Top, Bottom and Charm at Threshold

Alexander Penin

TTP Karlsruhe & INR Moscow

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Topics discussed

- Modern effective theory of nonrelativistic bound states
- Nonrelativistic renormalization group
- Status of high order perturbative analysis
- Phenomenology of top, bottom and charm
- Theoretical and experimental challenges

Why studying Heavy Quarkonium?

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- Theory:
  - *Possesses highly non-trivial multiscale dynamics*
  - *Allows for the first principle QCD predictions*
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- Phenomenology:
  - $\boxed{\textit{HEAVY QUARK MASSES}}$
  - $\alpha_s,\ \Gamma_t,\ y_t,\ M_H$

Heavy Quarkonium at NNLO

- Spectrum (Pineda, Yndurain)
- Leptonic/photonic width (Melnikov, Yelkhovsky; Penin, Pivovarov)
- Threshold production (Czarnecki, Melnikov; Beneke, Smirnov, Signer; Kühn, Penin, Pivovarov; Hoang, Teubner)

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- Apparent slow convergence:

$$\Gamma(\Upsilon(1S) \to l^+ l^-) = \Gamma^{\mathrm{LO}} \left(1 - 0.5_{\alpha_s} + 1.8_{\alpha_s^2} + \ldots\right)$$

$$M(\Upsilon(1S)) = M^{\mathrm{LO}} \left(1_{\alpha_s^2} - 0.1_{\alpha_s^3} - 0.05_{\alpha_s^4} + \ldots\right)$$

Bottom quark mass at NNLO

Pole mass $\rightarrow$ "short-distance mass"

(Beneke; Hoang; Melnikov, Yelkhovsky; Pineda)

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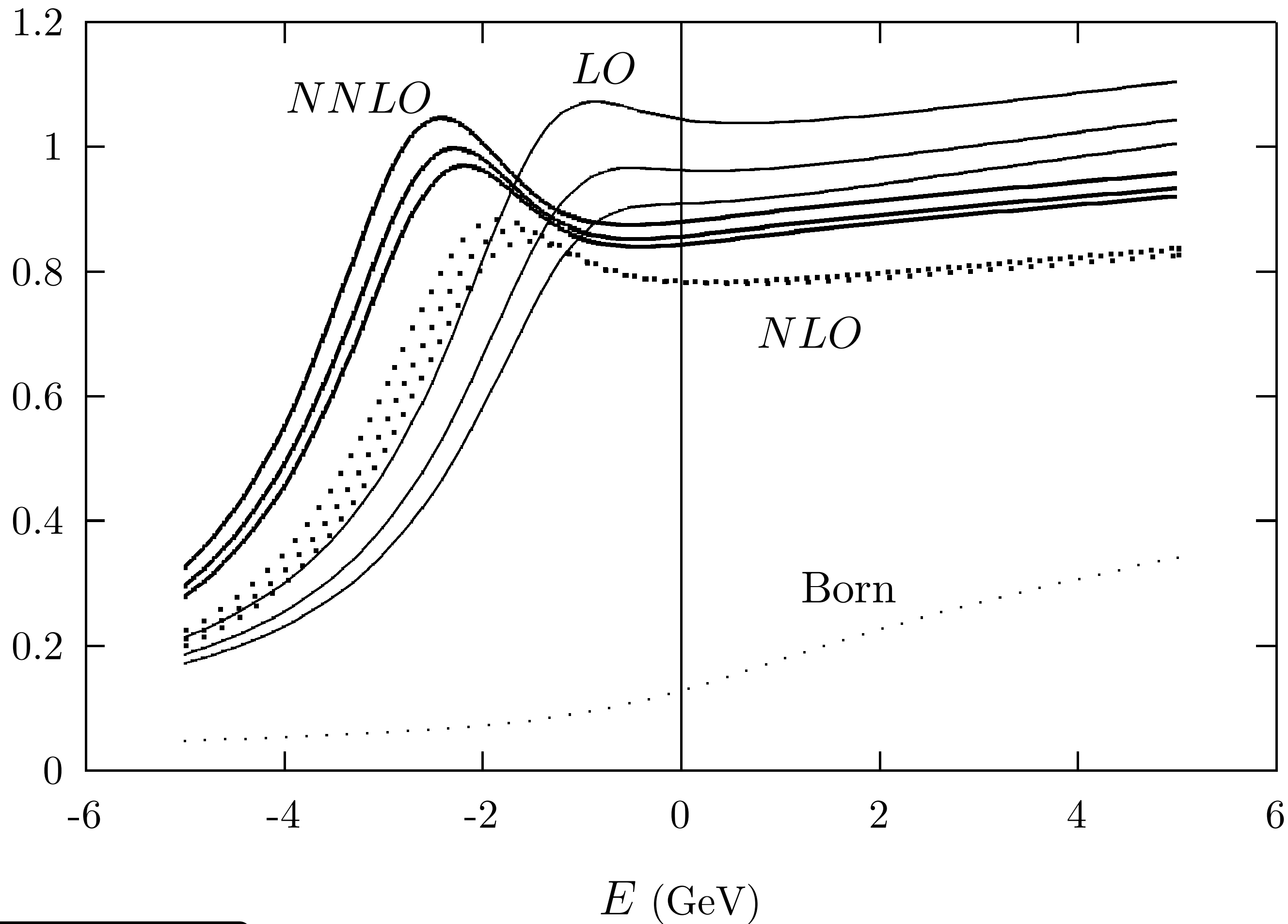
	Method	Order	$\overline{m}_b(\overline{m}_b)$ (GeV)
Penin, Pivovarov 98	nonrelativistic Υ sum rules	NNLO	4.21 ± 0.11
Melnikov, Yelkhovsky 99	"	"	4.20 ± 0.10
Beneke, Signer 99	"	"	4.25 ± 0.08
Hoang 00	"	"	4.17 ± 0.05
Kühn, Steinhauser 01	low moments sum rules	"	4.209 ± 0.050
Pineda 01	spectrum, $\Upsilon(1S)$ resonance	"	4.210 ± 0.115

$$\boxed{\overline{m}_b(\overline{m}_b) = 4.2 \pm 0.1 \text{ GeV}}$$

$e^+e^- \to t\bar{t}$ threshold production at NNLO



$\mu = 50,\ 75,\ 100$ GeV

- Precision b-physics (CKM,...): $\delta m_b < 50\ \mathrm{MeV}$
- Precision t-physics at ILC: $\delta R(e^+e^- \to t\bar{t}) < 3\%$
- General structure of PT: *renormalons,...*

$\boxed{\text{Full N}^3\text{LO analysis is necessary}}$

Nonrelativistic Effective Theory

- Characteristic momentum regions:

Hard	$p_0 \sim m_q, \quad \boldsymbol{p} \sim m_q$
Soft (static quarks)	$p_0 \sim v m_q, \quad \boldsymbol{p} \sim v m_q$
Potential (static gluons)	$p_0 \sim v^2 m_q, \quad \boldsymbol{p} \sim v m_q$
Ultrasoft	$p_0 \sim v^2 m_q, \quad \boldsymbol{p} \sim v^2 m_q$

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- Dynamical degrees of freedom

QCD $\rightarrow$ NQCD $\rightarrow$ pNRQCD (Caswell, Lepage; Pineda, Soto)

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$$\overline{q}\,(i\gamma^\mu D_\mu - m_q)\,q$$

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$\Big\downarrow$ hard modes integrated out

$$\psi^\dagger\left(iD_0 + \frac{\boldsymbol{D}^2}{2m_q}\right)\psi + \frac{1}{8m_q^3}\,\psi^\dagger\boldsymbol{D}^4\psi - \frac{c_F\,g_s}{2m_q}\,\psi^\dagger\boldsymbol{\sigma}\cdot\boldsymbol{B}\psi + \ldots$$

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$\Big\downarrow$ soft modes integrated out

$$\psi^\dagger\left(i\partial_0 + \frac{\boldsymbol{\partial}^2}{2m_q} - g_s\boldsymbol{r}\cdot\boldsymbol{E}\right)\psi + \ldots$$

Potential NRQCD

Schrödinger equation

$$(\mathcal{H}^{s,o} - E)\, G^{s,o}(\boldsymbol{r}\,, \boldsymbol{r}, E) = \delta(\boldsymbol{r} - \boldsymbol{r}')$$

Effective Hamiltonian

$$\mathcal{H}^{s,o} \quad = \quad -\frac{\boldsymbol{\partial}^2}{m_q} + V_C^{s,o}(\boldsymbol{r}) + \delta\mathcal{H}^{s,o}$$

Multipole interaction to ultrasoft gluons

$$g_s(\boldsymbol{r}_1 - \boldsymbol{r}_2) \cdot \boldsymbol{E} + g_s(\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2)\boldsymbol{B} + \ldots$$

Nonperturbative effects

- Perturbative quarkonium ($t\bar{t}$, $b\bar{b}$): $v^2 m_q \gg \Lambda_{QCD}$

  Local condensate expansion (Voloshin Leutwyler)

- Quasiperturbative quarkonium ($b\bar{b}$): $v^2 m_q \approx \Lambda_{QCD}$

  Time-nonlocal condensates (Brambilla, Pineda, Soto, Vairo)

- Nonperturbative quarkonium ($c\bar{c}$): $m_q \gg \Lambda_{QCD} \gg v m_q$

  NRQCD matrix elements (Bodwin, Braaten, Lepage)

Loops in the Effective Theory

Problem: Separation of regions in virtual momentum space

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$\boxed{\textit{Regions are separated in dimensional regularization!}}$

Effective theory in dimensional regularization

(Pineda, Soto; Czarnecki, Melnikov, Yelkhovsky; Beneke, Signer, Smirnov; Kniehl, Penin, Smirnov, Steinhauser)

Gauge, Lorenz invariance + automatic matching

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Threshold Expansion (Beneke,Smirnov)

Expansion of the Feynman integrals in $\boldsymbol{v}$

Effective Theory

Expansion of the QCD Lagrangian in $\boldsymbol{v}$

Loops in the Effective Theory

THRESHOLD EXPANSION	pNRQCD
Hard region	Wilson coefficients
Soft region	Potentials
Potential region	Quantum Mechanical PT
Ultrasoft region	Multipole radiation

NLO corrections to HFS

Spin-flip potential:

$$V_S(\boldsymbol{q}^2) = \frac{4\pi C_F \alpha_s \boldsymbol{S}^2}{3m_q^2}$$

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$$V_S(\boldsymbol{q}^2) \quad = \quad \frac{4\pi C_F \alpha_s \boldsymbol{S}^2}{3m_q^2}\left\{1 + \boxed{\frac{\alpha}{\pi}\left[\left(1+\frac{7}{8}\left(\frac{1}{\epsilon}+\ln\frac{\mu^2}{m_q^2}\right)\right)C_A - \frac{1}{2}C_F + \frac{6-6\ln 2+i3\pi}{4}T_F\right]}\right.$$

hard contribution

QCD on-shell on-threshold amplitude

NLO corrections to HFS

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$$\left. + \frac{\alpha}{\pi}\left[\left(-\frac{7}{18}-\frac{7}{8}\left(\frac{1}{\epsilon}+\ln\frac{\mu^2}{\boldsymbol{q}^2}\right)\right)C_A - \frac{5}{9}T_F n_l\right]\right\}$$

hard contribution

QCD on-shell on-threshold amplitude

soft contribution

NQCD, static heavy quark propagator

Coulomb potential:

$$V_C(\boldsymbol{q}^2) = -\frac{4\pi C_F\alpha_s(\boldsymbol{q}^2)}{\boldsymbol{q}^2}\left[1 + \frac{\alpha_s}{\pi}\left(\frac{31}{36}C_A - \frac{5}{9}T_F n_l\right)\right]$$

NLO corrections to HFS

Quantum Mechanical PT *(potential contribution)*

$$\delta G^s = G_C^s \otimes V_S^{1-\mathrm{loop}} \otimes G_C^s + 2 G_C^s \otimes V_S^{\mathrm{tree}} \otimes G_C^s \otimes V_C^{1-\mathrm{loop}} \otimes G_C^s$$

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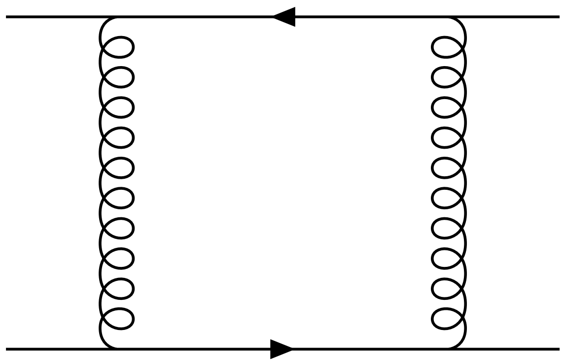
Result for general $\boldsymbol{n}$

$$\begin{aligned} E_{\mathrm{hfs}}^{NLO}(n) \quad = \quad & \frac{1}{3}\frac{C_F^4\alpha_s^4}{n^3}\left\{1 + \frac{\alpha_s}{\pi}\left[\frac{7\,C_A}{4}\ln\left(\frac{C_F\alpha_s}{n}\right) - \frac{C_F}{2} + \frac{-15 - 11\,n + 12\,n^2\,\Psi_2(n)}{9\,n}\right.\right. \\ & \left.\left. \times n_f\,T_F\; + \frac{393 + 150\,n + 126\,\gamma_E\,n + 126\,n\,\Psi_1(n) - 264\,n^2\,\Psi_2(n)}{72\,n}C_A\right]\right\} \end{aligned}$$

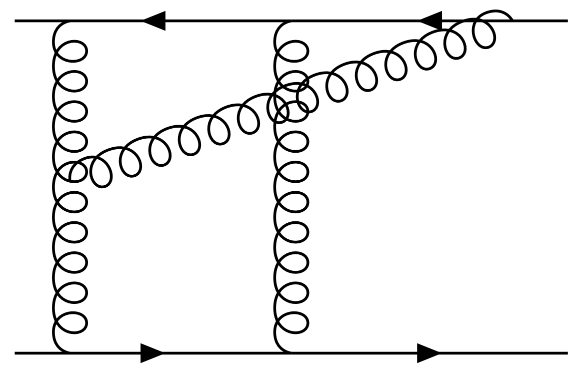
(Penin, Steinhauser)

Typical N^{3}LO contributions

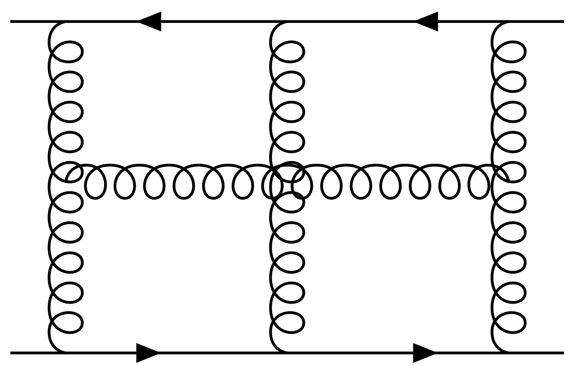
$\mathcal{O}(v^2)$



$\mathcal{O}(v)$



$\mathcal{O}(1)$



Heavy Quarkonium at N^{3}LO

- Spectrum

  $\mathcal{O}(\alpha_s^3)^*$ (Penin, Steinhauser)

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  $\mathcal{O}(\beta_0^3 \alpha_s^3)$, (Penin, Smirnov, Steinhauser; Beneke, Kiyo,Schuller)

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- **Threshold production**

  $\mathrm{N^3LO} \ln^2 v$ (Pineda)

  $\mathrm{N^3LO} \ln v$ (Manohar, Hoang, Stewart, Teubner)

  $\mathcal{O}(\beta_0^3 \alpha_s^3)$ (Beneke, Kiyo, Schuller)

$\mathrm{N^3LO}$ ground state energy (Penin, Steinhauser)

$$\frac{\delta E_1^{\mathrm{N^3LO}}}{E_1^{\mathrm{LO}}} = \alpha_s^3 \left[\begin{pmatrix} 70.590|_{n_l=4} \\ 56.732|_{n_l=5} \end{pmatrix} + 15.297 \ln(\alpha_s) + 0.001\, a_3 + \boxed{\begin{pmatrix} 34.229|_{n_l=4} \\ 26.654|_{n_l=5} \end{pmatrix}} \right],$$

Renormalon contribution

Padé estimate (Chishtie, Elias) $\quad 0.001\, a_3 = \begin{cases} 6.272|_{n_l=4} \\ 3.840|_{n_l=5} \end{cases}$

Top and bottom masses

$\Upsilon(1S)$ mass:

$$\boxed{M_{\Upsilon(1S)} = 2m_b + E_1^{\text{p.t.}} + \delta^{\text{n.p.}} E_1}$$

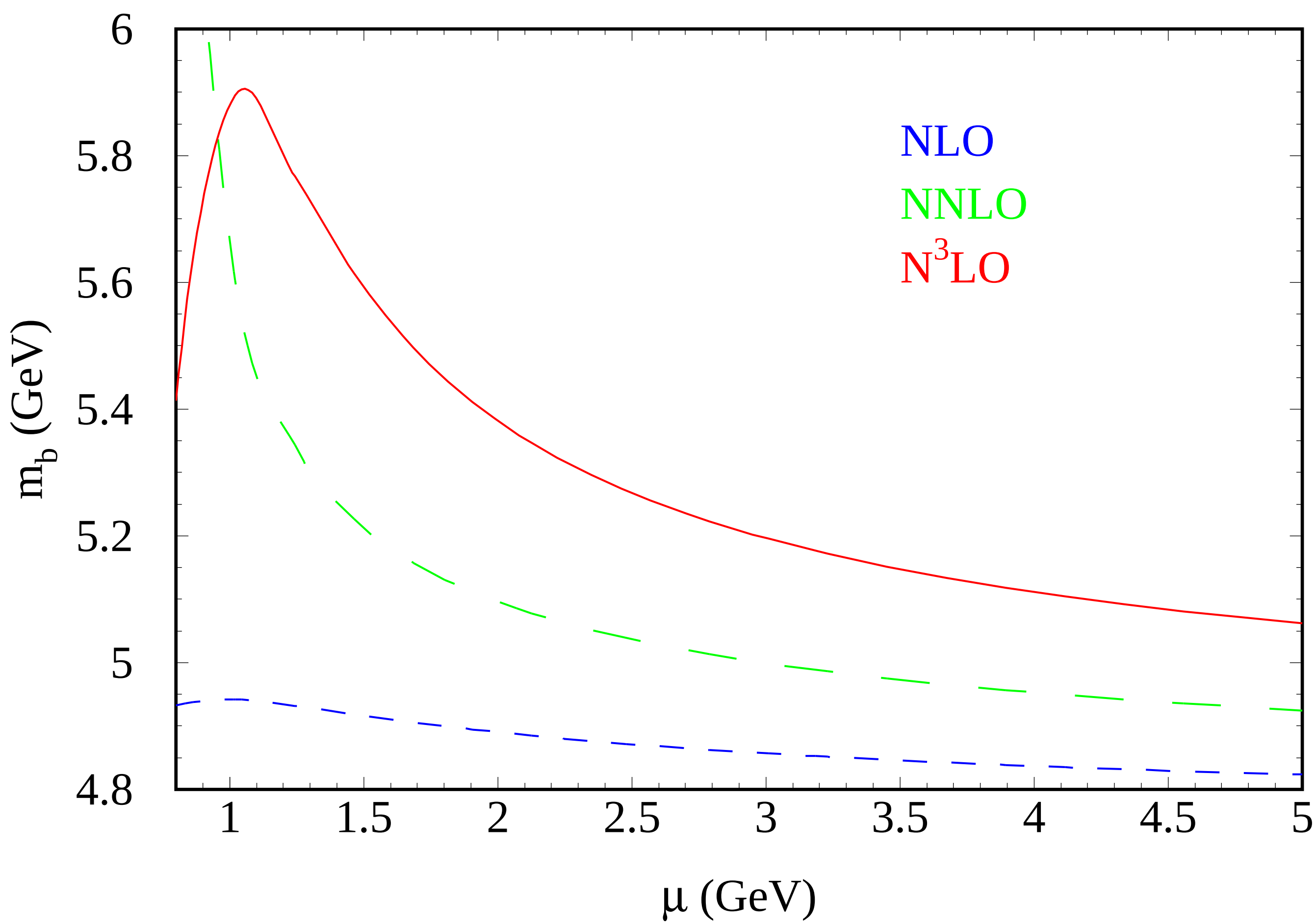
$\delta^{\text{n.p.}} E_1 = \frac{1872}{1275} \frac{m_q \left\langle \alpha_s G^a_{\mu\nu} G^{a\mu\nu} \right\rangle}{(C_F \alpha_s m_q)^4} = 60 \pm 30 \text{ MeV}$ (Voloshin, Leutwyler)

Toponium resonance energy:

$$\boxed{E_{\text{res}} = 2m_t + E_1^{\text{p.t.}} + \delta^{\Gamma_t} E_{\text{res}}}$$

$\delta^{\Gamma_t} E_{\text{res}} = 100 \pm 10 \text{ MeV}$

Bottom quark mass from $\Upsilon(1S)$



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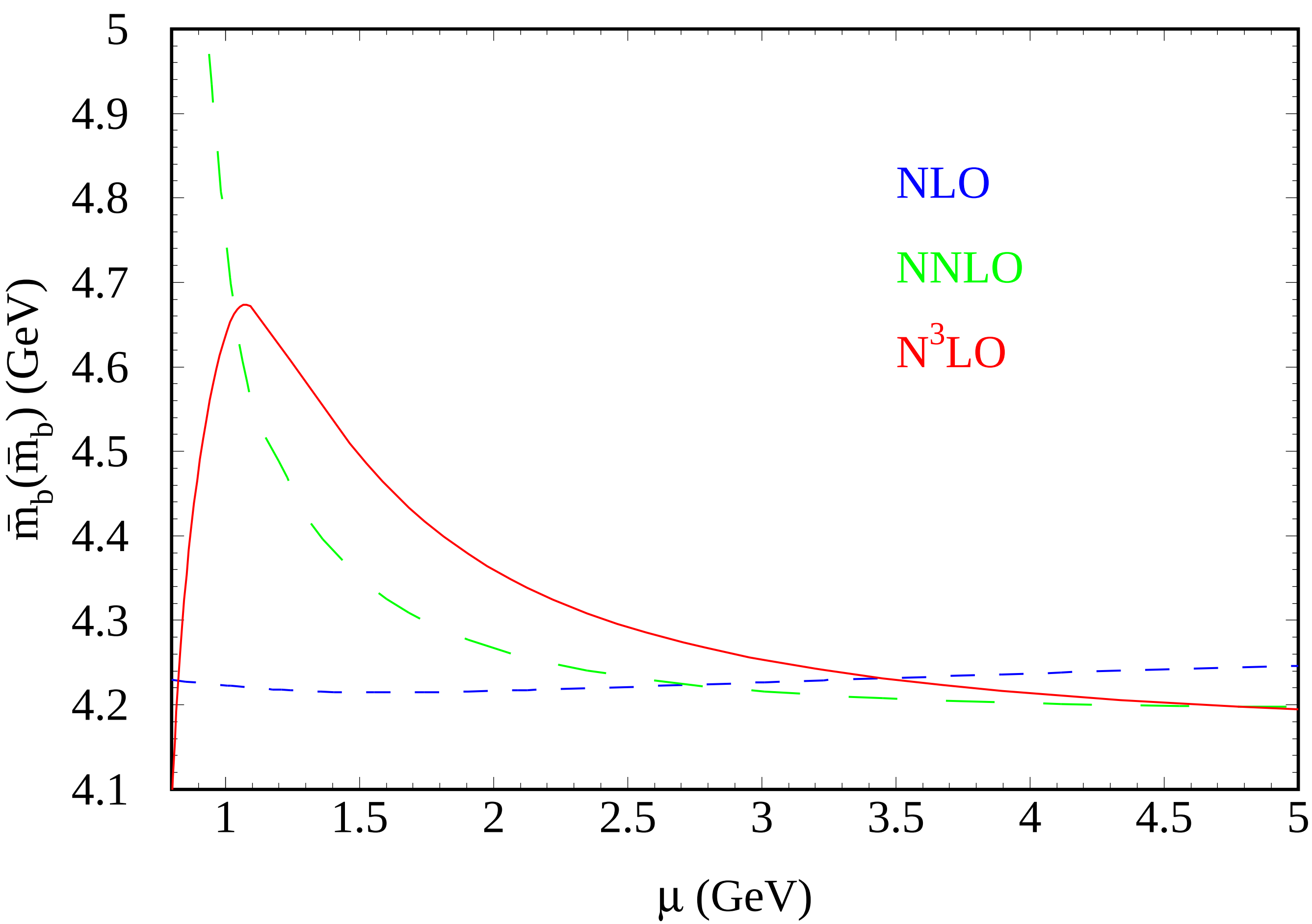
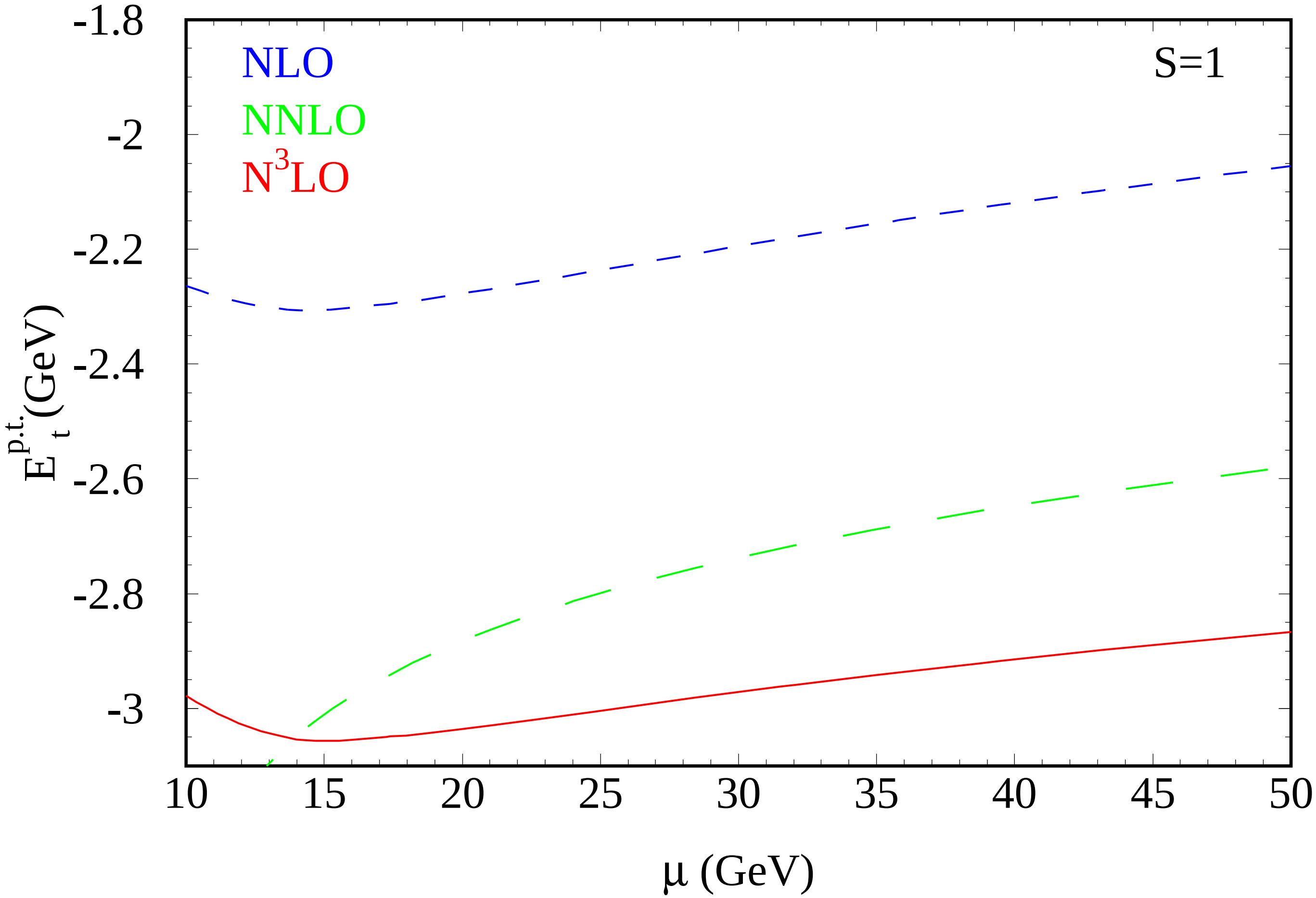


$e^+e^- \to t\bar{t}$ resonance energy



Top and bottom masses at N^{3}LO (Penin, Steinhauser)

- Bottom quark mass from $\Upsilon(1S)$:

$$\boxed{\overline{m}_b(\overline{m}_b) = 4.34 \pm 0.07 \text{ GeV}}$$

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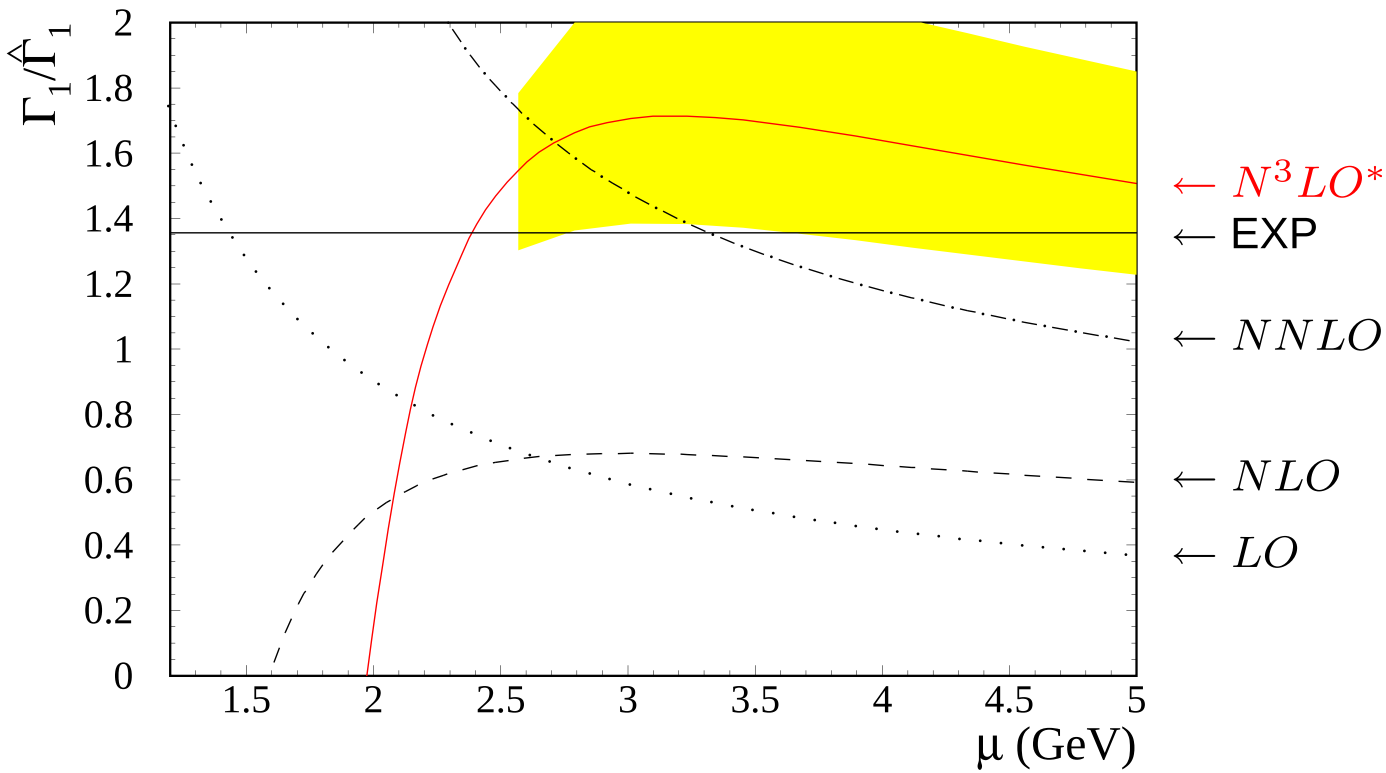
- Toponium resonance energy:

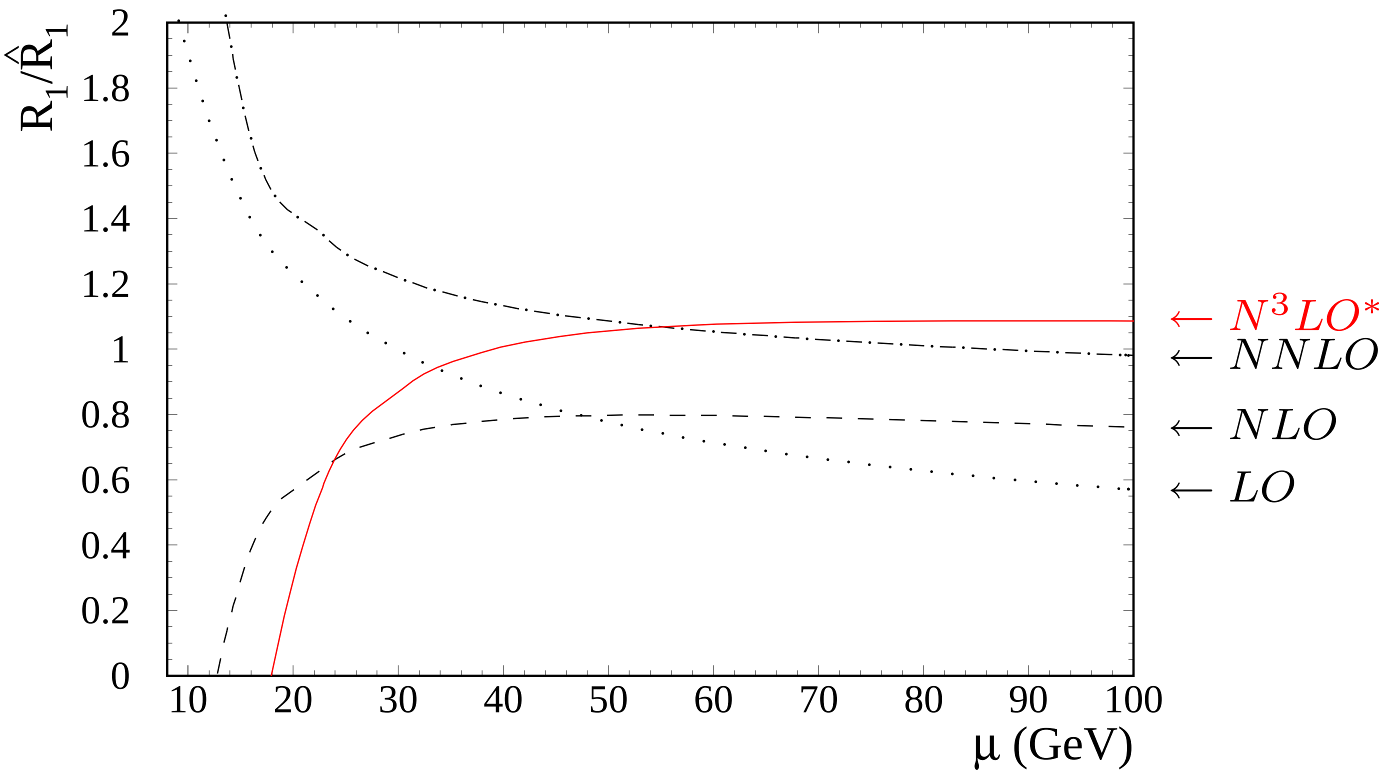
$$\boxed{E_{\rm res} = \left(1.9833 + 0.007\,\frac{m_t - 174.3 \text{ GeV}}{174.3 \text{ GeV}} \pm 0.0009\right) \times m_t}$$

Top quark mass with 80 *MeV accuracy*

$\Upsilon(1S)$ leptonic width



$e^+e^- \longrightarrow t\bar{t}$ resonance production



Nonrelativistic Renormalization Group

- Several scales: m_q, $m_q v$, $m_q v^2$
- Logarithmic integrals between the scales ⇨ $\ln v$ ⇨ $\ln \alpha_s$

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Logarithmic corrections ⟷ Singularities of effective theory

Renormalization of pNRQCD ⟶ NRG equations

Solution of NRG equations ⟶ Resummation of log corrections

Nonrelativistic Renormalization Group

- Spectrum:

  NNLL, $\alpha_s^n \ln^{n-2} \alpha_s$ (Pineda; Manohar, Hoang, Stewart)

  NLL HFS, $\alpha_s^n \ln^{n-1} \alpha_s$ (Kniehl, Penin, Pineda, Smirnov, Steinhauser)

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- Leptonic/photonic width and resonance production:

  NNLL spin dependence, $\alpha_s^n \ln^{n-2} \alpha_s$

  (Penin, Pineda, Smirnov, Steinhauser)

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  Resummation parameter $\ln v \sim 1$

  NRG can reduce scale dependence

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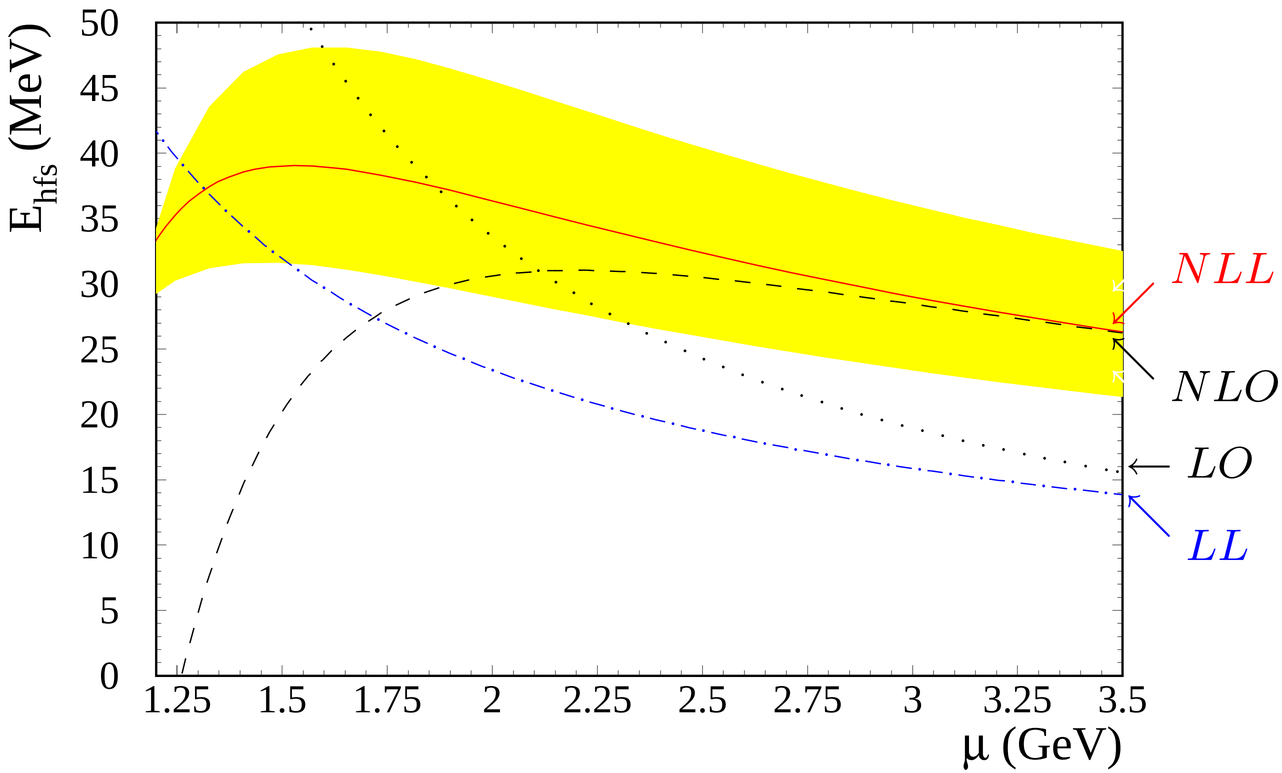
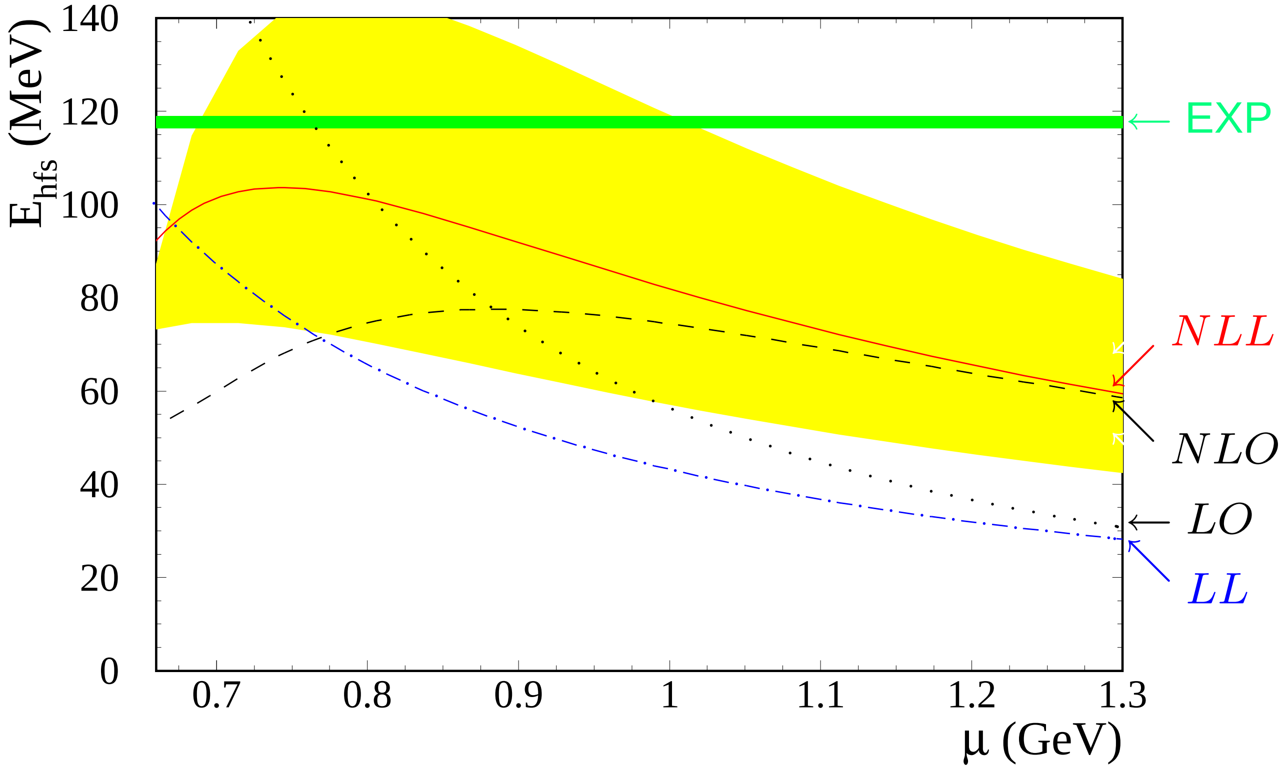
- Theoretical error estimate in (partial) NNLL analysis of $e^+e^- \to t\bar{t}$ threshold production (Manohar, Hoang, Stewart, Teubner)

  2% (2000) ⇨ 3% (2001) ⇨ 6% (2004)

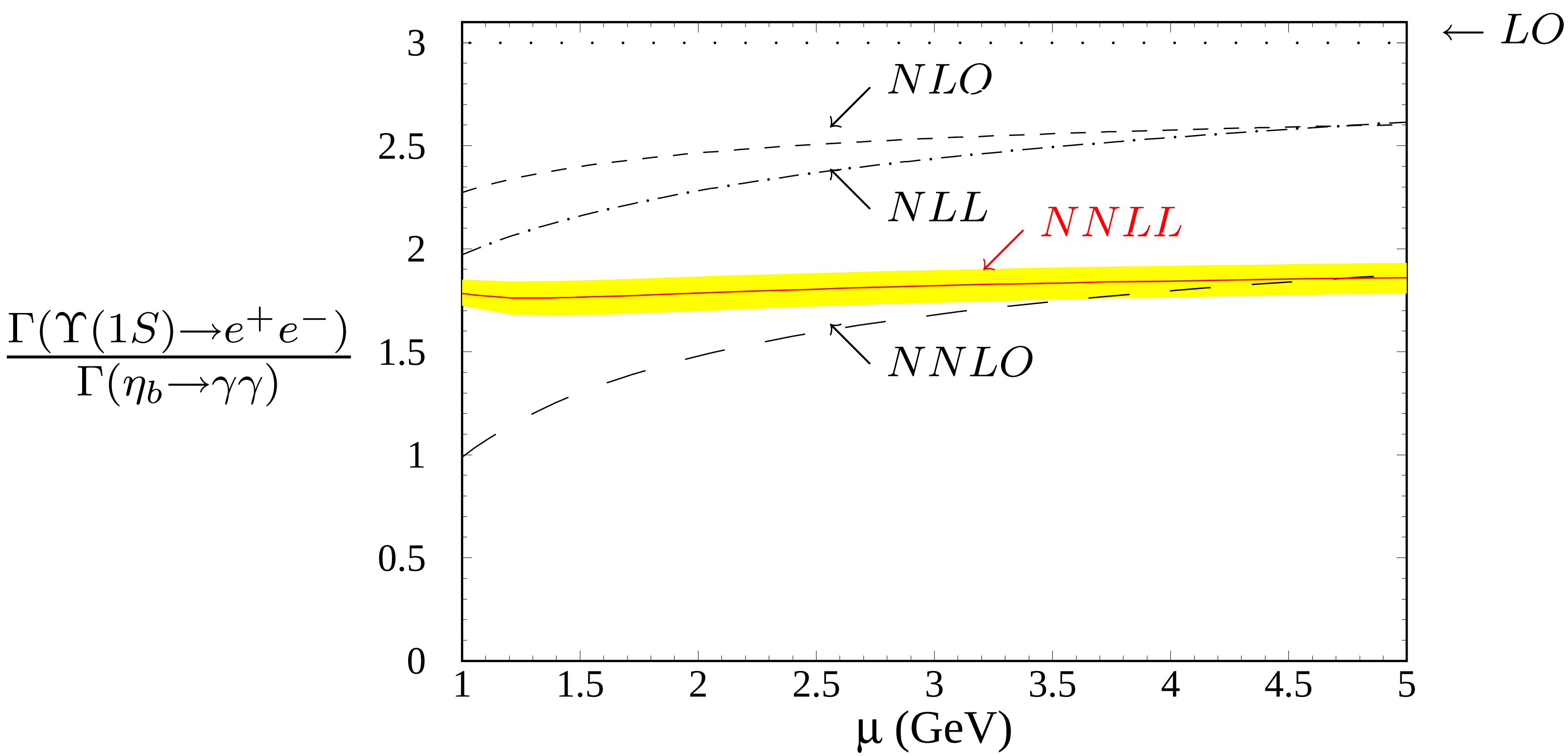
HFS in bottomonium



HFS in charmonium



$\Gamma_{\Upsilon(1S)}/\Gamma_{\eta_b}$ ratio



NRG analysis of η_b

- η_b mass from $M(\Upsilon(1S)) - M(\eta_b)$ in NLL:

$$\boxed{M(\eta_b) = 9421 \pm 11\,(\mathrm{th})\,{}^{+9}_{-8}\,(\delta\alpha_s)\ \mathrm{MeV}}$$

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- η_b photonic width from $\frac{\Gamma(\Upsilon(1S)\to e^+e^-)}{\Gamma(\eta_b\to\gamma\gamma)}$ in NNLL:

$$\boxed{\Gamma(\eta_b \to \gamma\gamma) = 0.659 \pm 0.089(\mathrm{th})^{+0.019}_{-0.018}(\delta\alpha_{\mathrm{s}}) \pm 0.015(\mathrm{exp})\ \mathrm{keV}}$$

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- $\alpha_s(M_Z)$ with the accuracy $\boxed{\pm 0.003}$

  best among the analyses based on low-energy data

Summary

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- Effective theory:
  - *Scale separation, power counting, NRG*
  - *Perturbative calculation through threshold expansion*
  - *First N^3LO, NNLL results on the market. Promising!*

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- Phenomenology:
  - m_b, m_t *from N^{3}LO spectrum*
  - η_b *properties from NRG*
  - α_s *from bottomonium HFS*

Challenging problems

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- Theory:
  - *Tree-loop static potental*
  - *N^3LO Υ-sum rules*
  - *N^3LO threshold production*
  - *pNRQCD + Lattice*

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- Theory:
  - *Tree-loop static potental*
  - *N^3LO Υ-sum rules*
  - *N^3LO threshold production*
  - *pNRQCD + Lattice*
- Experiment:
  - *Discovery of η_b, B_c^**
  - *Precise measurement of $\sigma(e^+e^- \to t\bar{t})$ at ILC*