

Light Quark Mediated Higgs + Jet Production at NNLO and beyond

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Topics discussed

- Introduction/warmup
 - *perturbative vs power corrections*
 - *light quark mediated $gg \rightarrow H$ through NLL*
- Light quark mediated Higgs+Jet production
 - *factorization structure of $gg \rightarrow Hg$*
 - *all-order amplitudes at LL*
 - *$pp \rightarrow H + j + X$ cross section*

Based on

*K. Melnikov, A.A. Penin, JHEP **05**, 172 (2016)*

*T. Liu, A.A. Penin, Phys.Rev.Lett. **119**, 262001 (2017)*

*C. Anastasiou, A.A. Penin, JHEP **07**, 195 (2020)*

*T. Liu, A.A. Penin, A. Reichman, JHEP **04**, 031 (2024)*

The advent of power corrections

● Power vs perturbative corrections

- Λ_X^2/Q^2 vs α_X^n

- e.g. $\Lambda_X = m_b$, $Q = m_H$, $\alpha_X = \alpha_s \Rightarrow n = 3$

● Logarithmically enhanced power corrections

- *phenomenologically relevant*

- *intriguing from QFT point of view*

- ➡ *weird RG structure, magic relations, etc.*

● Recent stream of the NLP results for

- *mass, angle, soft momentum, threshold, jetiness, ...*

Higgs production at the LHC

- Total cross section at 13 GeV

$$\sigma_{pp \rightarrow H+X} = 48.68 \text{ pb}$$

- Dominant theory uncertainties

- *scale choice* $+0.10$
 -1.15 pb

- *PDF N^3LO* $\pm 0.56 \text{ pb}$

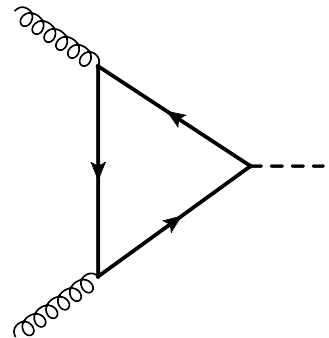
- $m_t < \infty$ *NNLO* $\pm 0.49 \text{ pb}$

- $m_b > 0$ *NNLO+* $\pm 0.40 \text{ pb}$

Anastasiou et al. JHEP **1605**, 058 (2016)

Bottom quark mass effect

● Leading contribution



A Feynman diagram showing a triangle loop of bottom quarks. Two gluon lines (represented by curly lines) enter the loop from the left, and a dashed line exits the loop to the right. The diagram is proportional to the following expression:

$$\propto \alpha_s \ln^2(m_H^2/m_b^2) \frac{m_b^2}{m_H^2}$$

➡ *large logs at subleading power*

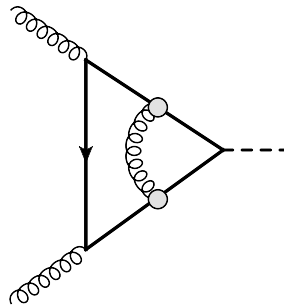
● *effective expansion parameter* $\alpha_s \ln^2(m_H^2/m_b^2) \sim 40\alpha_s$

➡ *resummation is mandatory*

$gg \rightarrow H$ amplitude at LL

● Non-Sudakov logs

T. Liu, A.A. Penin, Phys.Rev.Lett. **119**, 262001 (2017)



● Factorization formula

$$\mathcal{M}_{gg \rightarrow H}^b = Z_g^{2LL} g(z) \mathcal{M}_{gg \rightarrow H}^{b(0)}$$

● *gluon Sudakov factor* $Z_g^{2LL} = \exp \left[-\frac{C_A}{\varepsilon^2} \frac{\alpha_s}{2\pi} \frac{\mu^{2\varepsilon}}{Q^{2\varepsilon}} \right]$

● *non-Sudakov double logarithms*

$$g(z) = 2 \int_0^1 d\xi \int_0^{1-\xi} d\eta e^{2z\eta\xi} = {}_2F_2(1, 1; 3/2, 2; z/2)$$

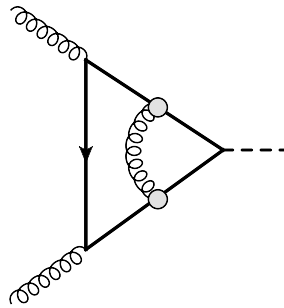
● *double-log variable* $z = (C_A - C_F) x$, $x = \frac{\alpha_s}{4\pi} L^2$, $L = \ln(m_H^2/m_q^2)$

eikonal color nonconservation

$gg \rightarrow H$ amplitude at LL

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T. Liu, A.A. Penin, Phys.Rev.Lett. **119**, 262001 (2017)



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➡ *Magic #1: same function for NLP QED scattering in Regge limit*

$gg \rightarrow H$ amplitude at NLL

C. Anastasiou, A.A. Penin, JHEP **07**, 195 (2020)

$$\mathcal{M}_{gg \rightarrow H}^{bNLL} = C_b \left(\frac{\alpha_s(m_H)}{\alpha_s(m_q)} \right)^{\gamma_m^{(1)}/\beta_0} Z_g^{2NLL} \left[-\frac{3}{2} \frac{m_q^2}{m_H^2} L^2 \mathcal{M}_{gg \rightarrow H}^{t(0)} \right]$$

Yukawa RG factor
gluon Sudakov form factor
LO amplitude

$$C_q = \left[g(z) + \frac{\alpha_s L}{4\pi} (2\gamma_q^{(1)} g_\gamma(z) - \beta_0 g_\beta(z)) \right] = 1 + \sum_{n=1}^{\infty} c_n$$

$$c_1 = \frac{z}{6} + C_F \frac{\alpha_s L}{4\pi}, \quad c_2 = \frac{z^2}{45} + \frac{z}{5} \frac{\alpha_s L}{4\pi} \left[\frac{3}{2} C_F - \beta_0 \left(\frac{5}{6} \frac{L_\mu}{L} - \frac{1}{3} \right) \right],$$

$$c_3 = \frac{z^3}{420} + \frac{z^2}{5} \frac{\alpha_s L}{4\pi} \left[\frac{5}{21} C_F - \beta_0 \left(\frac{2}{9} \frac{L_\mu}{L} - \frac{2}{21} \right) \right], \quad \dots$$

$$L = \ln(m_H^2/m_q^2), \quad L_\mu = \ln(m_H^2/\mu^2)$$

• *main NLL effects:*

$$\alpha_s(\mu) \rightarrow \alpha_s(m_H (\frac{m_q}{m_H})^{2/5})$$

$$m_q^2(\mu) \rightarrow m_q(m_q) m_q(m_H)$$

Higgs production

● Top-bottom interference through NNLO

● *NLL threshold*

$$\delta\sigma_{pp\rightarrow H+X}^{\text{NNLO}} = -2.18 \pm 0.20 \text{ pb}$$

C. Anastasiou, A.A. Penin, JHEP 07, 195 (2020)

● *full result*

$$\delta\sigma_{pp\rightarrow H+X}^{\text{NNLO}} = -1.99^{+0.30}_{-0.15} \text{ pb}$$

M. Czakon, F. Eschment, M. Niggetiedt, R. Poncelet, T. Schellenberger, 2312.09896 [hep-ph]

● Convergence of the logarithmic expansion

	LO	NLO	NNLO	N ³ LO
$\delta\sigma_{pp\rightarrow H+X}^{\text{LL}}$	-1.420	-1.640	-1.667	-1.670
$\delta\sigma_{pp\rightarrow H+X}^{\text{NLL}}$	-1.420	-2.048	-2.183	-2.204
$\delta\sigma_{pp\rightarrow H+X}$	-1.06	-1.64	-1.99	

Higgs production

● Top-bottom interference through NNLO

● *NLL threshold*

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C. Anastasiou, A.A. Penin, JHEP **07**, 195 (2020)

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● *Why does threshold approximation work so good?*

- *in principle it should not since the soft emission with $p_{\perp} \sim m_b$ is not suppressed by PDFs and resolves the bottom loop*

➡ *the reason is revealed in the rest of the talk*

Higgs + Jet production

● Top-bottom interference to NLO

J. M. Lindert, K. Melnikov, L. Tancredi and C. Wever, Phys. Rev. Lett. 118, 252002 (2017)

R. Bonciani, V. Del Duca, H. Frellesvig, M. Hidding, V. Hirschi, F. Moriello, G. Salvatori, G. Somogyi and F. Tramontano, Phys. Lett. B 843, 137995 (2023)

X. Chen, A. Huss, S. P. Jones, M. Kerner, J. N. Lang, J. M. Lindert and H. Zhang, JHEP 03, 096 (2022)

Higher orders?

Factorization, log structure?

● Brute force calculation of Abelian double logs

K. Melnikov, A.A. Penin, JHEP 05, 172 (2016)

➡ *full QCD?*

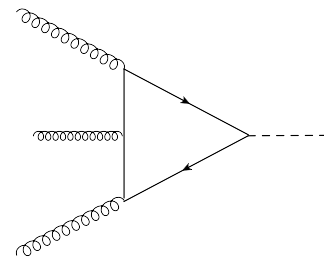
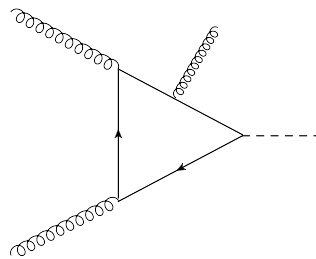
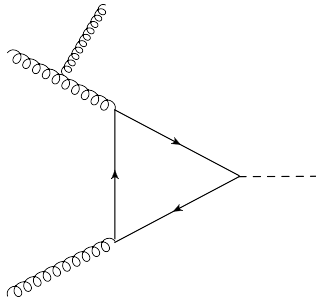
$gg \rightarrow Hg$ amplitudes

● Kinematics of $g(p_1) + g(p_2) \rightarrow g(p_3) + H(p_H)$

● $m_q^2 \ll p_\perp^2 \ll s, m_H^2$

● $s \approx m_H^2$

● One-loop amplitudes



● *single color structure*

● *two helicity structures*

$gg \rightarrow Hg$ amplitudes

● Amplitudes $M_{++\pm} \propto e^{\frac{\alpha_s}{2\pi} I} \sum_q A_{++\pm}^{(q)}$

● Form factors

● *large mass expansion* $A_{++\pm}^{(t)} = C_t \sum_{n=0} \left(\frac{\alpha_s}{2\pi}\right)^n \tilde{A}_{++\pm}^{(n)} + \mathcal{O}(m_t^{-2})$

● *small mass expansion* $A_{++\pm}^{(q)} = \frac{m_q^2}{s} \sum_{n=0} \left(\frac{\alpha_s}{2\pi}\right)^n A_{++\pm}^{(n+1)} + \mathcal{O}(m_q^4)$

● One-loop Sudakov exponent

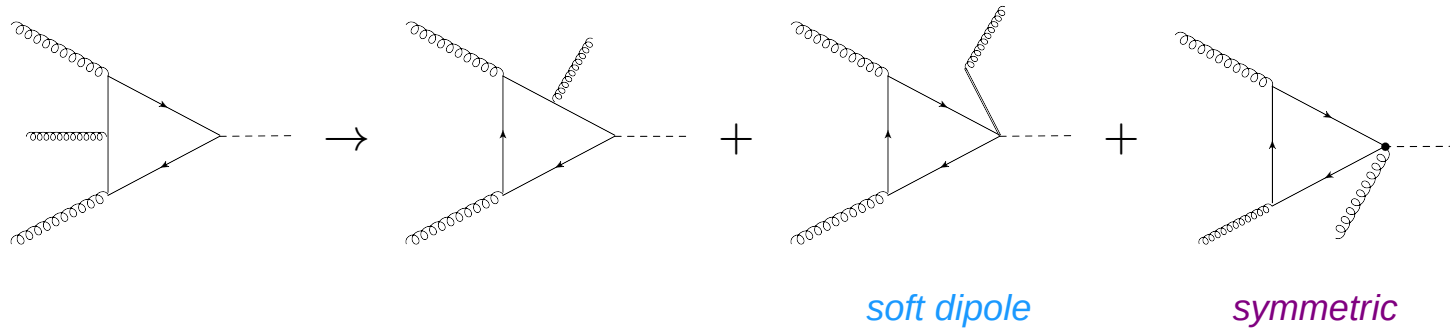
● *physical* $I_{\text{ph}}^{(1)} = -\frac{C_A}{2\epsilon^2} \left[2 \left(\frac{-s}{\mu^2}\right)^{-\epsilon} + \left(\frac{-tu}{s\mu^2}\right)^{-\epsilon} \right]$

➡ *accommodate all the double logs of $p_\perp$ and rapidity in $m_q \rightarrow \infty$ limit*

● *symmetric* $I_{\text{sym}}^{(1)} = -\frac{C_A}{2\epsilon^2} \left[\left(\frac{-s}{\mu^2}\right)^{-\epsilon} + \left(\frac{-t}{\mu^2}\right)^{-\epsilon} + \left(\frac{-u}{\mu^2}\right)^{-\epsilon} \right]$

Factorization

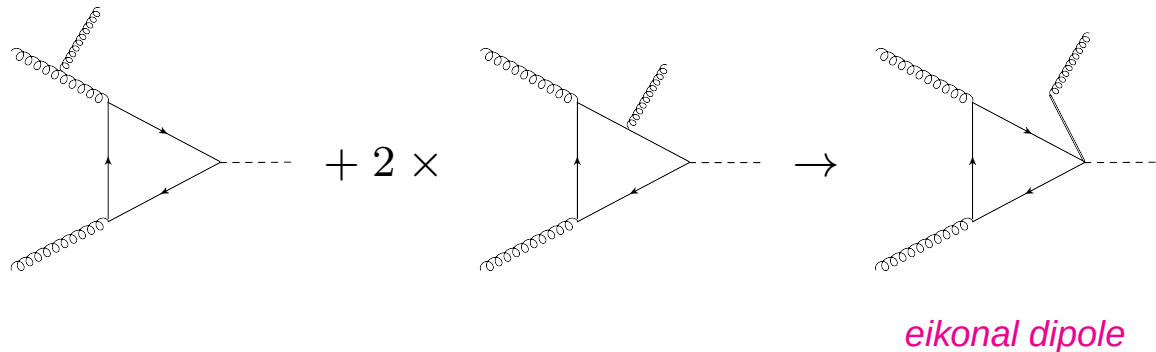
● Jet emission off the soft quark line



● *soft dipole with $l_{\perp}^2 < p_{\perp}^2$*

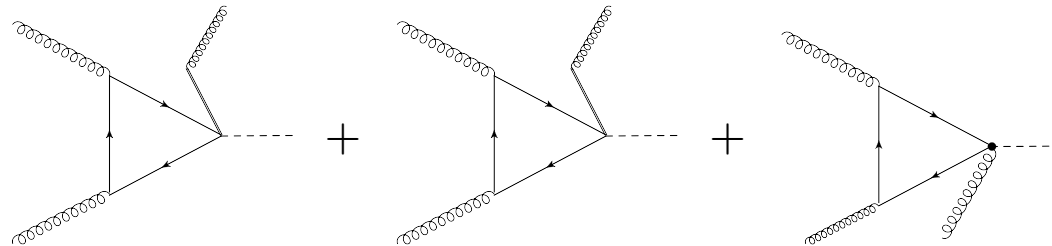
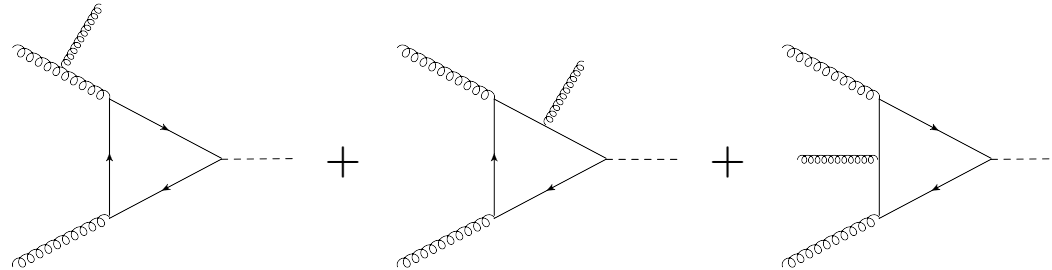
● *new symmetric structure $G_{\mu\nu}^a G_{\nu\lambda}^b G_{\lambda\mu}^c f^{abc}$ with $l_{\perp}^2 < p_{\perp}^2$*

● Jet emission off the eikonal line



Factorization

Effective theory decomposition



eikonal dipole

soft dipole

symmetric

Effective theory decomposition

● Eikonal dipole

- *standard dipole structure with $A_{+++} = -A_{++-}$*
- *jet completely factors out from the quark loop*

● Soft dipole

- *standard dipole structure with $A_{+++} = -A_{++-}$*
- *quark loop momentum cutoff $l_{\perp}^2 < p_{\perp}^2$*

● Symmetric

- *symmetric tensor structure with $A_{+++} = 0$*
- *quark loop momentum cutoff $l_{\perp}^2 < p_{\perp}^2$*

Double logs

- Soft emission from the factorized gluon line

- *color is conserved along gluon line*

- ➡ *only Sudakov double logs*

- Problem reduces to

- *eikonal dipole:* $g(p_1)g(p_2)H$ *form factor*

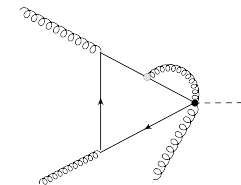
- *soft dipole:* $g(p_1)g(p_2)H$ *form factor with $p_\perp$ -dependent UV cutoff*

- *symmetric:*

- *unresolved vertex:* $g(p_3)g(p_{1,2})H$ *form factor with $p_\perp$ -dependent UV cutoff*

- *resolved vertex: additional Sudakov-Wilson contribution*

- ➡ *soft jet factors out*



Double logs

● Eikonal dipole

$$\left[A_{++\pm}^{\text{LL}}\right]_{\text{e.d.}} = \pm 2L^2 \int_0^1 d\eta \int_0^{1-\eta} d\xi e^{2z\eta\xi} = \pm L^2 g(z)$$

● Soft dipole

$$\left[A_{++\pm}^{\text{LL}}\right]_{\text{s.d.}} = \mp L^2 \int_{\tau_t - \tau}^{\tau_t} d\eta \int_{1-\tau_t}^{1-\eta} d\xi e^{2z\eta\xi}$$

● *transverse momentum variable* $\tau = \ln(p_\perp^2/m_q^2)/L$

● Symmetric

$$\left[A_{++-}^{\text{LL}}\right]_{\text{sym.}} = -L^2 \left[\int_{\tau_t - \tau}^{\tau_t} d\eta \int_{1-\tau_t}^{1-\eta} d\xi e^{2z(\eta - \tau_t + \tau)(\xi - 1 + \tau_t)} e^{2z(\tau_t - \tau)\xi} + (\tau_t \rightarrow \tau_u) \right]$$

Sudakov-Wilson factor



All-order amplitudes

● Leading logarithmic result

$$A_{+++}^{LL} = L^2 \left[g(z) - \int_{(1-\tau+\zeta)/2}^{(1+\tau+\zeta)/2} \frac{d\eta}{2z\eta} \left(e^{2z\eta(1-\eta)} - e^{z\eta(1-\tau-\zeta)} \right) \right]$$

$$A_{++-}^{LL} = -A_{+++}^{LL} - L^2 \left[\int_{(1-\tau+\zeta)/2}^{(1+\tau+\zeta)/2} \frac{d\eta}{2z\eta} e^{z((1-\tau)^2 - \zeta^2)/2} \left(e^{z\eta(1+\tau+\zeta-2\eta)} - 1 \right) + (\zeta \rightarrow -\zeta) \right]$$

● *rapidity variable* $\zeta = \ln(t/u)/L$

● Two loops

● *after infrared matching agrees with the explicit calculation*

K. Melnikov, L. Tancredi, C. Wever, JHEP 1611, 104 (2016)

All-order amplitudes

● Leading logarithmic result

$$A_{++++}^{LL} = L^2 \left[g(z) - \int_{(1-\tau+\zeta)/2}^{(1+\tau+\zeta)/2} \frac{d\eta}{2z\eta} \left(e^{2z\eta(1-\eta)} - e^{z\eta(1-\tau-\zeta)} \right) \right]$$

$$A_{++-}^{LL} = -A_{++++}^{LL} - L^2 \left[\int_{(1-\tau+\zeta)/2}^{(1+\tau+\zeta)/2} \frac{d\eta}{2z\eta} e^{z((1-\tau)^2 - \zeta^2)/2} \left(e^{z\eta(1+\tau+\zeta-2\eta)} - 1 \right) + (\zeta \rightarrow -\zeta) \right]$$

● *rapidity variable* $\zeta = \ln(t/u)/L$

● Boundary values

● *low transverse momentum* $p_{\perp} \sim m_q$: $A_{++\pm}^{LL} = \pm L^2 g(z)$

● *high transverse momentum, central rapidity* $p_{\perp} \sim m_H$, $t \sim u$:

$$A_{++++}^{LL} = \frac{1}{2} L^2 g(z), \quad A_{++-}^{LL} = -\frac{3}{2} L^2 g(z)$$

Higgs + Jet production

● Partonic cross section near threshold

$$d\sigma_{gg \rightarrow Hg+X}^{tb} = -\frac{3m_q^2}{m_H^2} L^2 C_t C_q(\tau, \zeta) d\tilde{\sigma}_{gg \rightarrow Hg+X}^{\text{eff}}$$

heavy top effective theory cross section

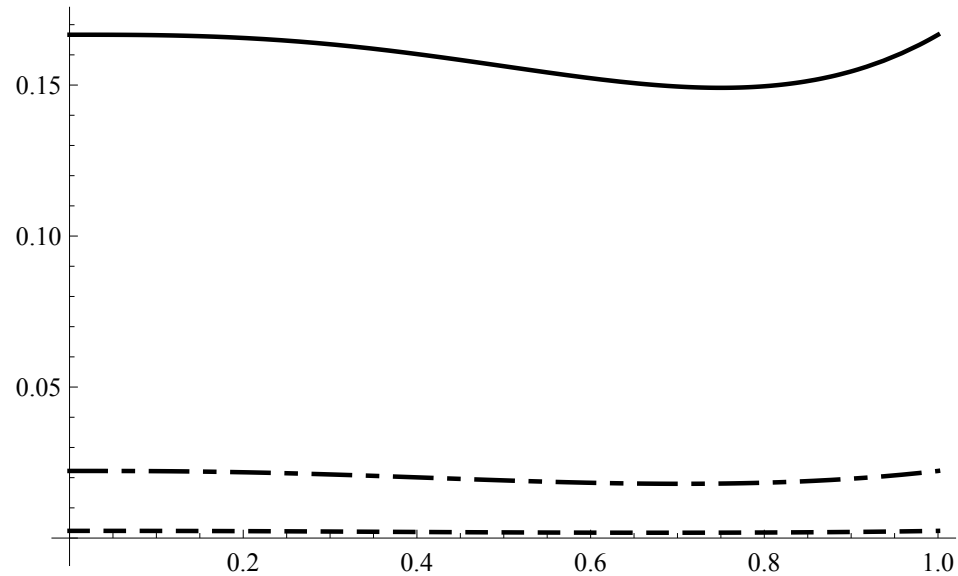
● *light quark coefficient*

$$C_q(\tau, \zeta) = \frac{A_{+++} - A_{++-}}{2L^2} = 1 + \frac{z}{6} (1 - \tau^3 + \tau^4) \\ + z^2 \left[\frac{1}{45} - \frac{\tau^3}{12} + \frac{\tau^4}{6} - \frac{7\tau^5}{60} + \frac{\tau^6}{30} + \frac{\zeta^2}{12} (\tau^3 - \tau^4) \right] + \dots$$

● *Magic # 2:* $C_q(\tau, \zeta) \approx C_q(0, 0) = C_q$

Jet factorization

dependence of the $z^{1,2,3}$ coefficients on $m_q < p_\perp < m_H$



➡ a jet with $m_q < p_\perp < m_H$ factors out as a soft jet with $p_\perp \ll m_q$

Higgs + Jet production

- Bottom quark contribution to hadronic cross section

$$d\sigma_{pp \rightarrow H j + X}^{tb} = \left[\frac{C_b}{C_t} \left(\frac{\alpha_s(m_H)}{\alpha_s(m_b)} \right)^{\gamma_m^{(1)}/\beta_0} \right] \left(\frac{d\sigma_{pp \rightarrow H j + X}^{tb}}{d\sigma_{pp \rightarrow H j + X}^{tt}} \right)^{\text{LO}} d\sigma_{pp \rightarrow H j + X}^{tt}.$$

- *numerically observed in NLO*

J. M. Lindert, K. Melnikov, L. Tancredi and C. Wever, Phys. Rev. Lett. 118, 252002 (2017)

- *explains the precision of the NLL total threshold cross section*

C. Anastasiou, A.A. Penin, JHEP 07, 195 (2020)

- *C_b is known to NLL*

Higgs + Jet production

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$$d\sigma_{pp \rightarrow H j + X}^{tb} = \left[\frac{C_b}{C_t} \left(\frac{\alpha_s(m_H)}{\alpha_s(m_b)} \right)^{\gamma_m^{(1)}/\beta_0} \right] \left(\frac{d\sigma_{pp \rightarrow H j + X}^{tb}}{d\sigma_{pp \rightarrow H j + X}^{tt}} \right)^{\text{LO}} d\sigma_{pp \rightarrow H j + X}^{tt}.$$

● $d\sigma_{pp \rightarrow H j + X}^{tt}$ is known to NNLO

X. Chen, T. Gehrmann, E. W. N. Glover and M. Jaquier, Phys. Lett. B 740, 147 (2015)

R. Boughezal, F. Caola, K. Melnikov, F. Petriello, M. Schulze, Phys.Rev.Lett. 115, 082003 (2015)

R. Boughezal, C. Focke, W. Giele, X. Liu and F. Petriello, Phys. Lett. B 748, 5 (2015)

F. Caola, K. Melnikov and M. Schulze, Phys. Rev. D 92, 074032 (2015)

X. Chen, J. Cruz-Martinez, T. Gehrmann, E. W. N. Glover and M. Jaquier, JHEP 10, 066 (2016)

➡ *get top-bottom interference through NNLO*

Higgs + Jet production

● Bottom quark contribution to hadronic cross section

$$d\sigma_{pp \rightarrow H j + X}^{tb} = \left[\frac{C_b}{C_t} \left(\frac{\alpha_s(m_H)}{\alpha_s(m_b)} \right)^{\gamma_m^{(1)}/\beta_0} \right] \left(\frac{d\sigma_{pp \rightarrow H j + X}^{tb}}{d\sigma_{pp \rightarrow H j + X}^{tt}} \right)^{\text{LO}} d\sigma_{pp \rightarrow H j + X}^{tt}.$$

● *higher multiplicities* $\Leftrightarrow$ *softer emissions* $\Leftrightarrow$ *better factorization*

➡ *complete the existing $p_\perp$ -resummations*

A. Banfi, P. F. Monni and G. Zanderighi, JHEP 01, 097 (2014)

F. Caola, J. M. Lindert, K. Melnikov, P. F. Monni, L. Tancredi and C. Wever, JHEP 09, 035 (2018)

Summary

Jets with $p_{\perp} \ll m_H$ decouple from the light quark loop

- explains some mystique phenomena
- provides access to higher orders