High-Energy Limit of QED beyond Sudakov Approximation

Alexander Penin

University of Alberta & TTP Karlsruhe

SLAC, Stanford

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Topics discussed

- Introduction
  - *high energy limit, Sudakov double logarithms*
  - *power corrections and renormalization group*
- QED at high energy to $\mathcal{O}(m_e^2/Q^2)$
  - *non-Sudakov double logarithms*
  - *soft electron pair emission and eikonal charge nonconservation*
  - *two-loop result, all order resummation and asymptotic formula for the vector form factor*
- Based on:

  A.A. Penin, Phys.Lett. **B745** *(2015) 69, arXiv:1412.0671 [hep-ph]*

Sudakov limit

- Sudakov limit
  - *on-shell*
  - *exclusive*
  - *high energy*
  - *fixed angle*
- Sudakov logarithms *(leading power in m_e^2/Q^2)*
  - *each external line gets* $e^{-\frac{\alpha}{4\pi}\frac{C_R}{2}\ln^2(Q^2)}$

    Sudakov (1956); Frenkel, Taylor (1976)
  - *subleading logs exponentiate as well*

    Mueller (1979); Collins (1980); Sen (1981), ...
- What about power suppressed logs?

High energy limit beyond Sudakov approximation

- Logarithmically enhanced power corrections
  - *can be phenomenologically important at moderate energies*
  - *can become asymptotically dominant*
  - *intriguing from QFT point of view*

High energy limit beyond Sudakov approximation

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- Power corrections and renormalization group

$$\Delta\mathcal{L} = \sum_i C_i O_i\,, \qquad \frac{d}{d\ln\mu} C_i = f_i(C_j)$$

- *OPE, large mass expansion* ⇨ *local composite operators*
- *threshold nonrelativistic limit* ⇨ *spartially nonlocal potentials*
- *Sudakov ultrarelativistic limit* ⇨ ?

QED form factor

- Dirac/Pauli components

$$\mathcal{F} = \bar{\psi}(p_1)\left(\gamma_\mu F_1 + \frac{i\sigma_{\mu\nu}q^\nu}{2m_e}F_2\right)\psi(p_2)$$

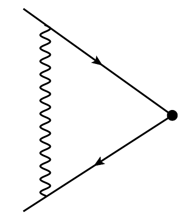
kinematics $\quad p_1^2 = p_2^2 = m_e^2, \qquad Q^2 = -(p_2-p_1)^2 \gg m_e^2 \gg \lambda^2$

- High-energy expansion

$$F_1 = e^{-\frac{\alpha}{2\pi}\ln\left(\lambda^2/m_e^2\right)\ln\rho}\sum_{n=0}^{\infty}\rho^n F_1^{(n)} \qquad (\rho = m_e^2/Q^2)$$

Dirac form factor at $\mathcal{O}(1)$

- Origin of double logs
  - *only vertex correction contribute in a covariant gauge*



  - *photon is soft and collinear to* p_i
  - *electrons are eikonal*

$$S = \frac{\not{p}_i - \not{l} + m_e}{(p_i - l)^2 - m_e^2} \approx -\frac{\not{p}_i}{2p_i l} \approx \frac{\gamma_\pm}{|\boldsymbol{l}|(\cos\theta \pm 1)}$$

- *Soft and collinear divergences*

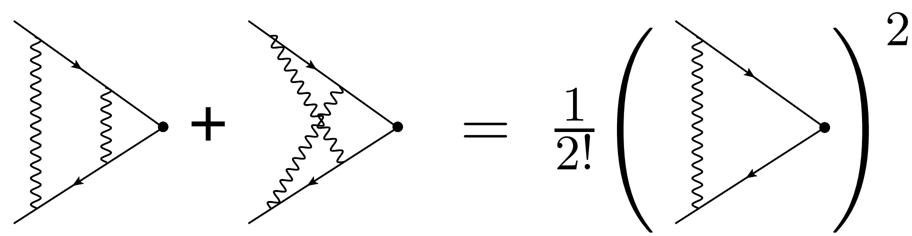
$$\int \frac{\mathrm{d}|\boldsymbol{l}|}{|\boldsymbol{l}|} \to \ln(\lambda) \qquad \int \frac{\mathrm{d}\cos\theta}{\cos\theta \pm 1} \to \ln(m_e)$$

Dirac form factor at $\mathcal{O}(1)$

- Eikonal factorization

$$\boxed{\frac{1}{p_i l_1}\,\frac{1}{p_i(l_1+l_2)} + \frac{1}{p_i(l_1+l_2)}\,\frac{1}{p_i l_2} = \frac{1}{p_i l_1}\,\frac{1}{p_i l_2}}$$

- Exponentiation



↪ $$\boxed{\text{n-loop} = \frac{(1\text{−loop})^n}{n!}}$$

- Sudakov double logarithmic result

$$\boxed{F_1^{(0)} = e^{-x}} \qquad \left(x = \frac{\alpha}{4\pi}\ln^2\rho\right)$$

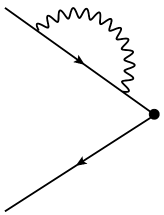
Dirac form factor at $\mathcal{O}(1)$

- Alternative picture: physical gauge

  - *forward radiation suppressed by helicity conservation*

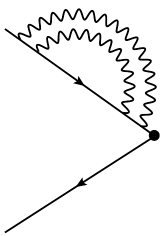
    ➪ only on-shell self-energies contribute

  ➥ universality of Sudakov DL



  - *ordering of virtual momenta ➪ rainbow diagrams*

  ➥ trivial color structure of Sudakov DL



- Alternative picture: evolution equation

$$\boxed{\frac{\partial}{\partial \ln Q} F_1 = \left[-\Gamma_{cusp} + \ldots\right] F_1}$$

  - *Cusp anomalous dimension* $\boxed{\Gamma_{cusp} = -\frac{\alpha}{\pi} \ln \rho}$

Dirac form factor to $\mathcal{O}(\rho)$

- Main idea
  - *expansion by regions* ⇨ *homogeneous integrals*
  - *leading singularity of a given region* ⇨ *double log*

Dirac form factor to $\mathcal{O}(\rho)$

Main idea

- *expansion by regions* ⇨ *homogeneous integrals*
- *leading singularity of a given region* ⇨ *double log*

Expansion by regions

I. identify the regions of virtual momentum

- *hard* $l \sim Q$
- *collinear* $l_+ \sim Q,\ l_- \sim m_e^2/Q$ *or* $l_- \sim Q,\ l_+ \sim m_e^2/Q$, $l_\perp \sim m_e$
- *soft* $l \sim m_e$

II. for each region

↪ *expand in small ratios*
integrate over all momentum space
use dimensional regularization

Dirac form factor to $\mathcal{O}(\rho)$

- *k-loop integral*

$$I_k = I_{c,\ldots,c} + I_{c,\ldots,h} + \ldots + I_{h,\ldots,h}$$

$$I_{c,\ldots,c} = \sum_{n=0}^{\infty} \left(\frac{m_e^2}{\tilde{p}_1 p_2}\right)^n I^{(n)}_{c,\ldots,c} + \ldots \qquad I^{(n)}_{c,\ldots,c} = \frac{1}{(m_e^2)^{k\varepsilon}} \left(\frac{c_k^{(n)}}{\varepsilon^{2k}} + \ldots\right)$$

- Sudakov logs ⇨ photon momentum $l \to 0$

soft photons *eikonal electrons*

$$\boxed{D = \frac{-g_{\mu\nu}}{l^2 - \lambda^2}} \qquad \boxed{S \approx -\frac{\not{p}_i + m_e}{2p_i l}}$$

$$c_k^{(0)} \propto \left(\frac{p_1 p_2}{\tilde{p}_1 p_2}\right)^k = 1 + \mathcal{O}(\rho^2) \qquad c_k^{(1)} = 0$$

Dirac form factor to $\mathcal{O}(\rho)$

- *either* $\mathcal{O}(\rho^2)$ *or subleading log*

- Cusp anomalous dimension

$$\boxed{\Gamma_{cusp} = -\frac{\alpha}{\pi}\ln\rho\left(1 + \mathcal{O}(\rho^2)\right)}$$

Korchemsky, Radyushkin (1987)

indeed there is no one-loop $x\rho$ *term ...*

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indeed there is no one-loop $x\rho$ *term ...*

but there is a two-loop $x^2\rho$ *one*

Dirac form factor to $\mathcal{O}(\rho)$

- Non-Sudakov logs ⇨ $l' = p_i - l \to 0$

eikonal photons *soft electrons*

$$D = \frac{g_{\mu\nu}}{2p_i l' - m_e^2 + \lambda^2}$$

$$S \approx \frac{m_e}{l'^2 - m_e^2}$$

Gorshkov, Gribov, Lipatov, Frolov (1968)

- Does not contribute at one-loop

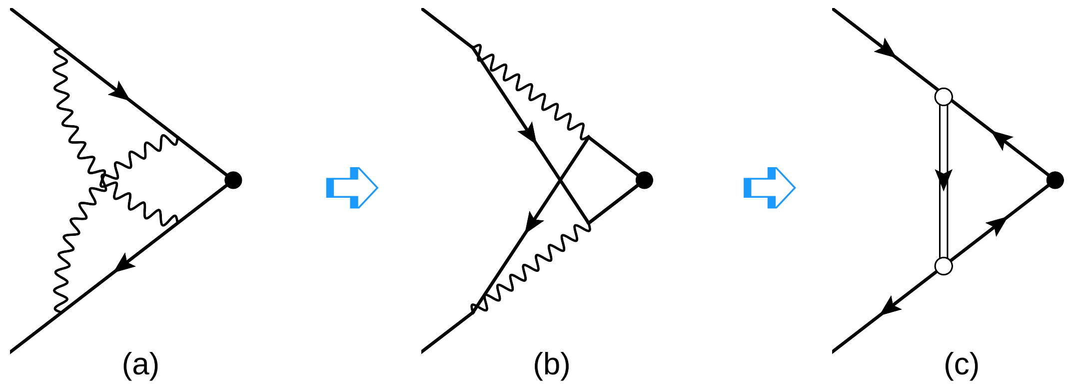
- Shows up in two-loop non-planar diagram

  - $l'_1 = p_1 - l_1 \to 0, \qquad l'_2 = p_2 - l_2 \to 0$

  ↪ *does not destroy eikonal structure*

Soft pair emission

- “Twist” the diagram



- *soft electon pair exchange*
- *eikonal charge nonconservation*

Evaluation of double logs

- *New variables*
  - *Sudakov parameters* $l = up_1 + vp_2 + l_\perp$
  - *hypercube coordinates* $\eta = \ln v/\ln\rho,\ \xi = \ln u/\ln\rho,\ 0 < \eta,\ \xi < 1$
  - *strict rapidity ordering* $\eta_i < \eta_j,\ \ldots$, *onshell condition* $\eta_i + \xi_i < 1$

- *Double log coefficient*
  - *volume under hypersurface*

$$F_1^{(1)}\Big|_{2-loop} = -4x^2 \int_{0<\eta_i,\ \xi_i<1} K(\eta_1,\eta_2,\xi_1,\xi_2)\mathrm{d}\eta_1\mathrm{d}\eta_2\mathrm{d}\xi_1\mathrm{d}\xi_2$$

  - *kernel* $K(\eta_1,\eta_2,\xi_1,\xi_2) = \theta(1-\eta_1-\xi_1)\theta(1-\eta_2-\xi_2)\theta(\eta_2-\eta_1)\theta(\xi_1-\xi_2)$
  - *result* $F_1^{(1)}\Big|_{2-loop} = -x^2/3$

Mastrolia, Remiddi (2003), Bernreuther *et al.* (2005)

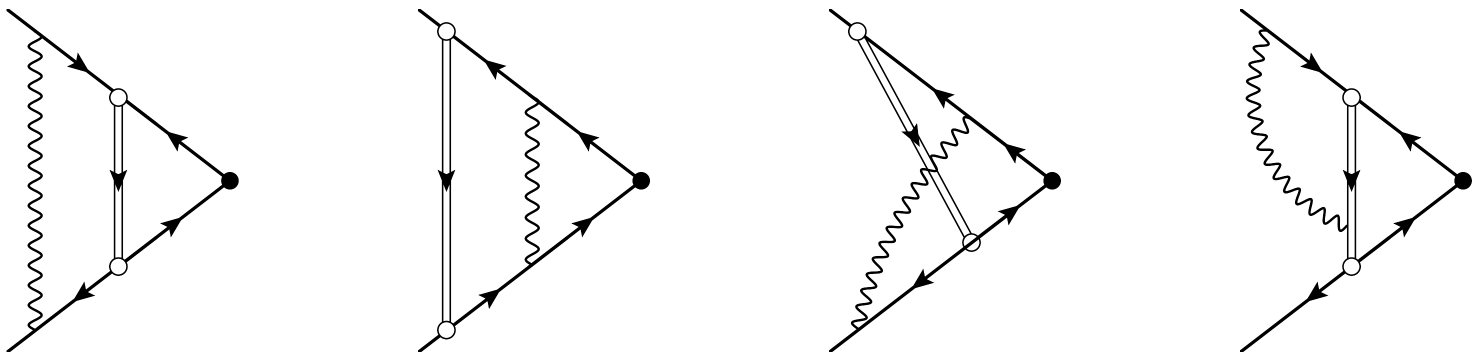
Universality

- *Quasi-universal character*
  - *leading power corrections are due to soft “scalar” electron exchange*
  - *the coefficient depends on scattering details*
  - ➥ *not determined by external particle charge only*

- *Two-loop form factors*
  - *Dirac, Pauli*
  - *scalar, axial*
  - ➥ *agree with known results*

Resummation

- *Dressing with Sudakov photons*



(a) (b) (c) (d)

Resummation

- *Multiple Sudakov exchange*

$$F_1^{(1)} = -4x^2 \int \mathrm{d}\eta_1 \mathrm{d}\eta_2 \mathrm{d}\xi_1 \mathrm{d}\xi_2 \, \phi^b(\eta_1,\xi_2)\phi^c(\eta_1,\xi_1)\phi^c(\xi_2,\eta_2)\phi^d(\eta_2)\phi^d(\xi_1) \times \Big[\phi^a(\eta_2,\xi_1)K_1(\eta_1,\eta_2,\xi_1,\xi_2) + K_2(\eta_1,\eta_2,\xi_1,\xi_2)\Big]$$

  - *form factors:*

$$\begin{aligned}
\phi^a(\eta,\xi) &= \exp\left[-x\,(1-\eta-\xi)^2\right], \\
\phi^b(\eta,\xi) &= \exp\left[-2x\eta\xi\right], \\
\phi^c(\eta,\xi) &= \exp\left[-x\eta\,(\eta+2\xi-2)\right], \\
\phi^d(\eta) &= \exp\left[-x\,(1-\eta)^2\right]
\end{aligned}$$

Resummation

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$$\times \Big[\phi^a(\eta_2, \xi_1) K_1(\eta_1, \eta_2, \xi_1, \xi_2) + K_2(\eta_1, \eta_2, \xi_1, \xi_2)\Big]$$

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Resummation

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$$\times\Big[\phi^a(\eta_2,\xi_1)K_1(\eta_1,\eta_2,\xi_1,\xi_2) + K_2(\eta_1,\eta_2,\xi_1,\xi_2)\Big]$$

  - *kernels:*

$$K_1(\eta_1,\eta_2,\xi_1,\xi_2) = \theta(1-\eta_2-\xi_1)\theta(1-\eta_2-\xi_2) \times\theta(\eta_2-\eta_1)\theta(\xi_1-\xi_2),$$
$$K_2(\eta_1,\eta_2,\xi_1,\xi_2) = \theta(1-\eta_1-\xi_1)\theta(1-\eta_2-\xi_2) \times\theta(\eta_2-\eta_1)\theta(\xi_1+\eta_2-1),$$

High-order double logarithms

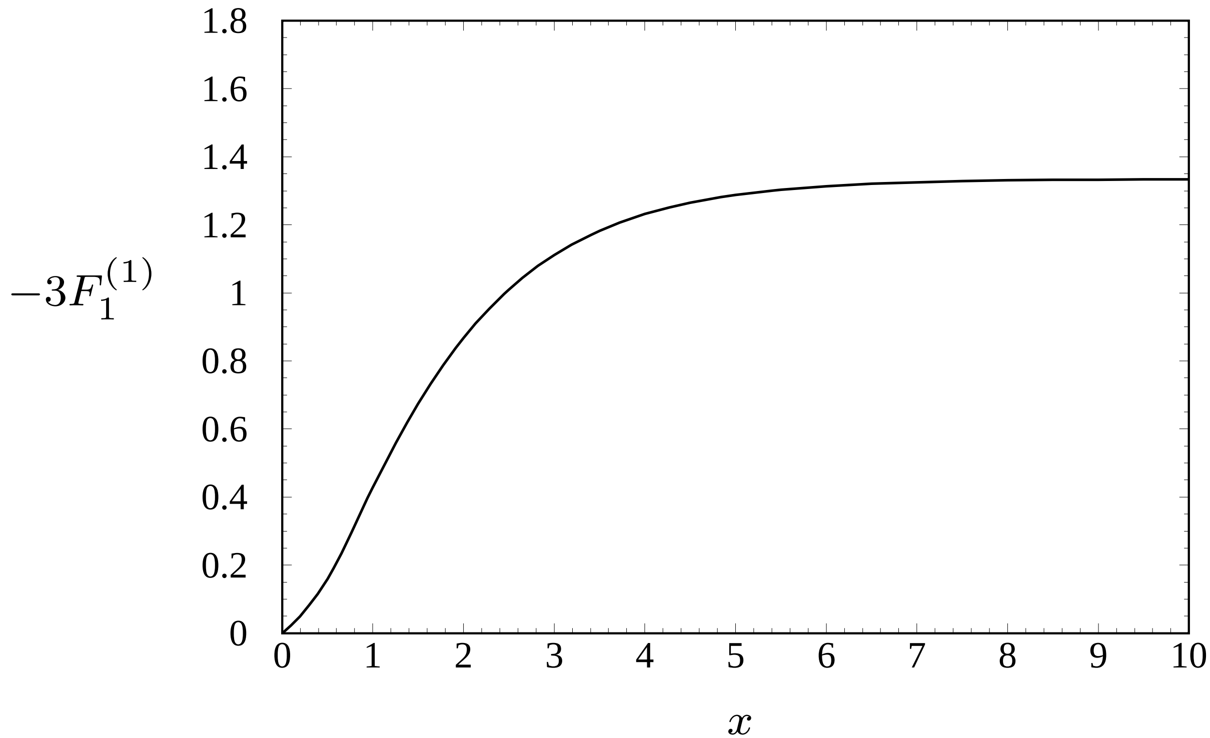
- *The series*

$$F_1^{(1)} = -\frac{x^2}{3}\left(1 + \sum_{n=1}^{\infty} c_n x^n\right)$$

- *The coefficients*

n	1	2	3	4	5	6	7
$(-1)^n n! c_n$	$\frac{29}{30}$	$\frac{257}{210}$	$\frac{1231}{630}$	$\frac{396581}{103950}$	$\frac{5531381}{630630}$	$\frac{72078311}{3153150}$	$\frac{4510839803}{68918850}$

High-energy limit



➥ *double logarithmic enhancement*

High-energy limit

- Asymptotic value

$$F_1^{(1)}(x=\infty) = -0.444988\ldots$$

  - *the approximation is valid for* $|\ln\rho| \ll 1/\alpha$ *i.e.* $x \ll 1/\alpha$

- Cf. electron-muon backward scattering

$$\sigma_{e^-\mu^-\to e^-\mu^-}(\theta=\pi) \sim e^{\sqrt{2x}}$$

(Gorshkov, Gribov, Lipatov, Frolov, 1968)

Summary

- The first DL resummation beyond Sudakov

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  - *soft pair exchange*

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- Drastically different from Sudakov logs
  - *eikonal charge nonconservation*
  - ↪ *DL enhancement*

Summary

- The first DL resummation beyond Sudakov
- New universal source of double logs
  - *soft pair exchange*
- Drastically different from Sudakov logs
  - *eikonal charge nonconservation*
  - ➥ *DL enhancement*
- To be done
  - *amplitudes beyond two legs*
  - *QCD beyond two loops*
  - *subleading logs*