Measuring Vacuum Polarization with Josephson Junctions

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Topics Discussed

- Exact results in quantum mechanics
  - *Flux quantization*
  - *Quantum Hall effect*
  - *Josephson effect*
- Vacuum polarization in strong magnetic field
- QED corrections to quantum Hall and Josephson effect
  - *Theory*
  - *Experiment*

Exact result: an example

Determination of the fine structure constant $\alpha \propto e^2/h$

- electron $g-2$
  - *experimental error:* 10^{-10}
  - *theoretical error:* 10^{-10} - *requires 4-loop calculation in QED*
- quantum Hall effect
  - *experimental error:* 10^{-8} *(could be* 10^{-12}*)*
  - *theoretical error:* $\mathbf{0}$ - *for free!*

Gauge invariance and quantum phase

$\hbar = c = 1, \ \alpha = \frac{e^2}{4\pi}$

- ✔ Schrödinger equation $\quad (i\partial_t - \mathcal{H})\,\Psi = 0$
- ✔ Wave function $\quad \Psi = |\Psi| e^{i\theta}$
- ✔ Hamiltonian $\quad \mathcal{H} = qA_0 + F(\boldsymbol{B}, \boldsymbol{E}, \boldsymbol{D})$

$$\boldsymbol{D} = \boldsymbol{\partial} - iq\boldsymbol{A}$$

➥ If $\boldsymbol{\nabla} \times \boldsymbol{A} = 0$ then

$$\theta(\boldsymbol{r}_1) - \theta(\boldsymbol{r}_2) = q\int_{\boldsymbol{r}_2}^{\boldsymbol{r}_1} \boldsymbol{A}(\boldsymbol{r}')\cdot d\boldsymbol{r}'$$

➥ Also $\phi(\boldsymbol{r}) = \theta(\boldsymbol{r}) - q\int^{\boldsymbol{r}} \boldsymbol{A}(\boldsymbol{r}')\cdot d\boldsymbol{r}'$ is gauge invariant

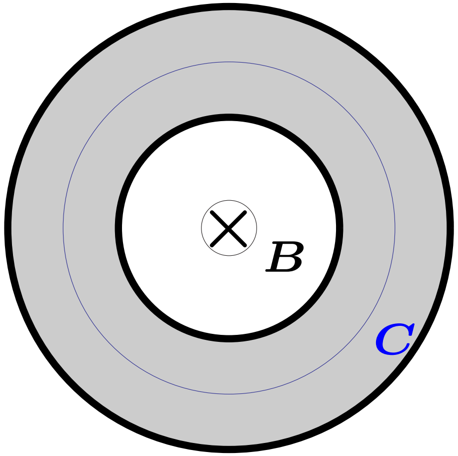
Flux quantization

F. London (1948)

Superconductivity ⇨ *coherent state of Cooper pairs* ⇨ $q = 2e$

Meissner effect ⇨ $\boldsymbol{B} = 0$ inside superconductor

Superconducting ring:



Single-valued wave function ⇨ $\Delta\theta = 2e \oint_C \boldsymbol{A} \cdot d\boldsymbol{x} = 2\pi n$

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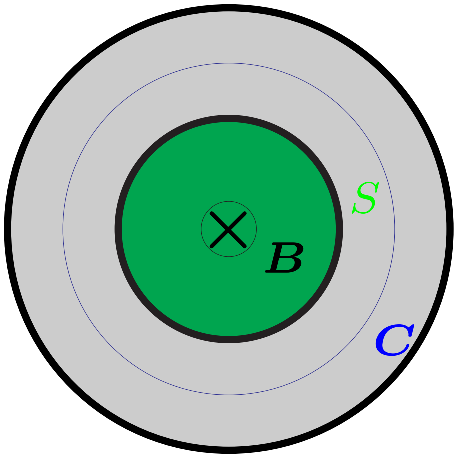


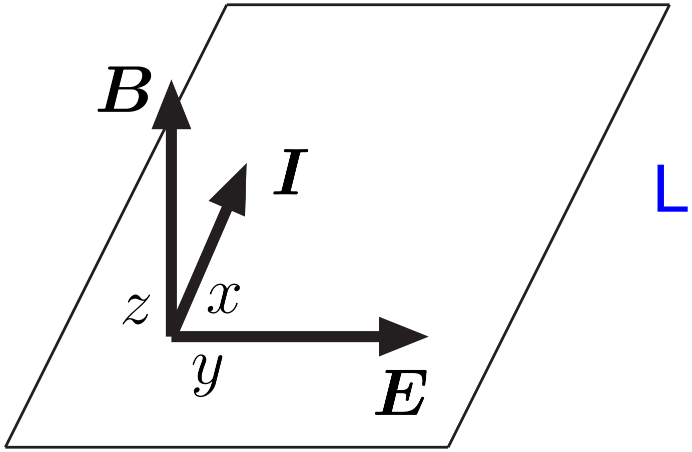
Single-valued wave function ⇨ $\Delta\theta = 2e \oint_C \boldsymbol{A} \cdot d\boldsymbol{x} = 2\pi n$

$\oint_C \boldsymbol{A}(\boldsymbol{r}) \cdot \boldsymbol{r} = \int_S \boldsymbol{B}(\boldsymbol{r}) \cdot d^2\boldsymbol{s} \equiv \Phi$ ⇨ $\Phi = \frac{\pi n}{e} \equiv n\Phi_0$

Flux quantum: $\Phi_0 = \frac{h}{2e}$

Hall effect

E. Hall (1879)



Lorentz force *vs* electrostatic force

$$e\boldsymbol{E} = -e\boldsymbol{v} \times \boldsymbol{B}$$

current density

$$j = \rho e v = \frac{\rho e}{B} E$$

total current per length

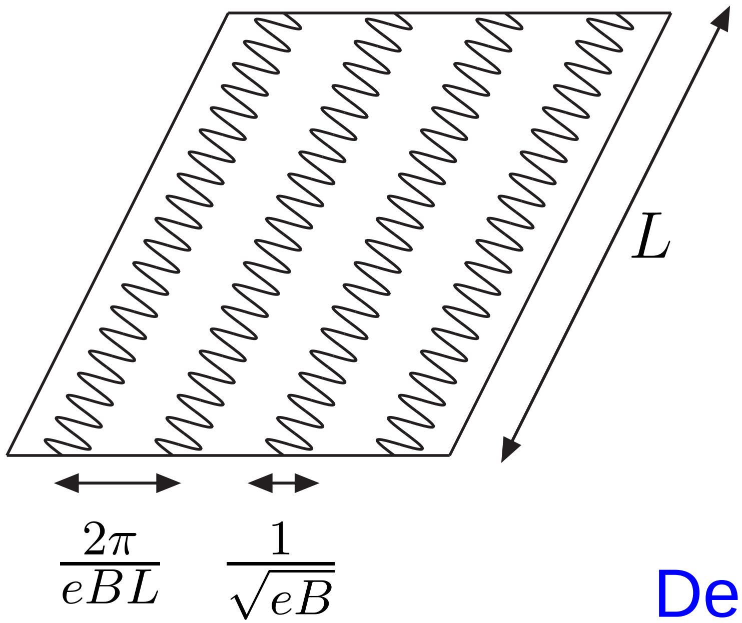
$$I = \frac{\rho e}{B} V$$

Hall conductivity

$$R^{-1} = \frac{\rho e}{B}$$

Quantum Hall effect

K. von Klitzing, G. Dorda, M. Pepper (1980)



Wave function:

$$\Psi(x,y) = e^{i2\pi m\frac{x}{L}}\ \psi(y-y_m)$$

$\psi(y-y_m)$ ⇨ *harmonic oscillator*

centered at $\quad y_m = \frac{2\pi m}{eBL}$

Density of quantum states with

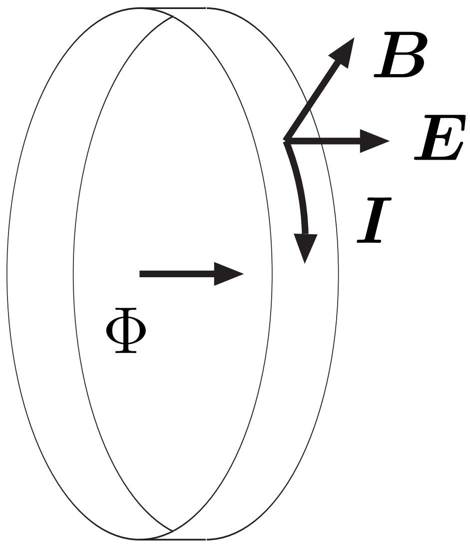
$\boldsymbol{n}$ Landau levels filled: $\quad \rho = n\frac{eB}{2\pi}$

Quantum Hall conductivity: $\quad R^{-1} = 2n\alpha = n/R_K$

von Klitzing constant: $\quad R_K = \frac{h}{e^2}$

Gauge invariance argument

R.B. Laughlin (1981)



current density $\boldsymbol{j} \propto \dfrac{\delta \mathcal{H}}{\delta \boldsymbol{A}}$

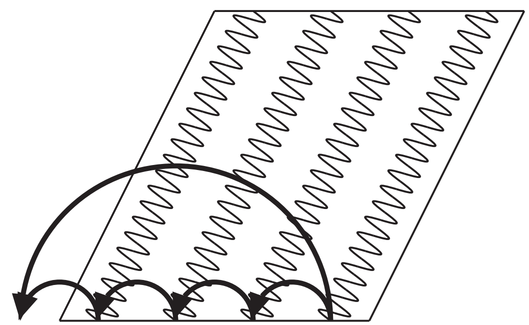
total current $I = \dfrac{d\mathcal{E}}{d\Phi}$

Flux quantization $\Phi = 4\pi|\boldsymbol{A}|/L = n2\Phi_0$

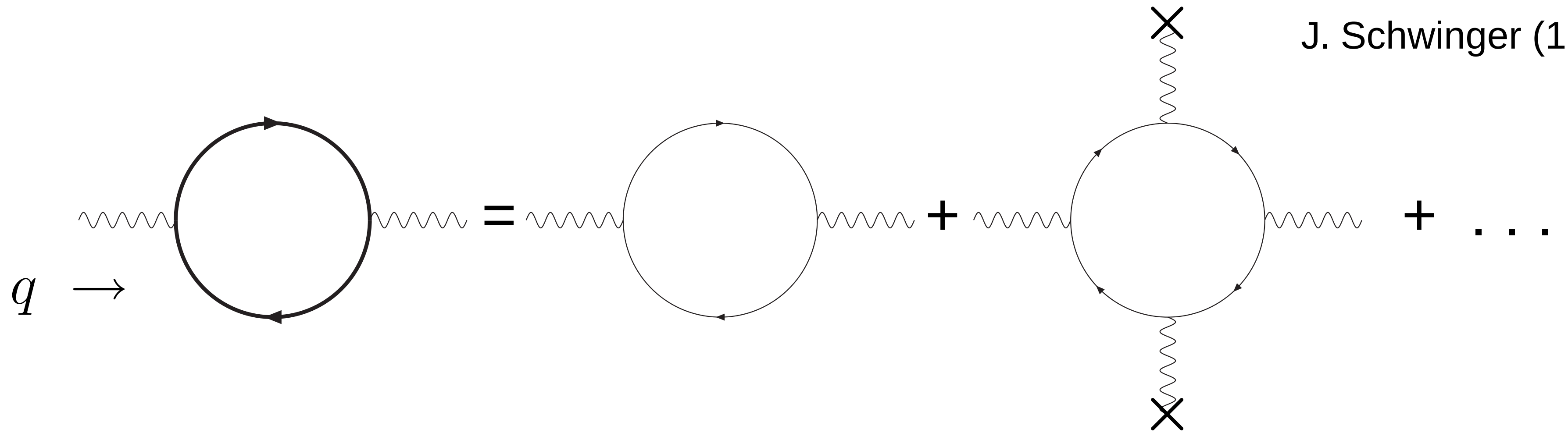
Flux dependence $y_m(\Phi + 2\Phi_0) = y_{m+1}(\Phi)$

for $d\Phi = 2\Phi_0$ ⇨ $d\mathcal{E} = neV$

➥ $I = \dfrac{neV}{2\Phi_0} = \dfrac{nV}{R_K}$



Vacuum polarization



J. Schwinger (1951)

Charge renormalization $\delta e \propto \Pi_{\mu\nu}(q)/q^2|_{q^2\to 0}$

Vacuum polarization in magnetic field

$$\delta\Pi_{\mu\nu}(q) = -\frac{\alpha}{\pi}\left(\frac{eB}{m^2}\right)^2\frac{1}{45}\left[2\left(g_{\mu\nu}q^2 - q_\mu q_\nu\right) - 7\left(g_{\mu\nu}q^2 - q_\mu q_\nu\right)_{\parallel} + 4\left(g_{\mu\nu}q^2 - q_\mu q_\nu\right)_{\perp}\right]$$

Lorentz invariance broken!

QED corrections to electromagnetic coupling

Coulomb potential

$$V_C(\boldsymbol{r}) = e^2 \int \left(1 - \frac{\delta\Pi_{00}(q)}{q^2}\right) \frac{e^{-i\boldsymbol{q}\cdot\boldsymbol{r}}}{\boldsymbol{q}^2} \frac{d^3\boldsymbol{q}}{(2\pi)^3} = \frac{\alpha}{|\boldsymbol{r}|}\left[1 + \frac{\alpha}{\pi}\left(\frac{eB}{m^2}\right)^2 \left(\frac{2}{45} - \frac{7}{90}\sin^2\theta \right)\right]$$

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correction to the photon dispersion

$$\frac{1}{q_0^2 - (1 + C_i\alpha)\boldsymbol{q}_i^2} \Rightarrow \nu\lambda \neq c$$

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local charge renormalization

$$e \Rightarrow \left(1 + \frac{\alpha}{\pi}\left(\frac{eB}{m^2}\right)^2 \frac{1}{45}\right) e \equiv e^*$$

QED corrections to R_K

A.P., Phys.Rev. B 79, 113303 (2009)

Origin of the corrections $R_K^{-1} \propto e^2 \Rightarrow (e^*)^2$

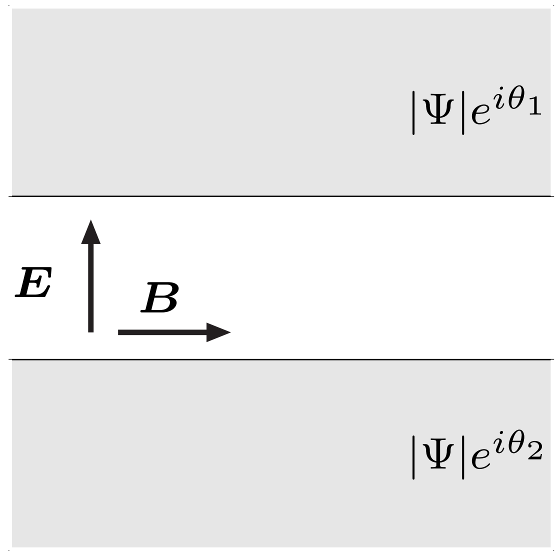
Result:

$$R_K^{-1} = \frac{e^2}{h}\left[1 + \frac{2}{45}\frac{\alpha}{\pi}\left(\frac{\hbar e B}{c^2 m^2}\right)^2\right] \approx \frac{e^2}{h}\left[1 + 10^{-20} \times \left(\frac{B}{10\,\mathrm{T}}\right)^2\right]$$

Josephson effect

B.D. Josephson (1962); S. Shapiro (1963)

Tunneling Josephson current I



- ✔ *no voltage drop if the current is time independent*
- ✔ 2π *periodic in* $\theta_1 - \theta_2$
- ✔ *simplest solution* $I = I_c \sin(\theta_1 - \theta_2)$
- ✔ *with voltage* V *across the junction the current oscillates at frequency* ν

Josephson effect

Time evolution of the phase:

$$\Psi(t) \propto e^{i\mathcal{E}t} \quad ➪ \quad \theta_1 - \theta_2 = (\mathcal{E}_1 - \mathcal{E}_2)t = 2eVt \quad ➪ \quad \omega = 2eV$$

Josephson frequency-voltage relation $\nu = K_J V$

Josephson constant $K_J = \frac{2e}{h}$

Gauge invariance argument

F. Bloch (1968)

Observables depend on $\Delta\phi = \theta_1 - \theta_2 - 2e\int_{\boldsymbol{r}_2}^{\boldsymbol{r}_1} \boldsymbol{A}(\boldsymbol{r}')\cdot d\boldsymbol{r}'$

Gauge transformation

$$A^{\mu\prime}(r) = A^{\mu}(r) - \partial^{\mu}\xi(r), \qquad \xi(r) = tA_0(\boldsymbol{r})$$
$$A_0'(\boldsymbol{r}) = 0, \qquad \boldsymbol{A}'(r) = \boldsymbol{A}(\boldsymbol{r}) + t\boldsymbol{\nabla} A_0(\boldsymbol{r})$$

Gauge invariant phase difference

$$\Delta\phi = \Delta\phi_0 - 2et\left(A_0(\boldsymbol{r}_1) - A_0(\boldsymbol{r}_2)\right)$$

➥ $\omega = 2eV$

QED corrections to K_J

A.P., Phys.Rev.Lett. 104, 097003 (2010)

Correction to the coupling with scalar potential

$$\delta_{v.p.}\mathcal{H} = -e\int \frac{\delta\Pi_{0\nu}(q)}{q^2}\tilde{A}^{\nu}(q)e^{iqr}\frac{d^4q}{(2\pi)^4} = 2\delta e A_0(\boldsymbol{r})$$

here $\delta e = e^* - e$

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gauge invariant!

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can be removed by $\delta\xi(r) = \frac{\delta e}{e}\xi(r)$

➥ $\omega = 2e^*V$

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here $\delta e = e^* - e$ *gauge invariant!*

can be removed by $\delta\xi(r) = \frac{\delta e}{e}\xi(r)$

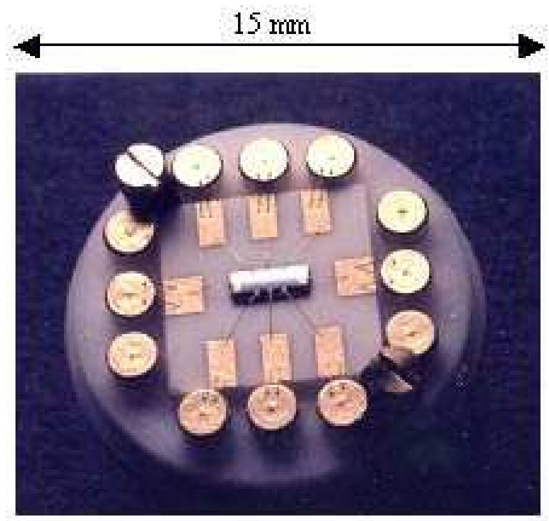
➥ $\omega = 2e^*V$

Result:

$$K_J = \frac{2e}{h}\left[1 + \frac{1}{45}\frac{\alpha}{\pi}\left(\frac{\hbar e B}{c^2 m^2}\right)^2\right] \approx \frac{2e}{h}\left[1 + 10^{-20}\times\left(\frac{B}{10\,\mathrm{T}}\right)^2\right]$$

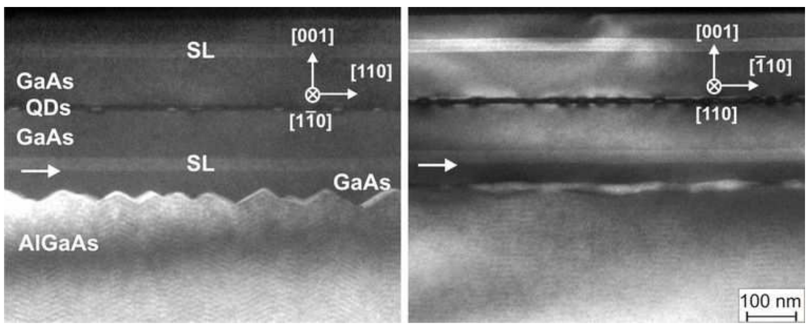
Experiment: quantum Hall effect

F. Schopfer, W. Poirier (2007)



gallium arsenide heterostructures

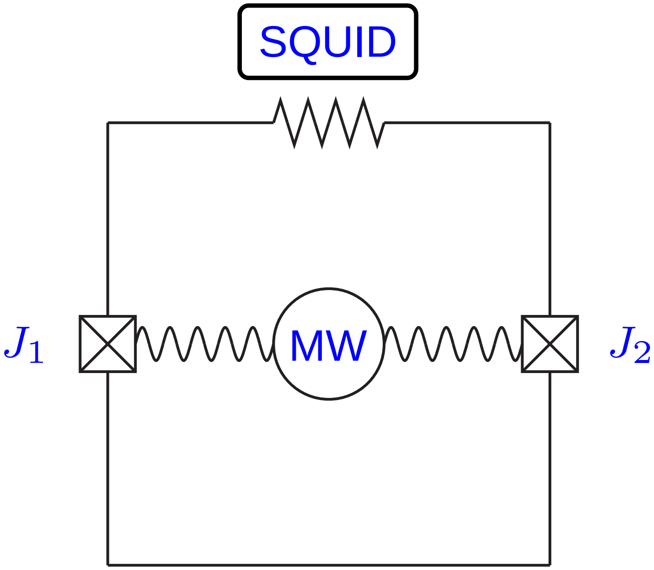
➥ *two-dimensional electron gas*

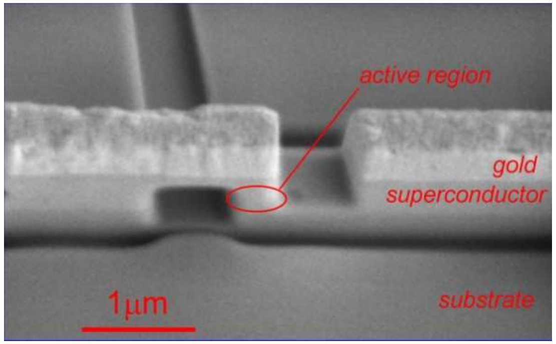


✘ thermal Johnson-Nyquist noise ➪ *accuracy limit* 10^{-12}

Experiment: Josephson effect

A.K. Jain, J.E. Lukens, J.-S. Tsai (1987)





linearly growing current $I = t(V_1 - V_2)/L$

✔ *accuracy* 10^{-19} – sensitive to gravitational red shift

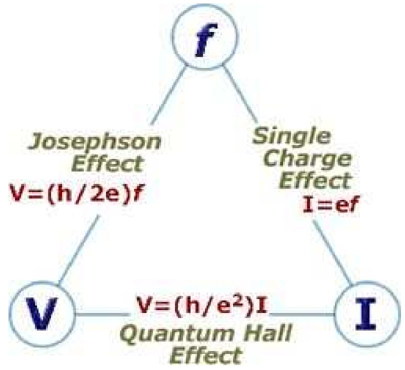
Metrology

- Fundamental relations
  - *electron charge* $e = \frac{2}{K_J R_K}$
  - *Planck constant* $h = \frac{4}{K_J^2 R_K}$

- Quantum metrology triangle

K.K. Likharev, A.B. Zorin (1985)

  - *quantum Hall effect* $V/I = R_K$
  - *Josephson effect* $V = \nu/K_J$
  - *Single electron tunneling* $I = e\nu$ **(?)**



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One literally can measure the vacuum polarization with a voltmeter!