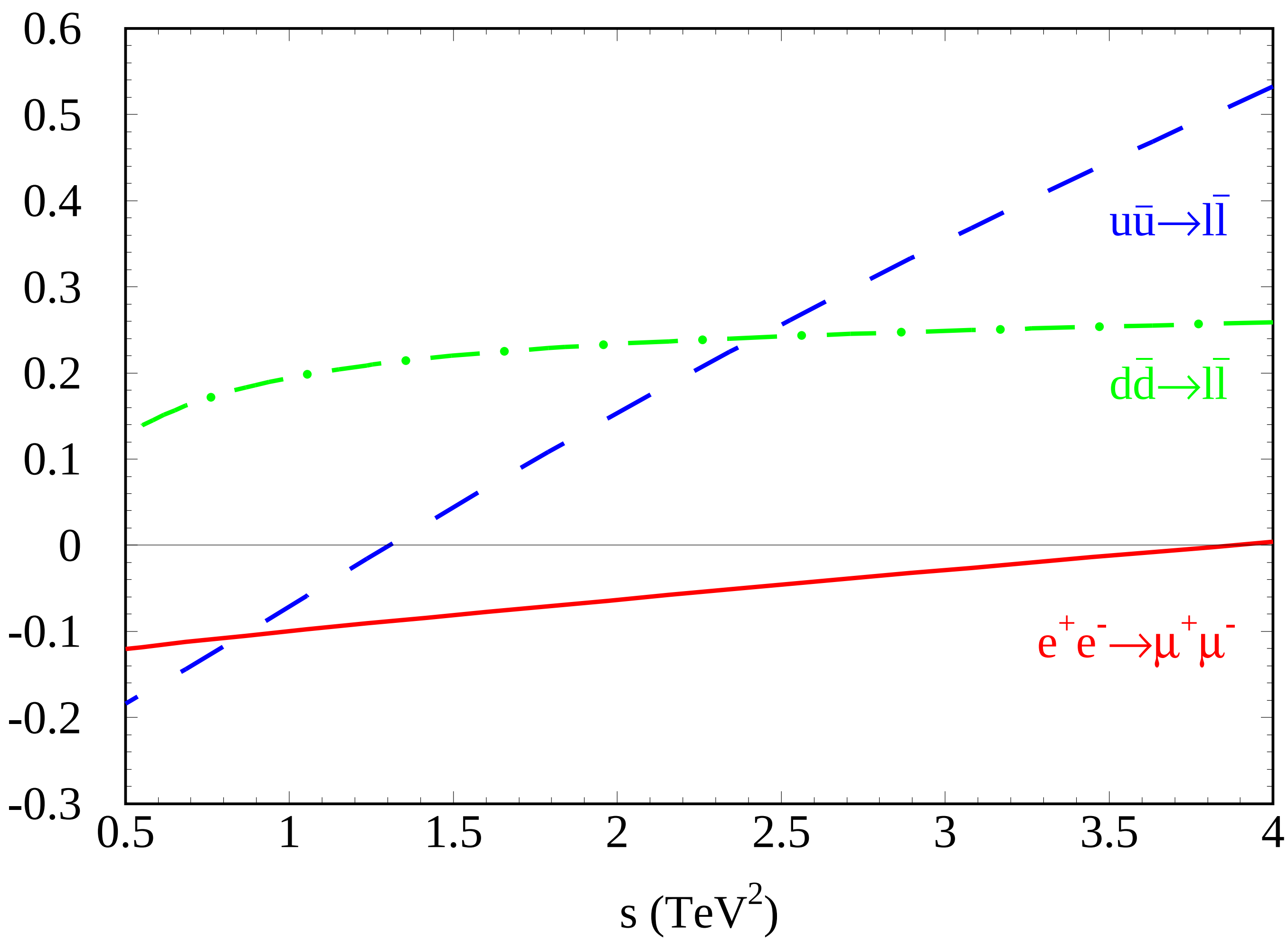
Two-loop corrections to $\sigma/\sigma_{\rm Born}(d\bar{d} \to l\bar{l})$



Two-loop corrections to $\sigma/\sigma_{\rm Born}(d\bar{d} \to l\bar{l})$



Two-loop corrections to $\sigma/\sigma_{\rm Born}(f\bar{f} \to f'\bar{f}')$



Summary

- For $\sigma(f\bar{f} \to f'\bar{f}')$ we have

Summary

- For $\sigma(f\bar{f} \to f'\bar{f}')$ we have
  - *two-loop quartic, cubic and quadratic logs*

Summary

- For $\sigma(f\bar{f} \to f'\bar{f}')$ we have
  - *two-loop quartic, cubic and quadratic logs*
  - *two-loop linear log with* 20% *error*

Summary

- For $\sigma(f\bar{f} \to f'\bar{f}')$ we have
  - *two-loop quartic, cubic and quadratic logs*
  - *two-loop linear log with 20% error*
  - *a few permill theoretical accuracy*

Summary

- For $\sigma(f\bar{f} \to f'\bar{f}')$ we have
  - *two-loop quartic, cubic and quadratic logs*
  - *two-loop linear log with* 20% *error*
  - *a few permill theoretical accuracy*
- Phenomenology

Summary

- For $\sigma(f\bar{f} \to f'\bar{f}')$ we have
  - *two-loop quartic, cubic and quadratic logs*
  - *two-loop linear log with 20% error*
  - *a few permill theoretical accuracy*
- Phenomenology
  - *Drell-Yan (LHC)*

Summary

- For $\sigma(f\bar{f} \to f'\bar{f}')$ we have
  - *two-loop quartic, cubic and quadratic logs*
  - *two-loop linear log with* 20% *error*
  - *a few permill theoretical accuracy*
- Phenomenology
  - *Drell-Yan (LHC)*
  - $e^+e^- \to \mu^+\mu^-$, *wide angle Bhabha (ILC)*

Summary

- For $\sigma(f\bar{f} \to f'\bar{f}')$ we have
  - *two-loop quartic, cubic and quadratic logs*
  - *two-loop linear log with* 20% *error*
  - *a few permill theoretical accuracy*
- Phenomenology
  - *Drell-Yan (LHC)*
  - $e^+e^- \to \mu^+\mu^-$*, wide angle Bhabha (ILC)*
- Problem to solve

Summary

- For $\sigma(f\bar{f} \to f'\bar{f}')$ we have
  - *two-loop quartic, cubic and quadratic logs*
  - *two-loop linear log with* 20% *error*
  - *a few permill theoretical accuracy*
- Phenomenology
  - *Drell-Yan (LHC)*
  - $e^+e^- \to \mu^+\mu^-$*, wide angle Bhabha (ILC)*
- Problem to solve
  - *mixed strong-electroweak corrections (Drell-Yan)*