Decay rate of Higgs to $\gamma\gamma$ mediated by light quarks

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Abstract

The amplitude for $H^0 \to \gamma\gamma$ mediated by a virtual quark loop is computed, and the leading logarithmic corrections due to strong interactions of virtual quarks are resummed to all orders in perturbation theory.The leading logarithmic corrections reduce the b-quark loop contribution to the total decay rate by $\approx 14\%$.

1 Background

The Higgs boson, predicted in the 1960s by Peter Higgs, Francois Englert and others, is a scalar particle in the Standard Model of physics which is responsible for giving mass to fermions as well as the vector bosons that mediate the weak nuclear force. It is an electrically neutral, spinless, colourless particle with a mass of approximately 125 GeV, detected for the first time in 2012 by the ATLAS and CMS experiments at the LHC at CERN [2, 3]. The Higgs field, of which Higgs bosons are excitations, has a non-zero vacuum expectation value, meaning that the field's lowest energy state has a non-zero value. This key feature permits the famous 'Higgs mechanism' to occur, whereby electroweak symmetry is broken, and the $W^{+/-}$ and Z^0 gauge bosons acquire mass.

Because the Higgs boson is the most recently discovered particle in the Standard Model, and one which plays such a key role in understanding the origin of mass, it is imperative to understand its decay channels and decay rates with high precision. Analyses of data collected at the LHC can use improved theoretical predictions to increase their precision in determining the Higgs mass as well as compare the predictions against experimentally determined values of its decay rates and coupling strength to other Standard Model particles.

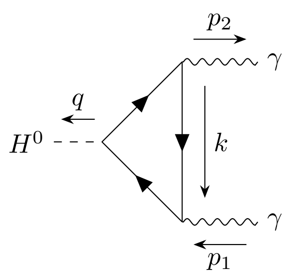
One of the cleanest decay channels for the Higgs boson is $H^0 \rightarrow \gamma\gamma$. This process is free of the complicated hadronic jets that commonly result from interactions whose products include quarks and gluons, or themselves decay quickly into quarks and gluons. The clear signature of such events in, for example, the ATLAS detector makes them a good candidate to study the Higgs with precision. In this work we consider the decay of a Higgs boson to two photons through a virtual quark loop. For the top quark, $m_t >> m_H$, and $\mathcal{M}$ scales with m_t^2. The top-quark loop is the dominant contribution to the decay rate. For quarks with mass much less than the Higgs, the amplitude is suppressed by $\frac{m_q}{m_H}$, but also proportional to $\log^2(\frac{m_q^2}{m_H^2})$, which is large and partially compensates for this suppression. Because of this, the amplitude of light-quark mediated decays is not negligible, and interferes with the amplitude from the top-quark loop, causing a change in the decay rate.

$$\begin{aligned}\delta\Gamma_{tot} &\propto |\mathcal{M}_{t\ loop} + \mathcal{M}_{b\ loop}|^2 - |\mathcal{M}_{t\ loop}|^2 = |\mathcal{M}_{b\ loop}|^2 + 2\mathcal{M}_{b\ loop}\mathcal{M}_{t\ loop} \\ &\approx 2\mathcal{M}_{b\ loop}\mathcal{M}_{t\ loop} = 2\frac{\mathcal{M}_{b\ loop}}{\mathcal{M}_{t\ loop}}\Gamma_{t\ loop}\end{aligned}$$

There are corrections that arise at the Higgs vertex dependent on $\log^2(\frac{m_q^2}{m_H^2})$ due to diagrams with exchange of virtual gluons between the quarks that are significant for the bottom-quark loop. The focus of the calculation is on finding these corrections.

2 Lowest-order diagram

We first consider the following, lowest-order diagram for $H^0 \to \gamma\gamma$ production.



The matrix element for this interaction is represented by

$$i\mathcal{M} = -\epsilon_\nu(p_1)\epsilon_\mu(p_2)\int \frac{d^4k}{(2\pi)^4} Tr\Big((-iQ)\gamma^\nu \frac{i(\not{k}+m_q)}{k^2-m_q^2+i\varepsilon}$$
$$\times(-iQ)\gamma^\mu \frac{i(\not{k}+\not{p}_2+m_q)}{(k+p_2)^2-m_q^2+i\varepsilon}$$
$$\times\frac{m_q}{v}\frac{i(\not{k}+\not{p}_1+m_q)}{(k+p_1)^2-m_q^2+i\varepsilon}\Big)$$

We will find that the most important range of integration is over small values of k, so we omit it in the numerator to obtain

$$i\mathcal{M} = -\,(iQ^2\frac{m_q}{v(2\pi)^4})\epsilon_\nu(p_1)\epsilon_\mu(p_2)Tr\Big(\gamma^\nu m_q\gamma^\mu(\not{p}_2+m_q)(\not{p}_1+m_q)\Big)$$
$$\times\int d^4k\frac{1}{k^2-m_q^2+i\varepsilon}\frac{1}{(k+p_2)^2-m_q^2+i\varepsilon}\frac{1}{(k+p_1)^2-m_q^2+i\varepsilon}$$

Following a procedure laid out in Landau & Lifshitz, we now introduce a change of variables to make this integration more convenient [1]. Let $k = k_\perp + k_\parallel$, where $k_\parallel = up_1 + vp_2$ lies in the p_1p_2 plane, and $k_\perp$ is orthogonal to this plane. Our new variables of integration are u, v, and $\rho = -k_\perp^2$. $k_\perp$ can be chosen spacelike so that $\rho > 0$. Then $d^4k = d^2k_\perp d^2k_\parallel$. $d^2k_\perp = |k_\perp|d|k_\perp|d\phi \to \frac{1}{2}d\rho(2\pi) = \pi d\rho$ since there is no ϕ dependence. Letting 0 and x indicate components in the p_1p_2 plane, $d^2k_\parallel = |\frac{\partial(k_0,k_x)}{\partial(u,v)}|dudv = |p_{10}p_{2x} - p_{20}p_{1x}|dudv = |p_{10}p_{20} - p_{2x}p_{1x}|dudv = |(p_1p_2)|dudv = \frac{1}{4}|q^2|dudv$, since $(p_1-p_2)^2 = (p_1)^2 + (p_2)^2 - 2(p_1p_2) = 0 + 0 - 2(p_1p_2) = (q)^2 \equiv t$. So $d^4k = \frac{1}{2}\pi|t|dudvd\rho$. Note that

$$2kp_1 = 2(up_1 + vp_2 + k_\perp)p_1 = 2v(p_2p_1) = -vt$$
$$2kp_2 = 2(up_1 + vp_2 + k_\perp)p_2 = 2u(p_1p_2) = -ut$$
$$k^2 = (up_1 + vp_2 + k_\perp)^2 = (k_\perp)^2 + 2uv(p_1p_2) = -\rho - uvt$$

In our calculation we will neglect k^2 when compared with kp_1 or kp_2. So we have

$$\int d^4k \frac{1}{k^2 - m_q^2 + i\varepsilon} \frac{1}{k^2 + p_2^2 + 2kp_2 - m_q^2 + i\varepsilon} \frac{1}{k^2 + p_1^2 + 2kp_1 - m_q^2 + i\varepsilon}$$
$$\approx \int d^4k \frac{1}{k^2 - m_q^2 + i\varepsilon} \frac{1}{2kp_2 - m_q^2 + i\varepsilon} \frac{1}{2kp_1 - m_q^2 + i\varepsilon}$$
$$= \frac{-\pi t}{2} \int \frac{dudvd\rho}{\rho + uvt + m_q^2 - i\varepsilon} \frac{1}{\rho + uvt + ut + m_q^2 - i\varepsilon} \frac{1}{\rho + uvt + vt + m_q^2 - i\varepsilon}$$

The range of values of u, v, ρ that allow us to discard extraneous terms and contribute to the double logarithm behaviour we seek lead us to impose the restrictions

$$|uvt| \ll |vt|, |ut| \Rightarrow |u|, |v| \ll 1$$
$$|\rho| \ll |ut|, |vt|$$
$$m_q^2 \ll |uvt| \Rightarrow \frac{m_q^2}{m_H^2|u|} \ll |v|$$
$$0 < \rho \text{ by def.}$$

Let $R \equiv \frac{m_q^2}{m_H^2}$. Using the above assumptions to set limits of integration and neglect subdominant terms, we arrive at

$$\frac{-\pi t}{2} \int_0^{min\{|ut|,|vt|\}} d\rho \left(\int_{-1}^{-R} + \int_R^1 \right) du \left(\int_{-1}^{-\frac{R}{|u|}} + \int_{\frac{R}{|u|}}^1 \right) dv \frac{1}{\rho + uvt - i\varepsilon} \frac{1}{ut} \frac{1}{vt}$$

We first perform the integration over ρ using residues.

$$\int_0^{min\{|ut|,|vt|\}} d\rho \frac{1}{\rho + uvt - i\varepsilon}$$
$$= \log\left(min\{|ut|,|vt|\} + uvt\right) - \log uvt + \begin{cases} i\pi, & \text{if } uvt < 0 \\ 0, & \text{if } uvt > 0 \end{cases}$$
$$\approx \log\left(min\{\frac{1}{|u|}, \frac{1}{|v|}\}\right) + \begin{cases} i\pi, & \text{if } uvt < 0 \\ 0, & \text{if } uvt > 0 \end{cases}$$

If we consider the integral over u and v with the first term, we see that the integrand is odd with respect to both u and v, and the integral vanishes over the range of integration. The integrals over u and v with the same sign are equal, and opposite to the integrals over u and v with opposite sign. Thus we are left with the integrals over u and v with opposite sign, with the integrand

$\frac{-i\pi}{uvt^2}$. This integrand is again odd for both u and v, so we can switch the order and sign of the negative limits of integration to obtain

$$\frac{-\pi t}{2}\frac{-2\pi i}{t^2}\int_R^1 du\int_{\frac{R}{|u|}}^1 \frac{dv}{uv} = \frac{i\pi^2}{t}\int_R^1 du\frac{\log\frac{|u|}{R}}{u} = \frac{i\pi^2}{t}\int_0^{\log\frac{1}{R}} dyy = \frac{i\pi^2}{2t}\log^2\frac{m_H^2}{m_q^2}$$

$$\Rightarrow i\mathcal{M} = -(iQ^2\frac{m_q}{v(2\pi)^4})\epsilon_\nu(p_1)\epsilon_\mu(p_2)Tr\Big(\gamma^\nu m_q\gamma^\mu(\not p_2+m_q)(\not p_1+m_q)\Big)\frac{i\pi^2}{2t}\log^2\frac{m_H^2}{m_q^2}$$

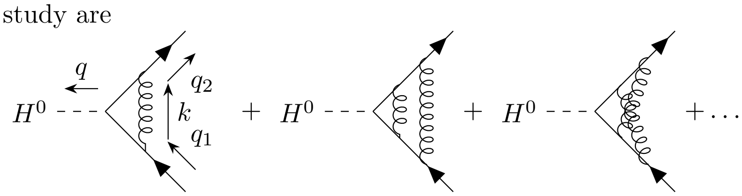
For completeness we evaluate the trace.

$$\begin{aligned}
&Tr\Big(\gamma^\nu m_q\gamma^\mu(\not p_2+m_q)(\not p_1+m_q)\Big) = Tr\Big(\gamma^\nu m_q\gamma^\mu \not p_2\not p_1\Big) + Tr\Big(m_q^3\gamma^\nu\gamma^\mu\Big)\\
&= 4m_q p_{2\alpha}p_{1\beta}(g^{\nu\mu}g^{\alpha\beta} - g^{\nu\alpha}g^{\mu\beta} + g^{\nu\beta}g^{\mu\alpha}) + 4m_q^3 g^{\mu\nu}\\
&= 4m_q^3 g^{\mu\neq} + 4m_q(g^{\mu\nu}(p_1p_2) - p_2^\nu p_1^\mu + p_1^\nu p_2^\mu)\\
&\approx -2m_q m_H^2 g^{\mu\nu}
\end{aligned}$$

where in the last step we have used $m_q \ll m_H$ and implicitly, the orthogonality of the photon momenta with their polarization vectors.

3 Quantum corrections

We now find the corrections to the Higgs-quark interaction vertex factor due to virtual gluons exchanged between the quarks. The relevant diagrams that we study are



There are other possible diagrams with internal gluons, including those with a three-gluon vertex and those with gluons between other legs of the quark loop, as well as self-energy loops. During the calculation we will show how these diagrams do not contribute to the double logarithmic behaviour we wish to find. We begin with the first diagram, and show how the result can be easily generalized to all orders, using methods outlined in Landau & Lifshitz [1]. Note that this diagram always appears with the external quarks as virtual, so we have

$$\begin{aligned}
i\mathcal{M}^1 = \int \frac{d^4k}{(2\pi)^4}Tr\Big(&\frac{i(\not q_2+m_q)}{(q_2)^2-m_q^2+i\varepsilon}(\frac{-ig_s}{2}\lambda^\alpha\gamma^\mu)\frac{i(\not q_2-\not k+m_q)}{(q_2-k)^2-m_q^2+i\varepsilon}\frac{m_q}{v}\\
&\times\frac{i(\not q_1-\not k+m_q)}{(q_1-k)^2-m_q^2+i\varepsilon}\frac{-i\delta^{\alpha\beta}g_{\mu\nu}}{k^2+i\varepsilon}(\frac{-ig_s}{2}\lambda^\beta\gamma^\nu)\frac{i(\not q_1+m_q)}{(q_1)^2-m_q^2+i\varepsilon}\Big)
\end{aligned}$$

The colour factor from $\lambda^\alpha \lambda^\alpha$ comes to $\frac{4}{3}$. We perform the same procedure here as for the earlier diagram. So, we assume $m_q^2 \ll |q_1 q_2| \approx \frac{1}{2}q^2$. This yields

$$
\begin{aligned}
i\mathcal{M}^1 =& \frac{m_q}{v} Tr\Big(\frac{i(\not{q}_2 + m_q)}{(q_2)^2 - m_q^2 + i\varepsilon}(\gamma^\mu)(\not{q}_2 + m_q)(\not{q}_1 + m_q)(\gamma_\mu)\frac{i(\not{q}_1 + m_q)}{(q_1)^2 - m_q^2 + i\varepsilon}\Big) \\
&\times \int \frac{d^4k}{(2\pi)^4}(\frac{-4ig_s^2}{3})\frac{1}{(q_2 - k)^2 - m_q^2 + i\varepsilon}\frac{1}{k^2 + i\varepsilon}\frac{1}{(q_1 - k)^2 - m_q^2 + i\varepsilon} \\
&\approx \frac{m_q}{v} Tr\Big(\frac{i(\not{q}_2 + m_q)}{(q_2)^2 - m_q^2 + i\varepsilon}\frac{i(\not{q}_1 + m_q)}{(q_1)^2 - m_q^2 + i\varepsilon}\Big) \\
&\times \int \frac{d^4k}{(2\pi)^4}(\frac{8ig_s^2 t}{3})\frac{1}{(q_2 - k)^2 - m_q^2 + i\varepsilon}\frac{1}{k^2 + i\varepsilon}\frac{1}{(q_1 - k)^2 - m_q^2 + i\varepsilon}
\end{aligned}
$$

Making the same change of variables to u, v, ρ, and using slightly amended conditions on the range of these variables,

$$
\begin{aligned}
&|uvt| \ll |vt|, |ut| \Rightarrow |u|, |v| \ll 1 \\
&|\rho| \ll |ut|, |vt| \\
&|q_1^2| \ll |vt|, |q_2^2| \ll |ut| \\
&0 < \rho \text{ by def.}
\end{aligned}
$$

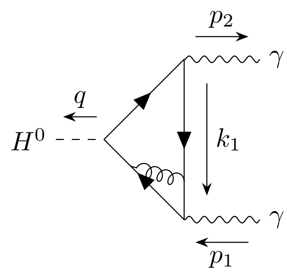
The integral above then becomes

$$
\begin{aligned}
I_1 =& -\frac{1}{2}\pi|t|(\frac{8ig_s^2 t}{3(2\pi)^4})\frac{1}{t^2}\int_0^{min\{|ut|,|vt|\}} d\rho \\
&\Bigg(\int_{-1}^{-q_2^2/t} + \int_{q_2^2/t}^{1}\Bigg)du\Bigg(\int_{-1}^{-q_1^2/t} + \int_{q_1^2/t}^{1}\Bigg)dv\frac{1}{\rho + uvt - i\varepsilon}\frac{1}{u}\frac{1}{v}
\end{aligned}
$$

Again the logarithmic term of the ρ integral disappears due to oddness of the function and symmetry of the range of integration, leaving us with two integrals proportional to $i\pi$ over u and v with opposite sign.

$$
\begin{aligned}
I_1 &= (-2i\pi)(\frac{i\pi}{2})(\frac{8ig_s^2}{3(2\pi)^4})\int_{q_2^2/t}^{1} du \int_{q_1^2/t}^{1} dv\frac{1}{uv} \\
&= -\frac{g_s^2}{6\pi^2}\log|\frac{q_1^2}{t}|\log|\frac{q_2^2}{t}|
\end{aligned}
$$

Before moving on to diagrams with more loops, it is necessary here to comment on diagrams of the following form:



We cannot here make the assumption that $p_1^2 \gg k_1^2, (p_1 - k_1)^2$, so the conditions for the double-logarithmic behaviour are not satisfied, and such diagrams do not contribute to our result [1]. For diagrams with self-energy loops, the resulting integrals do not have the correct balance of powers of k in the numerator and denominator to give double logarithmic corrections. Now consider a diagram with an arbitrary number n of virtual gluons, with an arbitrary number of crossings, indexed by the order of their lower vertices. Then the sum of the contributions of these diagrams, over interchanges of the order of the upper vertices, and again discarding all terms like q_1^2, q_2^2, k_i^2 compared to terms like $q_1 k_i, q_2 k_i$, is

$$
\begin{aligned}
i\mathcal{M}^n \approx & \frac{m_q}{v} Tr\Big(\frac{i(\not{q}_2 + m_q)}{(q_2)^2 - m_q^2 + i\varepsilon} \frac{i(\not{q}_1 + m_q)}{(q_1)^2 - m_q^2 + i\varepsilon} \Big) \big(\frac{8 i g_s^2 t}{3(2\pi)^4} \big)^n \\
& \sum_{int\ 2} \int \frac{d^4k_1 \ldots d^4k_n}{\prod_{i=1,2} 2(q_i k_1) 2(q_i k_1 + q_i k_2) \ldots 2(q_i k_1 + \ldots + q_i k_n) k_1^2 k_2^2 \ldots k_n^2} \\
& = \frac{m_q}{v} Tr\Big(\frac{i(\not{q}_2 + m_q)}{(q_2)^2 - m_q^2 + i\varepsilon} \frac{i(\not{q}_1 + m_q)}{(q_1)^2 - m_q^2 + i\varepsilon} \Big) \big(\frac{8 i g_s^2 t}{3(2\pi)^4} \big)^n J_n
\end{aligned}
$$

For diagrams with crossed gluon lines, the non-commutativity of the λ matrices is compensated by contributions from diagrams with three-gluon vertices, allowing us to interchange matrix factors and arrive at the one-loop coefficient raised to the power of n. Now, to solve the sum of integrals, consider the following: If we interchange any of the k_i in the terms with q_1, we are effectively only renaming the gluon momenta, and the integral will not change, so we can interchange k_i factors paired with both q_1 and q_2, and divide by $n!$ to achieve the same result.

$$
J_n = \frac{1}{n!} \sum_{int\ all} \int \frac{d^4k_1 \ldots d^4k_n}{\prod_{i=1,2} 2(q_i k_1) 2(q_i k_1 + q_i k_2) \ldots 2(q_i k_1 + \ldots + q_i k_n) k_1^2 k_2^2 \ldots k_n^2}
$$

We use the identity

$$\sum_{int} \frac{1}{(a_1)(a_1+a_2)\dots(a_1+\dots+a_n)} = \frac{1}{a_1}\frac{1}{a_2}\dots\frac{1}{a_n}.$$

$$\Rightarrow J_n = \frac{1}{n!}\int \frac{d^4k_1\dots d^4k_n}{2(q_1k_1)2(q_2k_1)k_1^2 2(q_1k_2)2(q_2k_2)k_2^2\dots 2(q_1k_n)2(q_2k_n)k_n^2} = \frac{J_1^n}{n!}$$

But we know from the 1-gluon case that

$$J_1 = (-\frac{1}{2}\pi|t|)(\frac{1}{t^2})(-2i\pi)\log|\frac{q_1^2}{t}|\log|\frac{q_2^2}{t}| = \frac{i\pi^2}{t}\log|\frac{q_1^2}{t}|\log|\frac{q_2^2}{t}|$$

$$\Rightarrow \sum_{n=0}^{\infty}\mathcal{M}^n = \frac{m_q}{v}Tr\Big(\frac{i(\not{q}_2+m_q)}{(q_2)^2-m_q^2+i\varepsilon}\frac{i(\not{q}_1+m_q)}{(q_1)^2-m_q^2+i\varepsilon}\Big)\sum_{n=0}^{\infty}\frac{(\frac{-g_s^2}{6\pi^2}\log|\frac{q_1^2}{t}|\log|\frac{q_2^2}{t}|)^n}{n!}$$

$$= \frac{m_q}{v}Tr\Big(\frac{i(\not{q}_2+m_q)}{(q_2)^2-m_q^2+i\varepsilon}\frac{i(\not{q}_1+m_q)}{(q_1)^2-m_q^2+i\varepsilon}\Big)\exp\Big(\frac{-2\alpha_s}{3\pi}\log|\frac{q_1^2}{t}|\log|\frac{q_2^2}{t}|\Big)$$

Thus we see that in the full diagram for $H^0 \to \gamma\gamma$, the Higgs vertex factor including corrections is $\frac{m_q}{v}\exp(\frac{-2\alpha_s}{3\pi}\log|\frac{q_1^2}{t}|\log|\frac{q_2^2}{t}|)$

4 Matrix element with corrections

We can insert this factor into our integral from the lowest-order case and solve the same way again. Note that now $q_1 = p_1+k,\ q_2 = p_2+k$, so $q_1^2 \approx 2kp_1 = -vt$ and $q_2^2 \approx 2kp_2 = -ut$. Therefore, we must calculate

$$i\mathcal{M}_{\mathcal{LL}} = -(iQ^2\frac{m_q}{v(2\pi)^4})\epsilon_\nu(p_1)\epsilon_\mu(p_2)Tr\Big(\gamma^\nu m_q\gamma^\mu(\not{p}_2+m_q)(\not{p}_1+m_q)\Big)$$

$$\times -\frac{\pi t}{2}\int_0^{min\{|ut|,|vt|\}} d\rho\Big(\int_{-1}^{-R}+\int_R^1\Big)du\Big(\int_{-1}^{-\frac{R}{|u|}}+\int_{\frac{R}{|u|}}^1\Big)dv$$

$$\frac{1}{\rho+uvt-i\varepsilon}\frac{1}{ut}\frac{1}{vt}\exp\Big(\frac{-2\alpha_s}{3\pi}\log|u|\log|v|\Big)$$

We perform the now familiar integration over ρ, noting that the exponential factor is even, so the overall function retains its oddness, leaving us with (omitting the already stated coefficients)

$$\frac{-2i\pi}{t^2}\int_R^1 du\int_{\frac{R}{|u|}}^1 dv\frac{1}{u}\frac{1}{v}\exp\Big(\frac{-2\alpha_s}{3\pi}\log|u|\log|v|\Big)$$

We now define $x = \frac{\log|u|}{\log R},\ y = \frac{\log|v|}{\log R}$, so that $dx = \frac{du}{\log R|u|},\ dy = \frac{dv}{\log R|v|}$ and our integral transforms to

$$
\begin{aligned}
&\frac{-2i\pi\log^2 R}{t^2}\int_0^1 dx\int_0^{1-x} dy\exp\frac{-2\alpha_s\log^2 R}{3\pi}xy\\
&=\frac{-2i\pi\log^2 R}{t^2}\int_0^1 dx\int_0^{1-x} dy\sum_{n=0}^{\infty}\frac{(\frac{-2\alpha_s\log^2 R}{3\pi}xy)^n}{n!}\\
&=\frac{-2i\pi\log^2 R}{t^2}\sum_{n=0}^{\infty}\int_0^1 dx\frac{(\frac{-2\alpha_s\log^2 R}{3\pi}x)^n(1-x)^{n+1}}{(n+1)!}\\
&=\frac{-2i\pi\log^2 R}{t^2}\sum_{n=0}^{\infty}\frac{(\frac{-2\alpha_s\log^2 R}{3\pi})^n}{(n+1)!}\mathrm{B}(n+1,n+2)\\
&=\frac{-2i\pi\log^2 R}{t^2}\sum_{n=0}^{\infty}\frac{(\frac{-2\alpha_s\log^2 R}{3\pi})^n}{(n+1)!}\frac{\Gamma(n+1)\Gamma(n+2)}{\Gamma(2n+3)}\\
&=\frac{-2i\pi\log^2 R}{t^2}\sum_{n=0}^{\infty}\Big(\frac{-2\alpha_s\log^2 R}{3\pi}\Big)^n\frac{n!}{(2n+2)!}\\
&=\frac{-2i\pi\log^2 R}{t^2}\frac{1}{2}\ {}_2F_2\big(1,1;\frac{3}{2},2;\frac{-\alpha_s\log^2 R}{6\pi}\big)
\end{aligned}
$$

Therefore, putting it all together, and with ${}_aF_b$ the generalized hypergeometric function,

$$
\begin{aligned}
&i\mathcal{M}_{LL}=-(iQ^2\frac{m_q}{v(2\pi)^4})\epsilon_\nu(p_1)\epsilon_\mu(p_2)Tr\Big(\gamma^\nu m_q\gamma^\mu(\not{p}_2+m_q)(\not{p}_1+m_q)\Big)\\
&\times\frac{i\pi^2\log^2\frac{m_H^2}{m_q^2}}{2t}\ {}_2F_2\big(1,1;\frac{3}{2},2;\frac{-\alpha_s\log^2 R}{6\pi}\big)
\end{aligned}
$$

The asymptotic behaviour of this hypergeometric function ${}_2F_2\big(1,1;\frac{3}{2},2;\frac{-x}{4}$, as $x\equiv\frac{2\alpha_s\log^2 R}{3\pi}\to\infty$, is ${}_2F_2\big(1,1;\frac{3}{2},2;\frac{-x}{4}\big)\sim\frac{2(\log x+\gamma_{EM})}{x}$, obtained using Mathematica.

5 Comparison to lowest-order diagram

Comparing $\mathcal{M}_{LL}$ to the uncorrected matrix element

$$
i\mathcal{M}_{LO}=-(iQ^2\frac{m_q}{v(2\pi)^4})\epsilon_\nu(p_1)\epsilon_\mu(p_2)Tr\Big(\gamma^\nu m_q\gamma^\mu(\not{p}_2+m_q)(\not{p}_1+m_q)\Big)\frac{i\pi^2}{2t}\log^2\frac{m_H^2}{m_q^2}
$$

we see that

$$
\frac{\mathcal{M}_{LL}}{\mathcal{M}_{LO}}=\ {}_2F_2\big(1,1;\frac{3}{2},2;\frac{-\alpha_s\log^2 R}{6\pi}\big)
$$

We can calculate the magnitude of this correction. First, we calculate the value of α_s at the energy scale of the bottom quark, $\mu=5$ GeV.

$$\frac{g_s^2}{4\pi} = \alpha_s(\mu^2) = \frac{4\pi}{\beta_0 \ln(\frac{\mu^2}{\Lambda_{QCD}^2})}$$

$$\Lambda_{QCD} \approx 0.1 \text{ GeV}$$

$$\beta_0 = 11 - \frac{2}{3}n,\ n = 5\ \Rightarrow \beta_0 = \frac{23}{3}$$

$$\Rightarrow \alpha_s((5\text{GeV})^2) = 0.21$$

$$\log^2(R) = \log^2(m_b^2/m_H^2) = \log^2(5 \text{ GeV})^2/(125.7 \text{ GeV})^2 = 41.59$$

$$\frac{-\alpha_s \log^2 R}{6\pi} = -0.463$$

The variation in the bottom-quark contribution to the overall decay rate from the top quark is:

$$\frac{\delta\Gamma_{LL}}{\delta\Gamma_{LO}} = \frac{\mathcal{M}_{LL}}{\mathcal{M}_{LO}} = {}_2F_2\big(1, 1; \frac{3}{2}, 2; -0.463\big) = 0.863$$

So the bottom-quark contribution is reduced by 14%.

6 Conclusion

The double-logarithmic corrections to the amplitude for $H^0 \to \gamma\gamma$ in the case of a bottom-quark loop with small loop momentum due to virtual quarks inside the loop were found, and the contributions from all orders were summed to give a total correction to the amplitude. It was found that these corrections have a significant effect on the contribution to the decay rate of the Higgs boson due to interference between the bottom- and top-loop amplitudes. That contribution is reduced by approximately 14%. This result demonstrates the need for precision in theoretical calculations of Higgs production and decay rates, in order to better interpret the experimental data obtained at the LHC.

References

[1] V.B. Berestetskii, E.M. Lifshitz, and L.P. Pitaevskii. *Quantum Electrodynamics, 2 ed.* Oxford, 1982.

[2] The ATLAS Collaboration. "Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC". In: *Physics Letters B* 716 (2012), pp. 1–29.

[3] The CMS Collaboration. "Observation of a new boson at a mass of 125 GeV with the CMS experiment at the LHC". In: *Physics Letters B* 716 (2012), pp. 30–61.