Effective Field Theory of Unstable Top

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Theory Seminar

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Topics discussed

- Top-antitop threshold production
  - *brief introduction and review*

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- Top width effect:
  - *beyond the complex energy shift*
  - *effective theory of unstable particles "ρNRQCD"*
  - *unstable top production in NLO and NNLO*
  - *spurious divergences in pNRQCD*

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- Top width effect:
  - *beyond the complex energy shift*
  - *effective theory of unstable particles "ρNRQCD"*
  - *unstable top production in NLO and NNLO*
  - *spurious divergences in pNRQCD*
- Based on: *A. Penin, J. Piclum, JHEP 1201 (2012) 034*

Top-antitop threshold production at the ILC

Would be an ideal place to study the top

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  - *Allows for the first principle QCD predictions*
  - *High order results are available (NNNLO is coming!)*

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  - $m_t,\ \alpha_s,\ \Gamma_t,\ y_t,\ M_H$

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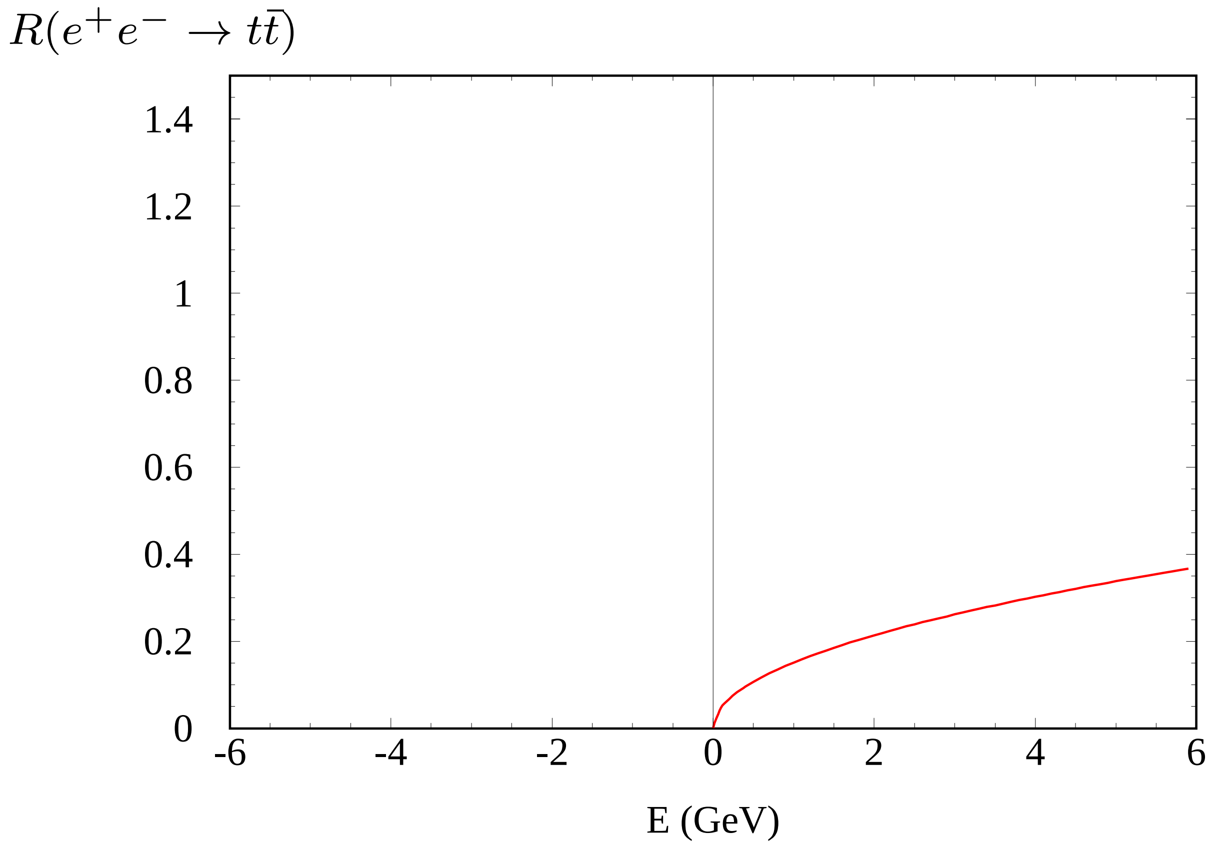
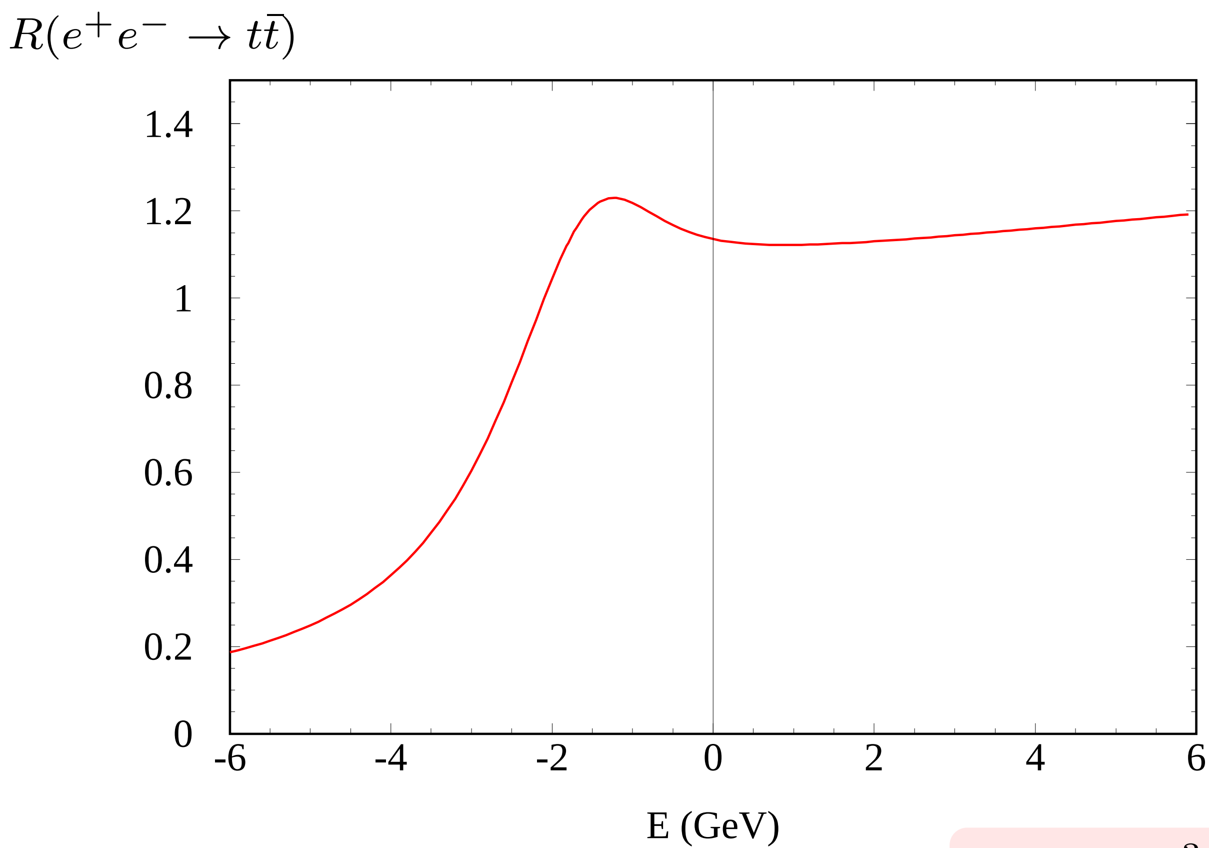
Higgs, EWSB, GUT, SUSE, ...

Born cross section



Coulomb and finite width effects



$$R_{\text{res}} \sim \frac{\alpha_s^3}{m_t \Gamma_t}, \quad E_{\text{res}} \sim \alpha_s^2 m_t$$

Perturbation theory for heavy quarkonium

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  - *Renormalons* $n!(\beta_0\alpha_s)^n$
  - *Threshold logs* $\alpha_s^n \ln^m \alpha_s$

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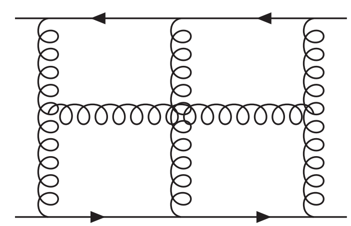
➥ $\boxed{\textit{Full N}^{\mathbf{3}}\textit{LO analysis is mandatory}}$

$\mathbf{N^3LO}$ ground state energy

(A. Penin, M. Steinhauser Phys.Lett. B538 (2002) 335-345)

$$\frac{\delta E_1^{\rm N^3LO}}{E_1^{\rm LO}} = \alpha_s^3 \left(58.205 + 15.297\ln(\alpha_s) + \boxed{26.654}\right)$$

Renormalon contribution



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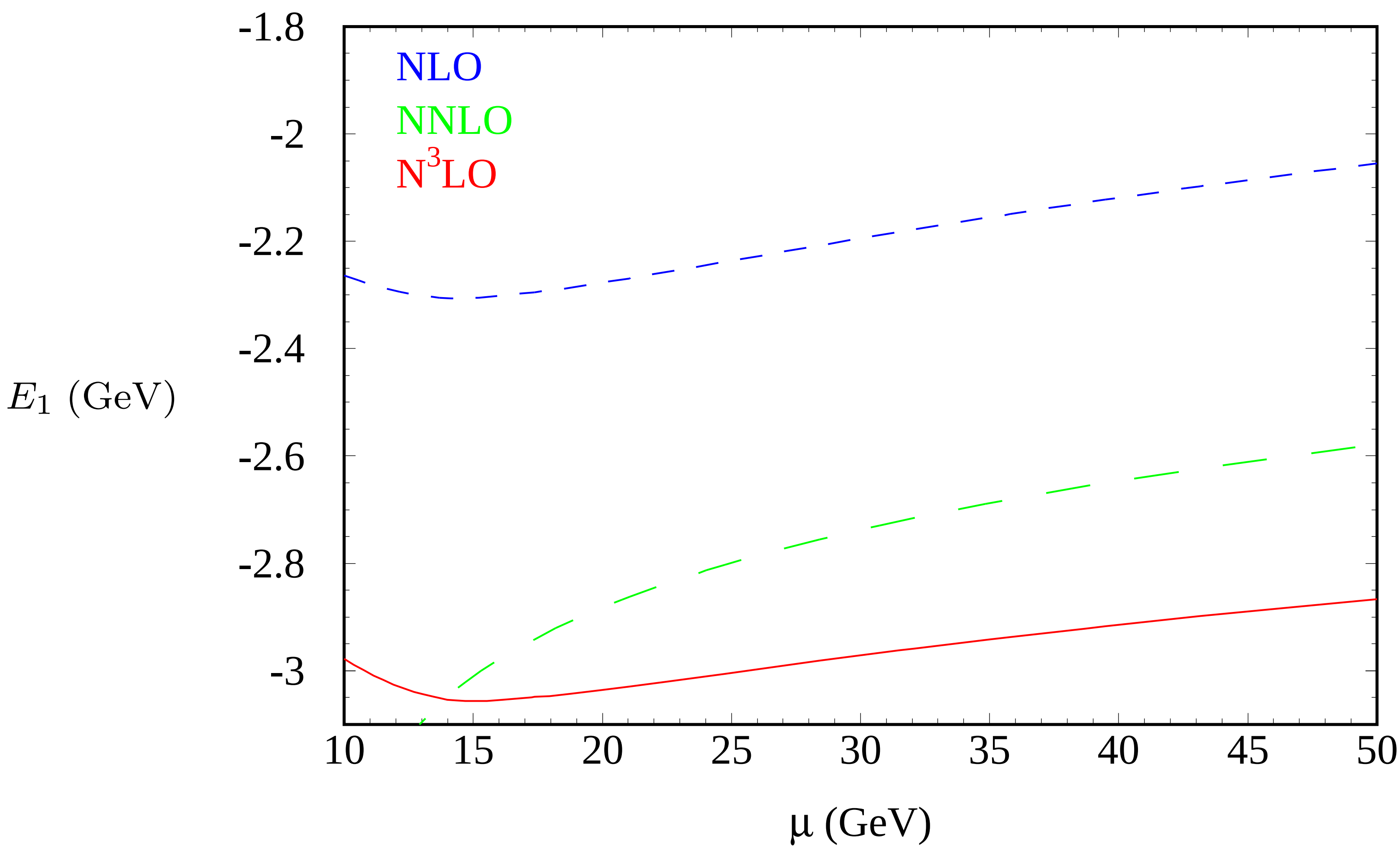

Finite top lifetime

- Resonant approximation
  - *complex energy shift* $E \to E + i\Gamma_t$

    (V.Fadin, V.Khoze, JETP Lett. 46 (1987) 525)
  - *not consistent in pNRQCD beyond LO!*

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- Nonresonant contribution
  - *Supression of nonfactorizible contribution*

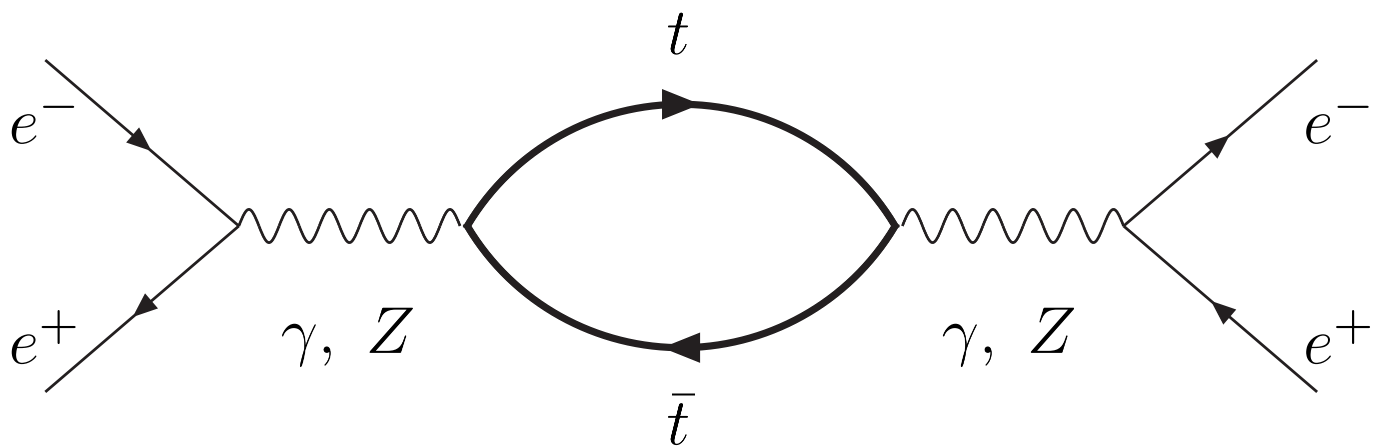
    (K. Melnikov, O. Yakovlev, Phys. Lett. B324 (1994) 217)
  - *Phase space matching (tight cuts on top invariant mass)*

    (A. Hoang, C. Reißer, P. Ruiz-Femenía, Phys. Rev. D82 (2010) 014005)
  - *QCD effective theory of unstable particles to NLO*

    (M. Beneke, B. Jantzen, P. Ruiz-Femenía, Nucl. Phys. B840 (2010) 186)
  - *NRQCD effective theory of unstable particles to NNLO*

    (A. Penin, J. Piclum, JHEP 1201 (2012) 034)

Stable top



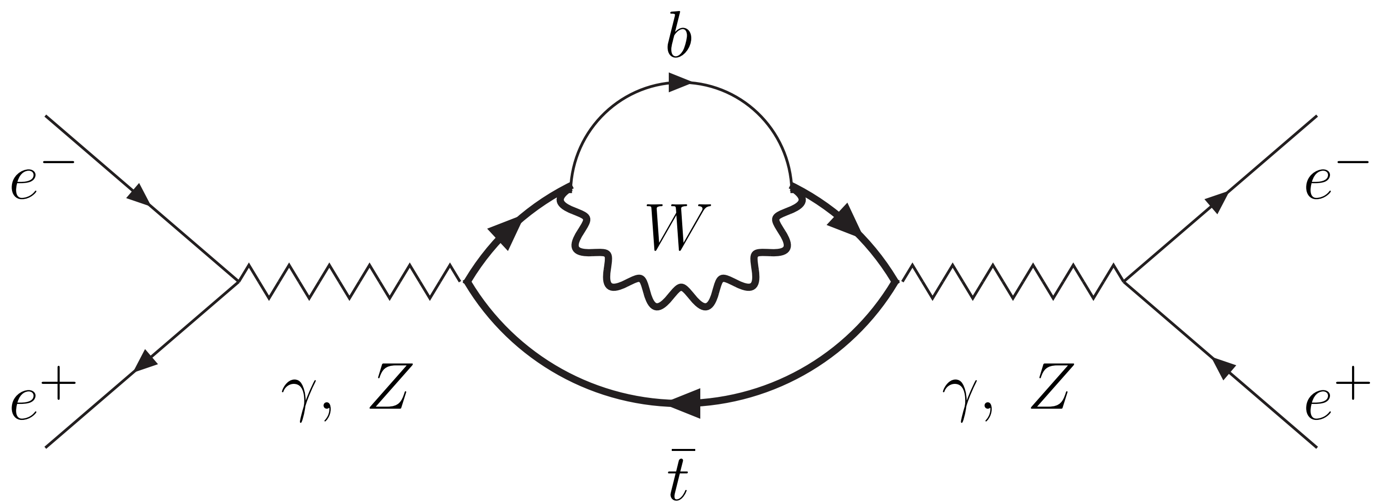
Optical theorem:

$$R_{res}^{Born} \sim \Im \int \frac{\mathrm{d}^3\boldsymbol{p}}{(2\pi)^3} \frac{1}{\boldsymbol{p}^2 - m_t E - i\epsilon} \sim \Im\sqrt{-E - i\epsilon}\,,$$

On-shell top:

$$\Im\left[\frac{1}{\boldsymbol{p}^2 - m_t E - i\epsilon}\right] \sim \delta(\boldsymbol{p}^2 - m_t E)\,,$$

Unstable top

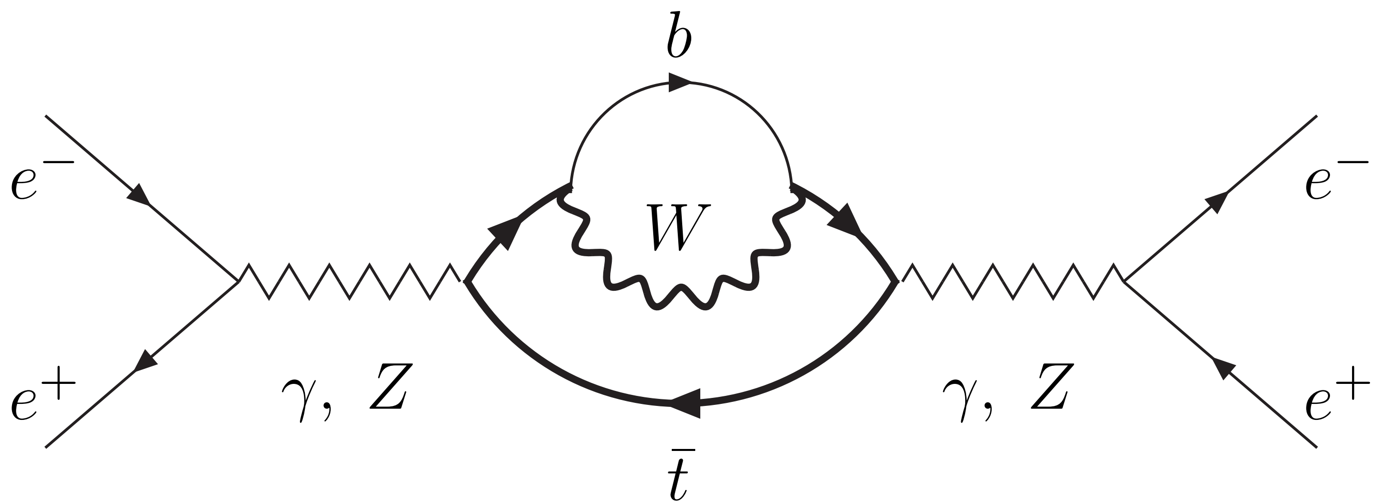


Imaginary part of mass operator:

here $\rho = 1 - M_W/m_t,\ z = (\boldsymbol{p}^2 - m_t E)/m_t^2 \ll 1$

$$\Im[\Sigma(z)] = \frac{\Gamma_t}{2} - \frac{\Gamma_t}{2}\left[\theta(z-\rho) + \left(\frac{2z}{\rho} - \frac{z^2}{\rho^2}\right)\theta(\rho - z) + \mathcal{O}(\rho, z)\right]$$

Unstable top



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Resonant contribution *Nonresonant contribution*

Resonant contribution

- Complex energy shift:
  - *Dyson resummation*

$$\frac{1}{\boldsymbol{p}^2 - m_t E - i\epsilon} \to \frac{1}{\boldsymbol{p}^2 - m_t E - i m_t \Gamma_t}$$

  - *Breit-Wigner resonance*

$$\delta(\boldsymbol{p}^2 - m_t E) \to \frac{1}{\pi} \frac{\Gamma_t}{(\boldsymbol{p}^2/m_t - E)^2 + \Gamma_t^2} \,,$$

  - *Born cross section*

$$R_{res}^{Born} \sim \Im\left[\sqrt{-E - i\Gamma_t}\right]$$

- Invariant mass distribution:
  - $$2\boldsymbol{p}^2 \approx m_t^2 - (p_W + p_b)^2$$

Nonresonant contribution

- On-shell t ⇨ on-shell W and b
  - *kinematical constraint* $M_W^2 < (p_W + p_b)^2 < m_t^2$
  - *natural cutoff on spatial momentum* $0 < \boldsymbol{p}^2 < \rho m_t^2$
  - $\Im[\Sigma] - \Gamma_t/2 \neq 0$ *for* $\boldsymbol{p}^2 \neq 0$ ⇨ *"nonresonant"*

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- Approximation $\rho \ll 1$
  - *nonrelativistic* t *and* W*, ultrarelativistic* b
  - *expansion in* ρ *similar to pNRQCD expansion in* $v^2 \sim E/m_t$
  - *actual value* $\rho = 0.53\ldots$

Nonrelativistic effective theory of unstable top

- Scales
  - *pNRQCD:*

hard	m_t	*soft*	vm_t	*ultrasoft*	v^2m_t

  - *ρNRQCD:*

hard	m_t	*ρ-soft*	$\rho^{1/2}m_t$	*ρ-ultrasoft*	ρm_t

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  - *pNRQCD:*

    hard m_t *soft* vm_t *ultrasoft* $v^2 m_t$

  - *ρNRQCD:*

    hard m_t *ρ-soft* $\rho^{1/2} m_t$ *ρ-ultrasoft* ρm_t

- Scale hierarchy and power counting
  - *pNRQCD scaling:* $\alpha_{ew}^{1/2} \sim \alpha_s \sim v \ll 1, \quad \Gamma_t/m_t \sim \alpha_{ew}$
  - *complimentary expansion in ρ with* $v \ll \rho^{1/2} \ll 1$
  - *ρ-Coulomb terms* $\alpha_s/\rho^{1/2} \ll 1$

Nonrelativistic effective theory of unstable top

- How to expand?
  - *ρ-pNRQCD Feynman rules*
  - *expansion by regions*

Nonrelativistic effective theory of unstable top

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  ✓ *expansion by regions*

NLO nonresonant contribution

- Power counting
  - *resonant contribution* $\Im\sqrt{-E - i\Gamma_t} \sim v$
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- Calculation steps
  - *treat* $\Im[\Sigma]-\Gamma_t/2$ *as a perturbation*
  - *add all the two-loop diagrams with* t-W-b *cut*
  - *expand in* $E/\rho m_t$, *expand in* ρ ⇨ *single region left:* t *and* W *are* ρ-*potential,* b *is* ρ-*ultrasoft*

↪ *recover nonrelativistic propagators and vertices*

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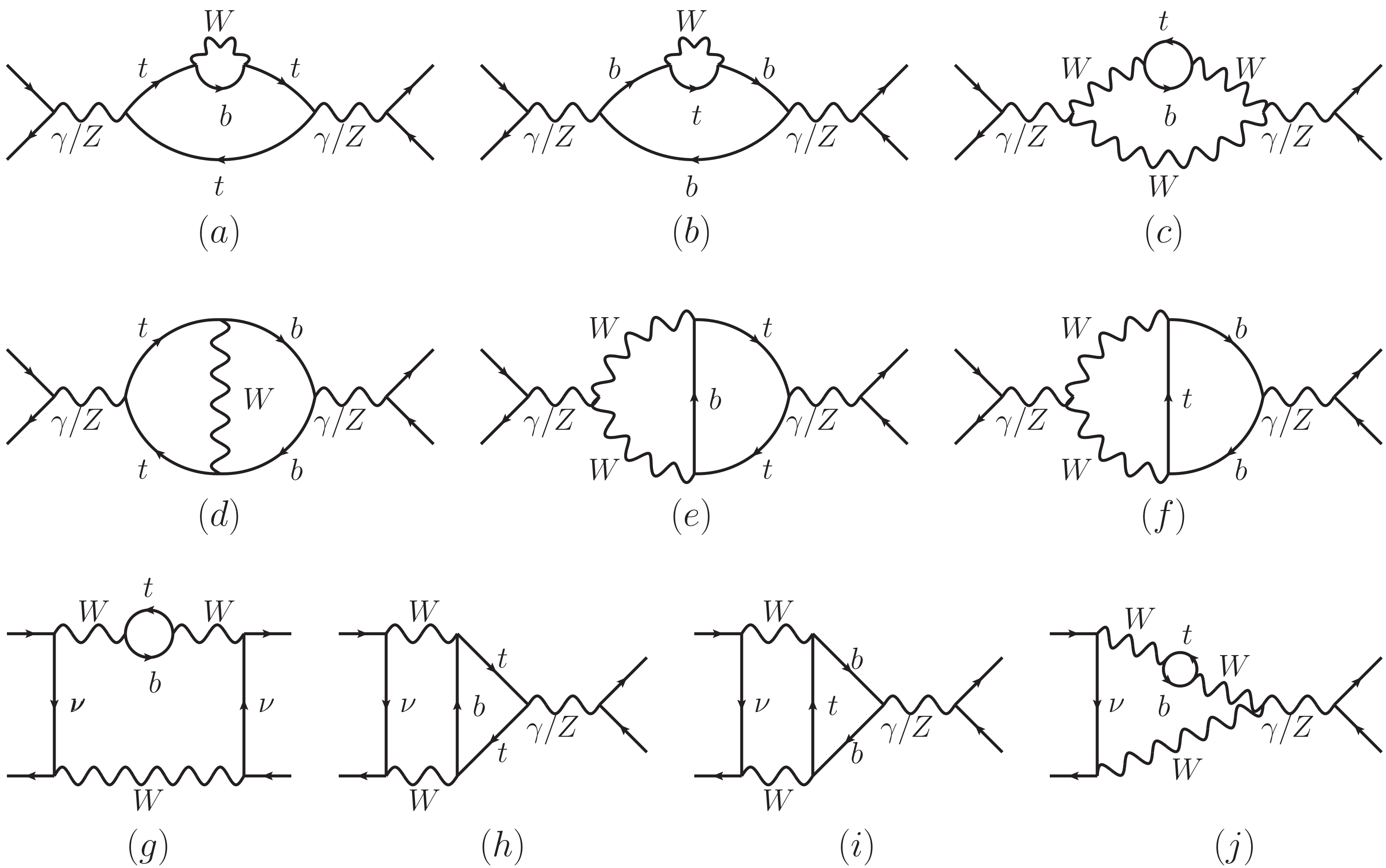
  ↪ *recover nonrelativistic propagators and vertices*
- *no expansion in* ρ ⇨ *fully relativistic calculation* *(M. Beneke et al.)*

NLO diagrams



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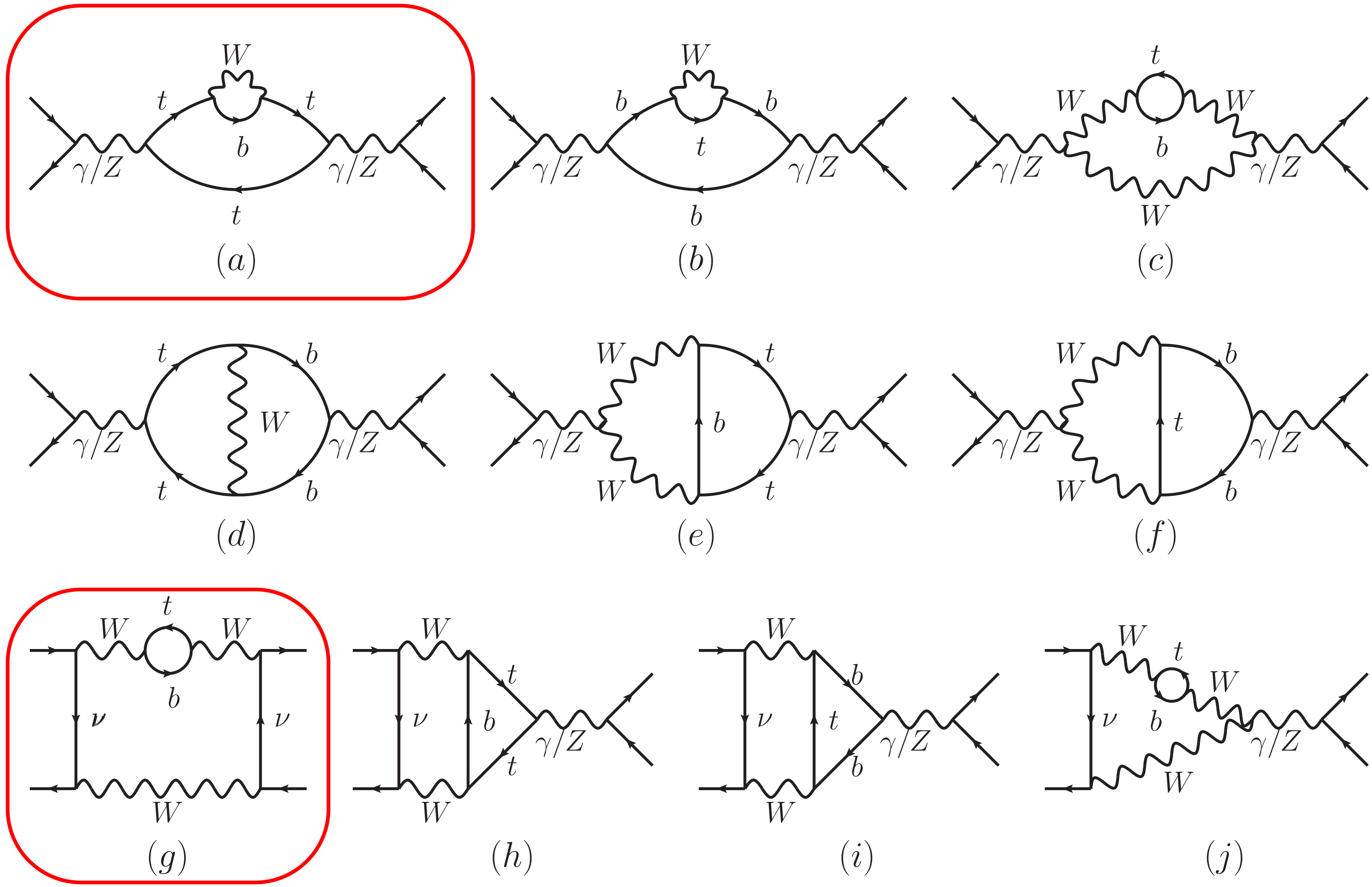

NLO result

- leading term of ρ-expansion

$$R_{nr}^{NLO} = -\frac{24}{\pi\rho^{1/2}}\frac{\Gamma_t}{m_t}\left[\frac{4}{9} + \text{“}Z\text{”} - \frac{1}{\sin^4\theta_W}\left(\frac{17}{48} - \frac{9\sqrt{2}}{32}\ln\left(1+\sqrt{2}\right)\right)\right]$$

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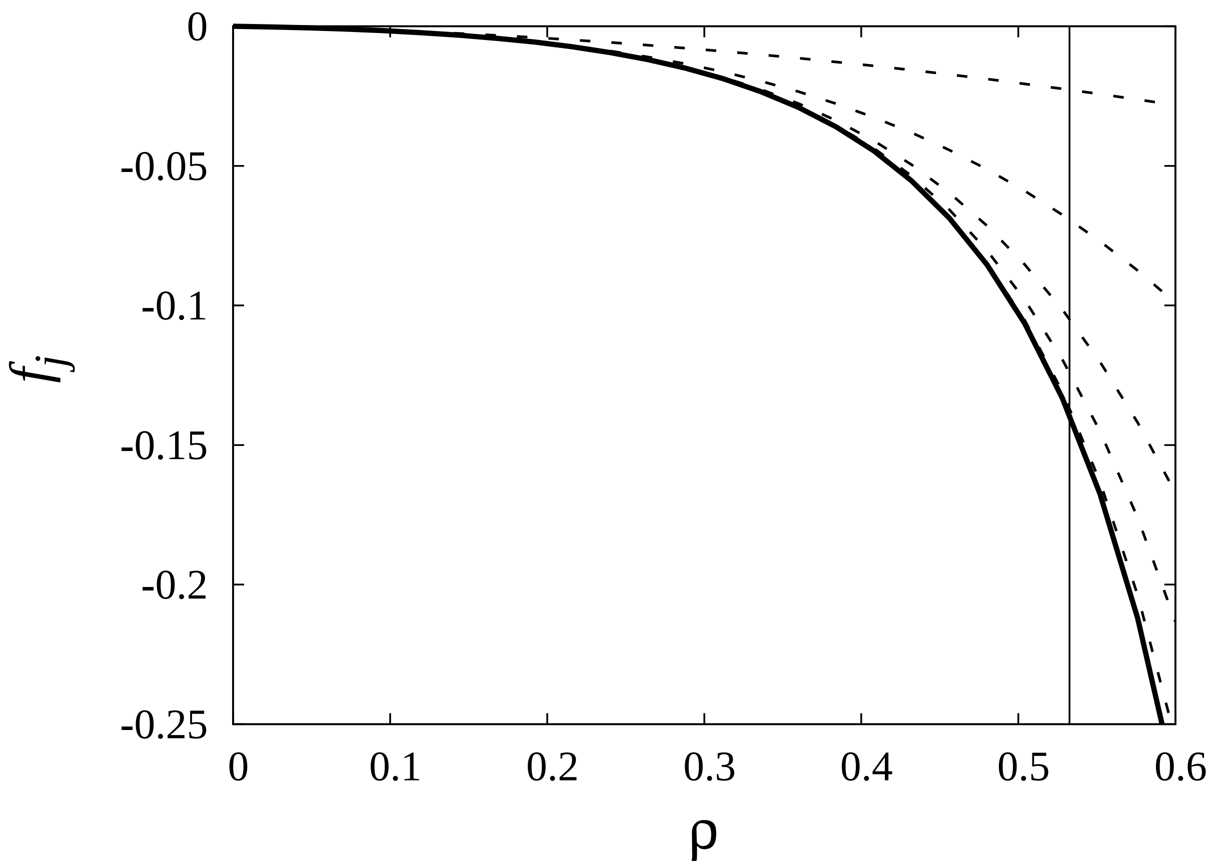
- Convergence?
  - *generally not bad*
  - *for some diagrams Padé is necessary*

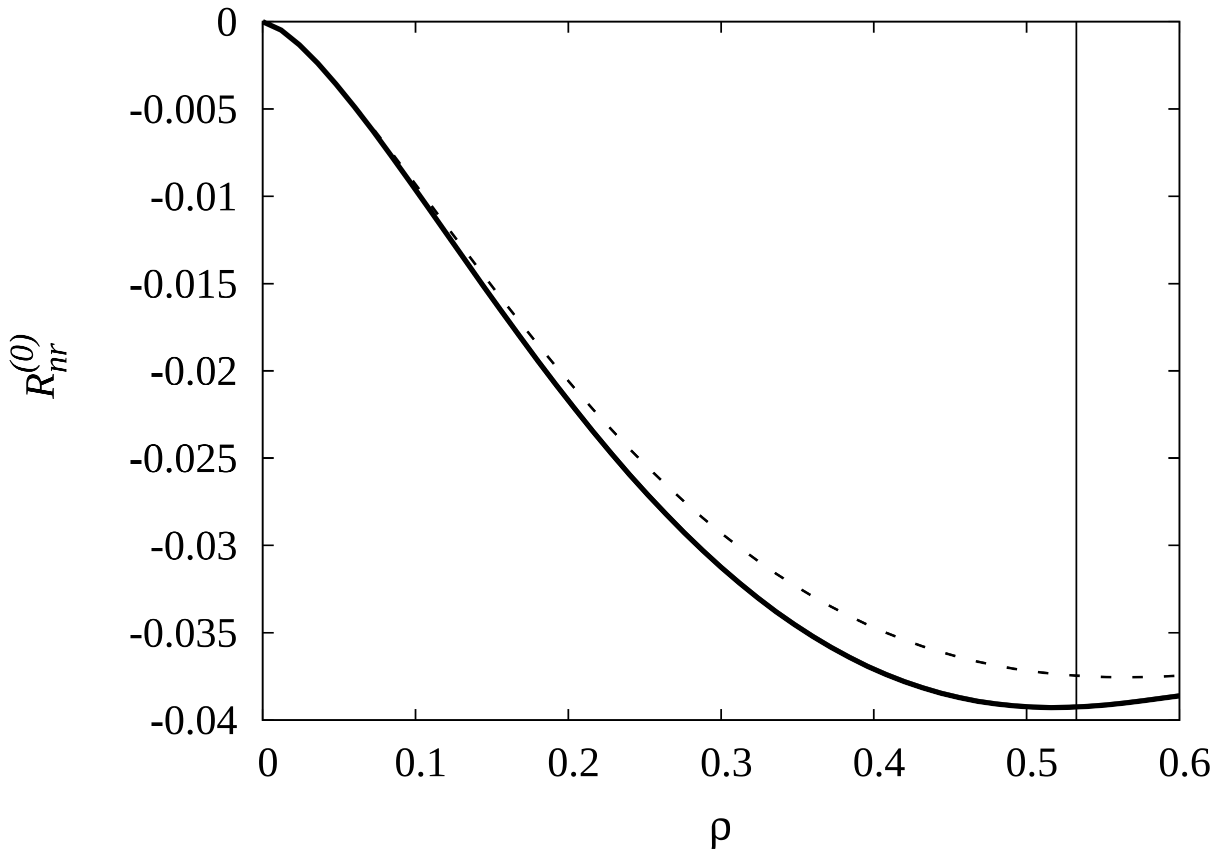
Convergence

diagram "j"



Convergence

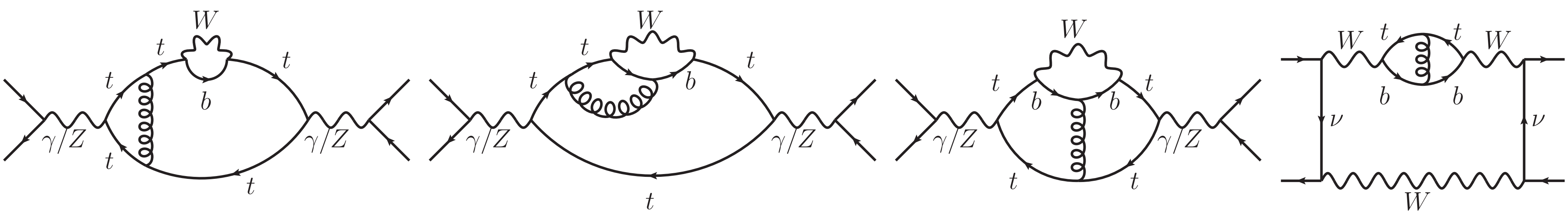
all diagrams



dash line - leading ρ-dependence
solid line - exact ρ-dependence

NNLO nonresonant contribution

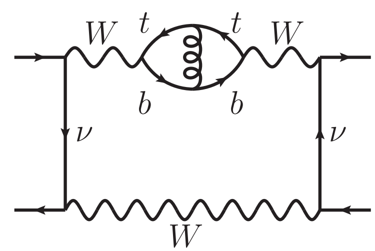
ρ-leading diagrams



(a) (b) (c) (d)

- Regions of gluon momentum
  - *(a) and (d) - hard, potential, ρ-potential*
  - *(b) and (d) - hard, ρ-soft*

NNLO nonresonant contribution



(d)

- Diagram (d):
  - *box momentum is ρ-potential*
  - *self-energy momentum is hard or ρ-soft*

$$\mathrm{Im}[\Pi^{(1)}(z)] = \frac{C_F\alpha_s}{\pi}\left[\frac{9}{4} + \frac{1}{3}\pi^2 - \frac{3}{2}\ln(2\rho) - \frac{3}{2}\ln\left(1 - \frac{z}{\rho}\right) + \mathcal{O}(\rho, z)\right]\mathrm{Im}[\Pi^{(0)}(z)]$$

  - $\ln(\rho)$ *term is absorbed into corrected* Γ_t

$$\Gamma_t = \left[1 + \frac{C_F\alpha_s}{\pi}\left(\frac{9}{4} - \frac{2}{3}\pi^2 - \frac{3}{2}\ln(2\rho) + \mathcal{O}(\rho)\right)\right]\Gamma_t^{LO}$$

NNLO result

- leading term of ρ-expansion

$$R_{nr}^{N^2LO} = \frac{3C_F\alpha_s}{\pi^2\rho^{1/2}}\,\frac{\Gamma_t}{m_t}\left\{\left[\frac{4}{9} + \text{“}Z\text{”}\right]\left[\frac{\pi^2}{\rho^{1/2}}\left(3\ln\left(\sqrt{E^2+\Gamma_t^2}/\rho m_t\right) + \frac{3}{2} + 6\ln 2\right) + (18 + 24\ln 2)\right]\right.$$
$$+\frac{1}{\sin^4\theta_W}\left[\frac{22}{3} + \frac{17\pi^2}{6} - \frac{17}{2}\ln 2 + (2 - 3\pi^2 + 9\ln 2)\,\frac{3\sqrt{2}}{4}\ln\left(1+\sqrt{2}\right)\right.$$
$$\left.\left.-\frac{27\sqrt{2}}{8}\left(\ln^2\left(1+\sqrt{2}\right) + \mathrm{Li}_2\left(2\sqrt{2}-2\right)\right)\right]\right\}.$$

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- *ρ-Coulomb term* $\alpha_s/\rho^{1/2}$
- *new type of logs* $\ln(E/\rho m_t) \sim \ln(v^2/\rho)$

Spurious divergences in pNRQCD

- Problem with complex energy shift in NNLO
  - *integral over potential momentum* $\boldsymbol{p}^2/m_t \sim E$
    is UV divergent $\delta R \sim \Im[E \ln(E/\mu)] \to \Gamma_t \ln(E/\mu)$

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- Physical cutoff
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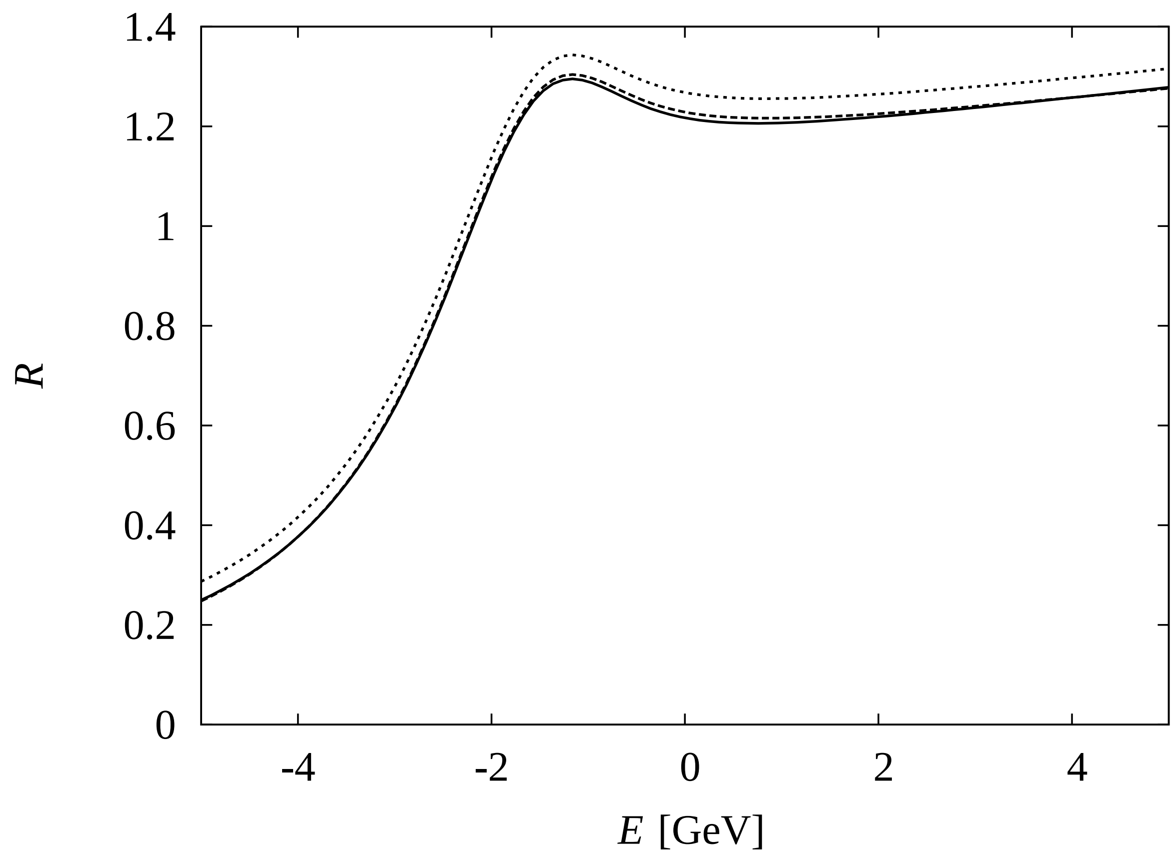
- Physical cutoff
  - $\Im[\Sigma] = 0$ *for* $\boldsymbol{p}^2/m_t > \rho m_t$
- Expansion by regions
  - *integral over* ρ*-potential region* $\boldsymbol{p}^2/m_t \sim \rho m_t$ *is IR divergent* $\delta R \sim \Gamma_t \ln(\mu/\rho m_t)$

➥ *finite sum* $\delta R \sim \Gamma_t \ln(E/\rho m_t)$

Numerics



dot line - *LO*

dash line - LO+NLO nonresonant

solid line - LO+NNLO nonresonant

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  - *based on nonrelativistic expansion in* $\rho = 1 - m_t/M_W$
  - *systematically accounts for finite width effects in threshold top-antitop production*
  - *optimized for high-order calculations*
  - *solve the problem of the spurious divergences*

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  - *systematically accounts for finite width effects in threshold top-antitop production*
  - *optimized for high-order calculations*
  - *solve the problem of the spurious divergences*
  - *conceptually clear and aesthetically appealing*