A Literature Review on Asymptotic Safety

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Abstract

This research project aims to provide a general overview for Asymptotic Safety (AS), a criterion for quantum gravity in theoretical physics. The report begins with an introduction to foundational concepts, such as renormalization group flows and fixed points, with a discussion on how it relates to Weinberg's criterion for AS. The subsequent part details analysis on the Einstein-Hilbert truncation and the qualitative behavior of its fixed points. Notably, an emphasis is placed on the UV-attractive fixed point that implies pure gravity may be nonperturbatively renormalizable. Finally, applications of AS to matter fields and the fine structure constant are discussed, and the report closes off with a brief discussion of the issues with the theory.

1 Introduction

Theoretical physics currently stands on foundations provided by two of the most empirically successful theories: the Standard Model (SM) of particle physics and Einstein's General Relativity (GR). The concept of effective field theory lies at the intersection of these theories, which provides a quantum interpretation of GR consistent with the SM at relatively low energy scales, accomplished through the introduction of an energy cutoff, Λ. This approach, however, begs the question of finding a universally applicable, ultraviolet (UV) complete theory that remains valid at arbitrarily high energy scales, thus eliminating the need for Λ.

One of the foremost challenges in unifying General Relativity with the Standard Model is its non-renormalizability; conventional renormalization techniques fail to address the infinities and divergences that emerge. Traditionally, perturbation theory attempts to mitigate these issues by introducing an infinite series of terms to counterbalance the divergences, resulting in a model rendered useless by the infinite number of free parameters. This problem of UV divergences and infinite free parameters can be resolved, in principle, using the framework of Asymptotic Safety. In 1978, Weinberg suggested a generalized condition for renormalizability, using ideas from Wilson's renormalization group [24]. The renormalization group (RG) flow represents the interaction strengths as a function of the energy, and using the functional RG equations, we can determine a"fixed point", where the RG flows halt. If such a fixed point exists, then it is possible recover the scale invariance of the system, consequently eliminating the need for a UV cutoff. With a fixed point, the coupling constants tend to a finite value, independent of the energy scale. The prototypical example of asymptotic safety is QCD, where in the high-energy limit the system converges to a Gaussian fixed point. In other words, this is known as asymptotic freedom.

The feasibility of applying Asymptotic Safety within the context of quantum gravity was first demonstrated by Reuter in 1998 [18] with a truncated version of the Einstein-Hilbert action. It was shown that fixed points indeed exists, and one of them is a nontrivial UV-attractive fixed point. This was a major breakthrough and set the example for many later calculations to follow. A significant amount of later work sought to generalize this result. This involved including higher order terms in the curvature polynomial $f(R)$ [4] or incorporating contributions from the Riemann tensor and Ricci tensor. It was also observed by [3] that couplings to matter fields significantly affected the behaviour of fixed points in the AS theory. Beyond this, AS has been successfully applied to a number of phenomenological scenarios, such as predicting the mass of the Higgs boson [22] and demonstrating the existence of a well-behaved ultraviolet limit for the fine structure constant [9]. At the same time, AS is not without critiques, which involve unitarity and performing calculations in Lorentzian metrics as opposed to Euclidean metrics [2, 5]. This paper is structured as follows: in Section 2, I provide a detailed introduction to renormalization groups (particularly the Wilsonian approach) and build on that to introduce Renormalization Group flows. From there, it is easy to understand β-functions and the role they play. After explaining background ideas, in Section 3, I present a case study of Asymptotic Safety for the Einstein-Hilbert truncation and discuss its implications. The last section gives an overview of the applications of Asymptotic Safety and the criticisms surrounding it.

2 Renormalization Group

Our description of the physical world is fundamentally dependent on the *scale* which we are considering; when studying chemical systems, we can safely ignore any details in the nuclei. Down to the hadrons and mesons scale, we ignore processes going on with the quarks. Therefore, all theories of physics amount to an *effective theory*, which gives a description of the physical world based on the length (or energy) scale [10]. At smaller scales, the effective theory breaks down. To describe our physical theories, we naturally need a scheme to transform between energy scales–this procedure is known as renormalization.

Here I introduce the Wilsonian renormalization group staring with the idea of renormalization as a demonstration of integrating out degrees of freedom to obtain an effective action. Noting the form of the action, it is possible to rewrite the result such that the form of the action is invariant, but it then induces a “flow” on the coupling constants, which naturally leads into the renormalization group flow and β-functions.

2.1 Integrating out Degrees of Freedom

To begin, consider one of the simplest actions in quantum field theory: φ^4-theory. The Lagrangian density is written as:

$$\mathcal{L}(\varphi) = \frac{1}{2}\left[(\partial_\mu \varphi)^2 + m^2\varphi^2\right] + \frac{\lambda}{4!}\varphi^4. \tag{1}$$

The action $\mathcal{S}$ is given by the integral of the Lagrangian density:

$$\mathcal{S} = \int d^d x \mathcal{L} = \int d^d x \left\{\frac{1}{2}\left[(\partial_\mu \varphi)^2 + m^2\varphi^2\right] + \frac{\lambda}{4!}\varphi^4\right\}, \tag{2}$$

where d is an arbitrary dimension of spacetime. The path integral, as written in Peskin and Schroeder [16] is given by the exponential of the action:

$$\mathcal{Z} = \int [\mathcal{D}\varphi]_\Lambda \exp\left\{-\int \mathrm{d}^d x \left[\frac{1}{2}(\partial_\mu \varphi)^2 + \frac{1}{2}m^2\varphi^2 + \frac{\lambda}{4!}\varphi^4\right]\right\}, \tag{3}$$

with

$$[D\varphi]_\Lambda = \prod_{|k|<\Lambda} d\varphi(k). \tag{4}$$

This means that instead of integrating over all possible momenta for the fields, we only integrate the fields up to a cutoff Λ. We now consider the system at a lower energy $b\Lambda$, where $b < 1$. This amounts to imposing a sharp-momentum cutoff that drops states with energy higher than the new cutoff. The *high-energy* degrees of freedom (or states) are labeled as follows:

$$\varphi_{\text{high}}(k) = \begin{cases} \varphi(k) & \text{for } b\Lambda < |k| < \Lambda \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

which comprises of all states with energies between $b\Lambda < |k| < \Lambda$. Consider now the low-energy modes $\varphi_{\text{low}}(k)$ which are identical to $\varphi(k)$, except that $\varphi_{\text{low}}(k) = \varphi(k)$ for $|k| < b\Lambda$, and 0 otherwise. These are the low-energy degrees of freedom. We can then write the original field $\varphi(k)$ as the sum of the high- and low-energy degrees of freedom $\varphi(k) = \varphi_{\text{high}}(k) + \varphi_{\text{low}}k$. Substituting this into the functional integral in 3 gives:

$$\begin{aligned} \mathcal{Z} = \int [D(\varphi_{\text{high}} + \varphi_{\text{low}})]_\Lambda \cdot \\ \cdot \exp\Bigg\{-\int \mathrm{d}^d x \left[\frac{1}{2}(\partial_\mu \varphi_{\text{high}} + \partial_\mu \varphi_{\text{low}})^2 + \frac{1}{2} m^2 (\varphi_{\text{high}} + \varphi_{\text{low}})^2 + \frac{\lambda}{4!}(\varphi_{\text{high}} + \varphi_{\text{low}})^4\right]\Bigg\}. \end{aligned} \tag{6}$$

Expanding out and noting that $\mathcal{D}[\varphi_{\text{high}} + \varphi_{\text{low}}]$ factors as $\mathcal{D}\varphi_{\text{high}}\mathcal{D}\varphi_{\text{low}}$ we can further simplify:

$$\begin{aligned} \mathcal{Z} = \int \mathcal{D}\varphi_{\text{low}} \exp\Bigg\{-\int \mathcal{L}(\varphi_{\text{low}})\Bigg\} \cdot \\ \cdot \int \mathcal{D}\varphi_{\text{high}} \exp\Bigg\{-\int \mathrm{d}^d x \left[\frac{1}{2}\left(\partial_\mu \varphi_{\text{high}}\right)^2 + \frac{1}{2} m^2 \varphi_{\text{high}}^2 + \right. \\ \left. \lambda \left(\frac{1}{6}\varphi_{\text{low}}^3 \varphi_{\text{high}} + \frac{1}{4}\varphi_{\text{low}}^2 \varphi_{\text{high}}^2 + \frac{1}{6}\varphi_{\text{low}} \varphi_{\text{high}}^3 + \frac{1}{4!}\varphi_{\text{high}}^4\right)\right]\Bigg\}. \end{aligned} \tag{7}$$

If one performs the integration over the high energy modes φ_{high} then (7) transforms to:

$$\mathcal{Z} = \int [\mathcal{D}\varphi_{\text{low}}]_{b\Lambda} \exp\left(-\int \mathrm{d}^d x \mathcal{L}_{\text{eff}}\right), \tag{8}$$

where the effective Lagrangian is given by

$$\mathcal{L}_{\text{eff}} = \frac{1}{2}(\partial_\mu \varphi_{\text{low}})^2 + \frac{1}{2} m^2 \varphi_{\text{low}}^2 + \frac{1}{4!}\lambda \varphi_{\text{low}}^4 + \text{other terms} \tag{9}$$

So we recover an identical form for the path integral, except we have a different Lagrangian, modified by our change of scales. In a sense, we preserve the form of the Lagrangian in the process of renormalization. As a result, the action of renormalization behaves like group addition, hence the naming of "renormalization group". It is important to note that this is not an actual group, as the action is not invertible.

For physical theories, their coupling constants flow with the energy or length scale, so it is important to understand the behavior of the coupling constants as the energy varies. We can always rescale the momenta and position, such that changing the variables $x' = xb$ and $k' = k/b$ simplifies $b\Lambda$ to Λ. As a result of integrating the high-energy modes, the action picks some extra terms as compared to (9):

$$\int d^d x \mathcal{L}_{\text{eff}} \\ = \int d^d x \left[\frac{1}{2}(1 + \Delta Z)(\partial_\mu \varphi)^2 + \frac{1}{2}(m^2 + \Delta m^2)\varphi^2 + \frac{1}{4!}(\lambda + \Delta\lambda)\varphi^4 + \Delta C(\partial_\mu \varphi)^4 + \cdots \right]. \quad (10)$$

In terms of the rescaled variable x' we have

$$\int d^d x' b^{-d} \mathcal{L}_{\text{eff}} = \\ \int d^d x \left[\frac{1}{2}(1 + \Delta Z)b^2(\partial_\mu \varphi)^2 + \frac{1}{2}(m^2 + \Delta m^2)\varphi^2 + \frac{1}{4!}(\lambda + \Delta\lambda)\varphi^4 + \Delta C b^4(\partial_\mu \varphi)^4 + \cdots \right]. \quad (11)$$

If we rescale the field by $\varphi' = \left[b^{2-d}(1 + \Delta Z)^{1/2}\right]$ we recover the initial form of the effective Lagrangian, except now in terms of the rescaled field φ':

$$\int d^d x' \frac{1}{2} \left[\frac{1}{2}(\partial_\mu \varphi')^2 + \frac{1}{2}(m'\varphi')^2 + \frac{\lambda'}{4!}\varphi'^4 \right]. \quad (12)$$

Note that this transformation also changes the coupling constants m and λ

$$m' = (m^2 + \Delta m^2)(1 + \Delta Z)^{-1} b^{-2}, \qquad \lambda' = (\lambda + \Delta\lambda)(1 + \Delta Z)^{-2} b^{d-4}. \quad (13)$$

The process of integrating out the higher-energy modes and performing a rescaling on variables returns us with the same form of the Lagrangian, but with the coupling constants modified. If we successively iterate this process while taking b close to 1, (so that the momentum shell of the integration becomes infinitesimally thin), this process becomes a continuous way to change between energy scales. As mentioned above, this idea is historically understood as the *renormalization group.*

2.2 Renormalization Group Flow

2.2.1 β-functions and the Callan-Symanzik Equation

Further observe that while the form of the Lagrangian is preserved, the coupling constants are certainly not — so the change in scale modifies coupling constants. Therefore, the renormalization procedure induces a “flow” on the coupling constants. It is natural to question the behavior of such a flow. To do this, we turn to the Callan-Symanzik equation, which describes the behavior of correlation functions as the energy scale varies.

$$\left[M\frac{\partial}{\partial M} + \beta(\lambda)\frac{\partial}{\partial \lambda} + n\gamma(\lambda) \right] G^{(n)}\left(\{x_i\}; M, \lambda\right) = 0 \quad (14)$$

Here M is the renormalization scale and λ is the coupling constant in φ^4-theory. $G^{(n)}$ is the n point correlation function. Intuitively, the Callan-Symanzik equation tells us that for a shift associated with the change in renormalization scale, we must have corresponding shifts in the coupling constant and field strength to compensate (to get zero overall). This equation is an example of a renormalization group equation. Using an explicit perturbative expansion for the correlation functions in (14) returns a system of equations that can be used to solve for β and γ as well. The β-function is *defined* as the rate of change of coupling constant with respect to the scale:

$$\beta(\lambda) = M\frac{\partial}{\partial M}\lambda\Big|_{\lambda_0,\Lambda}. \tag{15}$$

Eq.(15) can also be written as:

$$\frac{d}{d\log k/M}\bar{\lambda} = \beta(\bar{\lambda}), \tag{16}$$

with k as the momentum. Furthermore, $\bar{\lambda}$ is also known as the *running coupling constant* and (16) is known as the *renormalization group equation*. If β is positive, we know that for large momenta the coupling increases, and for negative β, large momenta correspond to decreasing the coupling constant. For a Lagrangian with multiple coupling constants, (15) corresponds to a system of coupled autonomous differential equations (i.e. without time dependence).

2.2.2 β-function for QED

The β-function for QED at one-loop correction for the fine structure constant reads [23]

$$\beta(\alpha) = \frac{2\alpha^2}{3\pi}. \tag{17}$$

From this, it is immediately evident that $\beta(\alpha)$ is strictly positive if $\alpha \neq 0$. This implies that the coupling constant is forever *increasing*, since the β function describes how the coupling constant (in this case α) varies as a function of scale. This implies the existence of a Landau pole, where the coupling constant runs to infinity in the ultraviolet limit.

2.2.3 β-function for QCD

For QCD, interestingly, it is the opposite scenario. The coupling constant for the strong force actually *decreases* as a function of the energy scale, leading to what is known as asymptotic freedom. Let us consider the β-function for QCD at the lowest non-trivial order [8]:

$$\beta(\alpha_s) = -\left(11 - \frac{n_s}{6} - \frac{2n_f}{3}\right)\frac{\alpha_s^2}{2\pi}, \tag{18}$$

where α_s is the coupling constant for the strong force, $n_s = 3$ for 3 gluons, and n_f is the number of flavours of quarks, known to be 6 . The N corresponds to the degree of the underlying symmetry group $SU(N)$, and since $SU(3)$ is the gauge symmetry for QCD here $N = 3$. Substituting $N = 3$ and $n_f = 6$, we see that β is negative definite. As a result, α_s *decreases* if the energy scale increases. This is known as asymptotic freedom. In the language of renormalization group flow, we see that there is an attractive trivial fixed point. The β evaluates to zero only at $\alpha_s = 0$, and $\beta < 0$ implies that the fixed point is attractive.

2.2.4 Fixed Points and the UV Critical Surface

In analogy to dynamical systems theory, the RG flow can have equilibrium or "fixed points", where the flow halts. Mathematically, the β functions evaluate to zero at the fixed point. Physically, the coupling constants converge to a finite value. To convert to a more generalized notation, let $\{g^j(k)\}$ denote the set of coupling constants for a system, with k describing the running coupling scale. The system of ordinary differential equations arising from the RG flow is given by:

$$k\partial_k g^i(k) = \beta^i(\{g^j\}), \tag{19}$$

which is the multi-component form of (15) [21]. As RG flows halt at the fixed points $\{u_*^j\}$, we obtain the equation

$$\beta^i(\{g_*^j\}) = 0. \tag{20}$$

The β functions describe the rate of flow of the coupling constants, and at a fixed point the rate of flow is zero (i.e. there is no flow) so β evaluates to zero at a fixed point. Similarly to dynamical systems, we analyze the qualitative behavior of RG flow via the linearization of (19) near fixed points $\{u_*^j\}$.

$$k\partial_k g^i(k) = \sum_j B^i{}_j (g^j(k) - g_*^j) + \mathcal{O}\big(g^2\big), \tag{21}$$

where

$$B^i{}_j \equiv \frac{\partial}{\partial g^j}\beta^i \bigg|_{g=g_*}, \tag{22}$$

is the Jacobian evaluated at the fixed point (i.e. this is the linearization matrix). The eigenvalues of this matrix describe stability of fixed points and eigenvectors characterize the flow direction, which may be repulsive or attractive. Let V_I be the eigenvectors, and θ_I be the *stability coefficients* associated with a fixed point. The stability coefficients are the *negative* of the eigenvalues for the stability matrix. We denote by C_J the integration constants and by k_0 the reference scale. Then the solution to the linearized system is given by:

$$g^i(k) = g_*^i + \sum_J C_J V_J^i \left(\frac{k_0}{k}\right)^{\theta_I}. \tag{23}$$

If $\mathrm{Re}(\theta) > 0$, then as $k \to \infty$ (UV-limit) the sum approaches zero, corresponding to an UV-attractive fixed point. In contrast, flow trajectories are repelled if $\mathrm{Re}(\theta) > 0$, so this case is a UV-repulsive fixed point. Fixed points with $\theta_I > 0$ are *relevant* while $\mathrm{Re}(\theta_I) < 0$ fixed points are *irrelevant*. If $\mathrm{Re}(\theta_I) = 0$ then the theory is marginal. The set of all coupling constants that flow towards a relevant (stable) fixed point is known as the **UV critical surface**. 2.2.4 shows what this might look like. As coupling constants flow with scale towards a stable fixed point, they may approach zero, in which case we have a *trivial*, or *Gaussian* fixed point. The obvious example of this is asymptotic freedom, to which the coupling constants for QCD tend towards zero at ultraviolet energies. On the other hand, if the couplings run toward some finite constant, this is regarded as a *nontrivial* fixed point, and more often this is the scenario of interest.

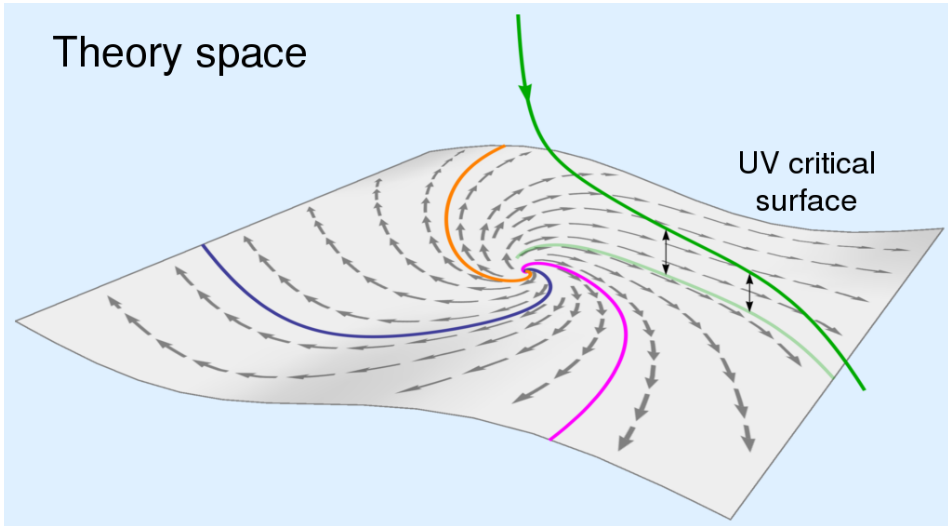

Figure 1: Depiction of UV critical surface as the set of theories with coupling constant flowing towards an attractive fixed point. Theories not part of the UV critical surface are generally unphysical due to their divergences. The arrows are drawn from UV to IR, by convention. [17]

2.2.5 Asymptotic Safety and Weinberg's Criterion

In *Critical Phenomena for Field Theorists* (1978), Weinberg argued that physical theories should not have coupling constants approaching infinity such as $k \to \infty$ [24]. Therefore, theories with a physical interpretation must lie on the UV critical surface (i.e. they approach a fixed point in the UV limit). Such a theory is free from divergences, and hence asymptotically ”safe”. Furthermore, Weinberg noted that there exist *finitely* many *attractive* eigenvectors near a fixed point. As a result, this provides a criterion for choosing a physically meaningful quantum field theory (i.e. one that is asymptotically safe). This ”may either explain renormalizability or else replace it” [25]. It is particularly interesting that gravity, while perturbatively nonrenormalizable, may nonperturbatively renormalizable if it is asymptotically safe.

3 The Einstein Hilbert Truncation

When considering Asymptotic Safety in the context of gravity, it is particularly insightful to study the Einstein-Hilbert truncation, the first concretely worked-out example of Steven Weinberg's suggestion. In 1998, Reuter presented his ”Nonperturbative Evolution Equation for Quantum Gravity” [19], where he proposed a general framework for studying quantum gravity based off the Wilsonian renormalization group and tested on the Einstein-Hilbert action as a first application. His work with Saueressig in 2001 analyzed the qualitative behavior of RG flows for the Einstein-Hilbert truncation using numerical approximations [20]. Here, I present some of the results he obtained and the numerical solutions he derived.

Considering a general scale-dependent effective action, Reuter derived 1998 an exact renormalization group equation, which includes explicit dependence on the infrared (low-energy) cutoff. Using that, he was able to obtain an equation describing the evolution of the effective average action Γ_k:

$$\partial_t \Gamma_k[g,\bar{g}] = \frac{1}{2}\,\mathrm{Tr}\left\{\left(\kappa^{-2}\Gamma_k^{(g)}[g,\bar{g}] + R_k^{\mathrm{grav}}[\bar{g}]\right)^{-1}\partial_t R_k^{\mathrm{grav}}[\bar{g}]\right\} - \mathrm{Tr}\left\{\left(-\mathcal{M}[g,\bar{g}] + R_k^{\mathrm{gh}}[\bar{g}]\right)^{-1}\partial_t R_k^{\mathrm{gh}}[\bar{g}]\right\}, \quad (24)$$

where $\Gamma_k[g,\bar{g}]$ is the effective average action depending on a scale k, fixed background metric $\bar{g}$, and the full metric g. $R_k^{\mathrm{grav}}[\bar{g}]$ and $R_k^{\mathrm{gh}}[\bar{g}]$ are cutoff functions that suppress low-momentum modes, and $\mathcal{M}$ is the Faddeev-Popov ghost operator. Here $\partial_t = k\frac{\mathrm{d}}{\mathrm{d}k}$ is the logarithmic derivative of the length scale (that is, $t = \ln k$). This equation essentially evolves the system to different energy scales. Importantly, the effective action satisfies a set of Ward identities such that simple truncations can be made to the action while still preserving essential physics. A first application of (24) was to study the (full) Einstein-Hilbert action

$$S = \frac{1}{16\pi\bar{G}}\int d^dx\sqrt{g}\left[-R(g) + 2\bar{\lambda}\right]. \quad (25)$$

where $\bar{G}$ here is the bare Newton constant (i.e. value of gravitational constant before including effects from renormalization) and similarly $\bar{\lambda}$ is the bare cosmological constant. $R(g)$ is the Ricci scalar describing the scalar curvature. The later paper by Reuter and Saueressig in 2001 examined the renormalization group flow for this action with truncations from cutoff functions (arising from the IR cutoff), which for this paper was chosen to have the form

$$R_k^{\mathrm{grav}}[\bar{g}] = \mathcal{Z}_k^{\mathrm{grav}}k^2R^{(0)}(-\bar{D}/k^2), \qquad R_k^{\mathrm{gh}}[\bar{g}] = k^2R^{(0)}(-\bar{D}/k^2). \quad (26)$$

Here $(\mathcal{Z}_k^{\mathrm{grav}})^{\mu\nu\rho\sigma}$ is a matrix that acts on metric fluctuations $h_{\mu\nu} \equiv g_{\mu\nu} - \bar{g}_{\mu\nu}$, and $\bar{D}^2$ would be the covariant Laplacian. The shape function $R^{(0)}$ is in general arbitrary with constraints

$$R^{(0)} = 1, \qquad R^{(0)}(z\to\infty) = 0. \quad (27)$$

This form corresponds to a cutoff of "TYPE A" (i.e. a cutoff formulated in terms of the metric fluctuation $h_{\mu\nu}$), and the cutoff functions are included in the IR-cutoff term $\Delta_k S$ to suppress low-energy/momentum modes.

$$\Delta_k S[h,C,\bar{C};\bar{g}] = \frac{1}{2}\kappa^2\int d^dx\sqrt{\bar{g}}h_{\mu\nu}(R_k^{\mathrm{grav}}[\bar{g}])^{\mu\nu\rho\sigma}h_{\rho\sigma} + \sqrt{2}\int d^dx\sqrt{\bar{g}}\bar{C}_\mu R_k^{\mathrm{gh}}[\bar{g}]C^\mu. \quad (28)$$

C^μ and $\bar{C}_\mu$ correspond to ghost fields which are not of particular relevance here. To obtain the flow equation, one substitutes the effective average action into (24). For the problem at hand, we approximate $\Gamma_k[g,\bar{g}]$ by the Einstein-Hilbert truncation anstaz.

$$\Gamma_k[g,\bar{g}] = (16\pi G_k)^{-1}\int d^dx\sqrt{\bar{g}}\left(-R + 2\bar{\lambda}_k\right) + \text{ classical gauge fixing}. \quad (29)$$

Substituting this into to (24) gives us the flow equations of the form in(19) for the dimensionless cosmological constant $\bar{\lambda}_k$ and Newton's constant G_k. They are related to λ_k and g_k through

$$g_k \equiv G_k k^{d-2}, \qquad \lambda_k \equiv \bar{\lambda}_k k^{-2}. \quad (30)$$

Using the dimensionless couplings, the flow equation is given by

$$\partial_t \lambda_k = \beta_\lambda(\lambda_k, g_k), \qquad \partial_t g_k = \beta_g(\lambda_k, g_k). \tag{31}$$

The β-functions are given as

$$\beta_\lambda(\lambda_k, g_k) = -(2-\eta_N)\lambda + \frac{1}{2}(4\pi)^{1-d/2} g \cdot \\ \cdot \left[2d(d+1)\Phi^1_{d/2}(-2\lambda) - 8\Phi^1_{d/2}(0) - d(d+1)\eta_N \tilde{\Phi}^1_{d/2}(-2\lambda)\right], \tag{32}$$

$$\beta_g(\lambda_k, g_k) = (d - 2 + \eta_N), \tag{33}$$

Here η_N is the anomalous dimension of the operator $\int d^d x \sqrt{g} R$, describing the changes in the field dimension due to scale variations. The expression for η_N reads

$$\eta_N(g, \lambda) = \frac{g B_1(\lambda)}{1 - g B_2(\lambda)}, \tag{34}$$

with $B_1(\lambda), B_2(\lambda)$ defined through

$$B_1(\lambda) \equiv \frac{1}{3}(4\pi)^{1-d/2}\left[d(d+1)\Phi^1_{d/2-1}(-2\lambda) - 6d(d-1)\Phi^2_{d/2}(-2\lambda) - 4d\Phi^1_{d/2-1}(0) - 24\Phi^1_{d/2}(0)\right], \tag{35}$$

$$B_2(\lambda) \equiv -\frac{1}{6}(4\pi)^{1-d/2}\left[d(d+1)\tilde{\Phi}^1_{d/2}(-2\lambda) - 6d(d-1)\tilde{\Phi}^2_{d/2}(-2\lambda)\right]. \tag{36}$$

The functions $\Phi^p_n(w)$, $\tilde{\Phi}^p_n(w)$are threshold functions depending on the shape functions $R^{(0)}$.

$$\Phi^p_n(w) = \frac{1}{\Gamma(n)}\int_0^\infty dz z^{n-1} \frac{R^{(0)}(z) - z R^{(0)\prime}(z)}{[z + R^{(0)}(z) + w]^p}, \tag{37}$$

$$\tilde{\Phi}^p_n(w) = \frac{1}{\Gamma(n)}\int_0^\infty dz z^{n-1} \frac{R^{(0)}(z)}{[z + R^{(0)}(z) + w]^p}. \tag{38}$$

We now have the equations (and tools) to study the numerical solutions of that set of horribly nonlinear differential equations. We set $d = 4$ (i.e. back to the real world). Let us begin with identifying the fixed points that occur where the β-functions evaluate to zero. By inspection, (33) and (32) evaluate trivially to zero at $\lambda^* = g^* = 0$, and this corresponds to a *trivial* fixed point. However, the β-functions can also evaluate to zero for $g^* \neq 0$, but this would resemble a non-Gaussian fixed point. To study behaviour near the fixed points, we linearize the system of differential equations using the following general form

$$\partial_t g_i \approx \sum_j B_{ij}(g_j - g_j^*), \qquad \mathbf{B} \equiv [B_{ij}] = \begin{bmatrix} \frac{\partial \beta_\lambda}{\partial \lambda} & \frac{\partial \beta_\lambda}{\partial g} \\ \frac{\partial \beta_g}{\partial \lambda} & \frac{\partial \beta_g}{\partial g} \end{bmatrix} \tag{39}$$

In the case of the Einstein-Hilbert action, g_i refers to λ and g. Calculating the required derivatives for the Jacobian, we obtain

$$\frac{\partial \beta_\lambda}{\partial \lambda} = -(2-\eta_N) + \left(\lambda - g/2(4\pi)^{1-d/2} d(d+1)\tilde{\Phi}^2_{d/2}(-2\lambda)\right)\frac{\partial \eta_N}{\partial \lambda} \\ + \frac{g}{2}(4\pi)^{1-d/2}\left(4d(d+1)\Phi^2_{d/2}(-2\lambda) - 2d(d+1)\tilde{\Phi}^2_{d/2}(-2\lambda)\right), \tag{40}$$

$$\frac{\partial \beta_\lambda}{\partial g} = \left(\lambda - g/2(4\pi)^{1-d/2} d(d+1)\tilde{\Phi}^1_{d/2}(-2\lambda)\right)\frac{\partial \eta_N}{\partial g} + \frac{1}{2}(4\pi)^{1-d/2}\left(2d(d+1)\Phi^1_{d/2}(-2\lambda) - d(d+1)\tilde{\Phi}^1_{d/2}(-2\lambda)\right), \tag{41}$$

$$\frac{\partial \beta_g}{\partial \lambda} = \frac{g^2}{1-gB_2(\lambda)}(B_1'(\lambda) + \eta_N B_2'(\lambda)), \tag{42}$$

$$\frac{\partial \beta_g}{\partial g} = d - 2 + \left(2 + \frac{gB_2(\lambda)}{1-gB_2(\lambda)}\right)\eta_N. \tag{43}$$

The derivatives of η_N reads

$$\frac{\partial \eta_N}{\partial g} = \left(2 + \frac{gB_2(\lambda)}{1-gB_2(\lambda)}\eta_N\right). \tag{44}$$

3.1 Trivial Fixed Point

Now, substituting in the trivial fixed point to the Jacobian gives the following stability matrix

$$\mathbf{B} \equiv \begin{bmatrix} -2 & (4\pi)^{1-d/2} d(d-3)\Phi^1_{d/2}(0) \\ 0 & d-2 \end{bmatrix}. \tag{45}$$

Diagonalizing this matrix produces two eigenvalues (i.e. stability coefficients).

$$\theta_1 = 2, \qquad \theta_2 = 2 - d, \tag{46}$$

and the corresponding eigenvectors

$$V^1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \qquad V^2 = \begin{pmatrix} (4\pi)^{1-d/2} d(d-3)\Phi^1_{d/2}(0) \\ 1 \end{pmatrix}. \tag{47}$$

This linearization tells us the qualitative behavior of the RG trajectories near the trivial fixed point, and using this we obtain the linearized solutions for λ_k and g_k

$$\lambda_k = \alpha_1 \frac{M^2}{k^2} + \alpha_2 (4\pi)^{1-d/2} d(d-3)\Phi^1_{d/2}(0)\frac{k^{d-2}}{M^{d-2}}, \tag{48}$$

$$g_k = \alpha_2 \frac{k^{d-2}}{M^{d-2}}, \tag{49}$$

with α_1, α_2 as integrating constants or initial conditions. Since we live in a 4-dimensional spacetime, setting $d = 4$ gives us $\theta_2 = -2$. Now the eigenvalues are opposite in sign, so the trivial fixed point has a saddle-point like behavior, where it is driven *away* along the V^1 direction (i.e. repulsive), but is attractive in the V^2 direction. The behavior of λ_k itself depends on the initial conditions, with the sign on α_1 playing an especially important role. If we restore the dimensions, the coupling constants $\bar{\lambda}_k$, G_k reads

$$G_k = G_0, \qquad \bar{\lambda}_k = \bar{\lambda}_0 + (4\pi)^{1-d/2} d(d-3)\Phi^1_{d/2}(0) G_0 k^d, \tag{50}$$

with α_2 chosen to be 1 and $\bar{\lambda}_0 = \alpha_1 m_{Pl}^2$ and note that M is identified as the Planck mass (i.e. $M = m_{Pl}$). Here it is evident that for $k \to 0$ the coupling constants approach fixed values (G_0, λ_0), which depend on an external parameter α_1.

3.2 The Non-trivial Fixed Point

It was identified earlier that we have a nontrivial fixed point for $g^* \neq 0$, so now let us consider the qualitative behaviour of trajectories near this fixed point. The β-function for g in (33) implies that $d-2+\eta_N(\lambda^*, g^*) = 0$, and using the expression for η_N from (34) gives g^* in terms of λ^*

$$g^*(\lambda^*) = \frac{d-2}{(d-2)B_2(\lambda^*) - B_1(\lambda^*)}, \tag{51}$$

which may be further used to remove the dependence on g in β_λ at the fixed point. Unfortunately, solving for λ^* in $\beta_\lambda(\lambda^*, g^*) = 0$ analytically is impossible. The matrix entries for the stability matrix is given through

However, we can impose a "sharp cutoff" for the shape functions $R^{(0)}$ and use that to simplify calculations for the nontrivial fixed point. The function $R_k(p^2)$ is defined through

$$R_k(p^2) \equiv k^2 R^{(0)}\left(\frac{p^2}{k^2}\right), \tag{52}$$

so that the sharp cutoff reads

$$R_k(p^2)^{\text{sc}} \equiv \hat{R}\Theta\left(1 - \frac{p^2}{k^2}\right), \tag{53}$$

where $\hat{R}$ is a parameter that we send to infinity after performing the integral over p in the threshold functions. Here Θ is the Heaviside step function. This choice of cutoff suppresses modes with momentum $p^2 < k^2$. The nontrivial fixed point is then numerically determined to be

$$\lambda^* = 0.330, \qquad g^* = 0.403. \tag{54}$$

Following the same procedure as analyzing the trivial fixed point, the stability coefficients are determined to be

$$\theta^1 \equiv \theta' + i\theta'' = 1.941 + i\,3.147, \tag{55}$$

$$\theta^2 \equiv \theta' - i\theta = 1.941 - i\,3.147, \tag{56}$$

which are clearly complex. Without explicitly determining the associated eigenvectors, we can express the general solution as

$$\begin{pmatrix} g(t) - g^* \\ \lambda(t) - \lambda^* \end{pmatrix} = \alpha_1 \sin(-\theta'' t) e^{-\theta' t}\,\text{Re}(V) + \alpha_2 \cos(-\theta'' t) e^{-\theta' t}\,\text{Im}(V), \tag{57}$$

where V corresponds to the associated complex eigenvector. Here $\theta' = 1.941 > 0$, so the nontrivial fixed point is attractive, with solutions spiraling into the fixed point. As a result of the positive stability coefficient, taking $t \to \infty$ the trajectories will be UV-attracted towards the fixed point. Furthermore, this implies that between the trivial and nontrivial fixed point, the nontrivial one will dominate for large t.

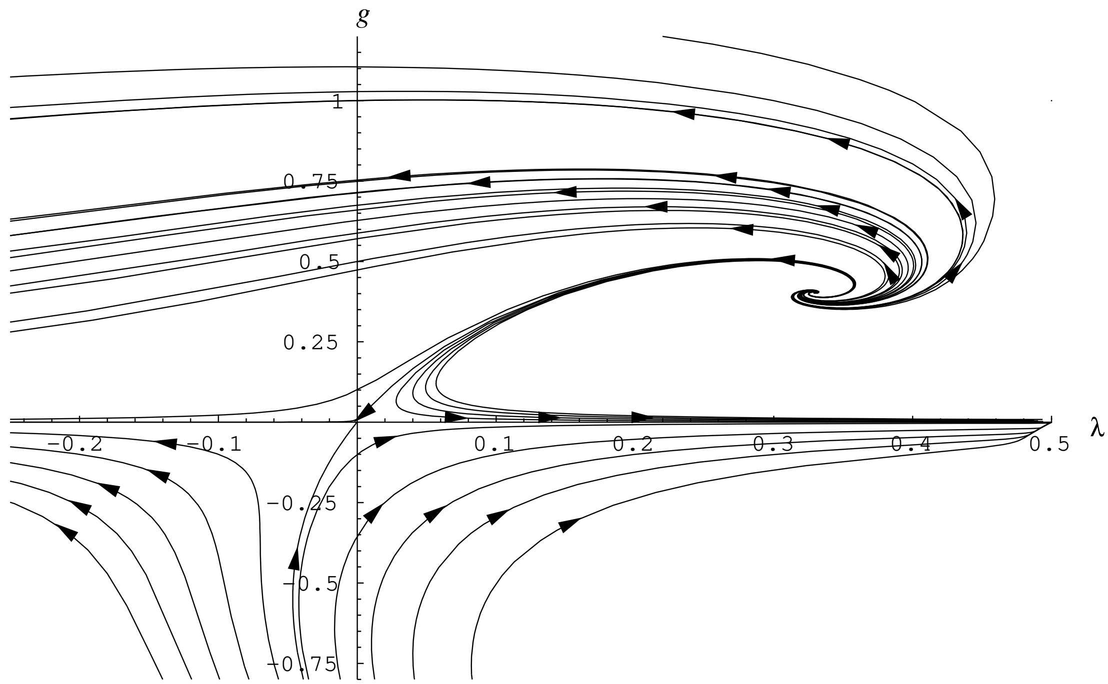

Figure 2: Phase portrait of RG flow in the λ-g phase plane. There are two fixed points, a trivial one at the origin and a nontrivial one in the positive λ-g region. The arrows indicate the flow as $k \to 0$.

3.3 The Phase Portrait

Earlier we have determined the autonomous differential equations that govern the RG flow of g, λ. Now, we consider the phase portrait describing the RG flow and try to get a qualitative understanding of the solutions. The phase portrait (see Fig. 2) is obtained by numerically solving the differential equations for g, λ in equation (31). Examining Fig.2 reveals the existence of two fixed points – a trivial one at the origin and a nontrivial fixed point in the upper right quadrant, in complete agreement with our earlier analysis. The former (if one looks near the origin) has a saddle-like nature, corresponding to the two opposing-sign eigenvalues obtained. The latter fixed point is clearly dominant for large t as solutions in the upper half are strongly attracted toward that fixed point, with the telltale spiral behavior predicted by the complex stability coefficients. More interestingly, there is a trajectory connecting the nontrivial fixed point to the trivial fixed point in the $k \to 0$ limit, implying that the renormalized cosmological constant $\bar{\lambda}_0$ vanishes in the limit of $k \to 0$. On a note about the more general behavior, we can see that trajectories starting to the left of the separation line (where g = 0) are driven towards negative infinity. At the same time, those beginning to the right of the separation line halt at $\lambda = 1/2$ at a *finite* value of k.

The important consequence of the results presented in Fig.2 is the non-Gaussian fixed point that attracts all trajectories with $g > 0$ in the UV limit. Physically, this would correspond to a *stable* constant value for the gravitational and cosmological constants in the high-energy limit, and this would be a *non-perturbative* result. This reveals extremely interesting physical behavior previously unobserved using perturbative calculations. The remaining question is whether or not this behavior is present in the full theory for the Einstein-Hilbert action without the truncation. If so, gravity would have a non-perturbatively renormalizable interpretation, described by taking the cutoff to infinity and recovering the fixed point–this is the critical thinking behind Asymptotic Safety. This would allow (pure) Einstein gravity to be interpreted as a complete and fundamental theory valid at *all* energy scales, in opposition to its current effective theory description valid at low energies only. [15, 26].

3.4 Extensions of the Einstein-Hilbert Truncation

Since the establishment of two fixed points for the Einstein-Hilbert truncation, extensive efforts have been made to further analyze the Einstein-Hilbert truncation. This involved considering higher-order terms in the truncation, which would require incorporating more complicated terms involving the metric and its curvature tensors. A notable example is from[11], where the Einstein-Hilbert truncation is generalized by including a higher derivative term R^2. This introduces a third coupling constant in addition to λ and g. When investigating the resulting three-dimensional RG flow, it was observed that the original Gaussian fixed point vanished, as $(0, 0, 0)$ was not a simultaneous zero of all three β-functions. On the other hand, there remains a non-Gaussian fixed point, and surprisingly it continues to be UV-attractive. Moreover, it yields a positive physical value for the constants λ^*, g^*. Further analysis in extensions of the Einstein-Hilbert truncation included a contribution from the Riemann tensor $R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$, and beyond this, the action involving $R^2, R_{\mu\nu}R^{\mu\nu}$, $R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$ was also examined [1, 7, 13]. More recently, there has been work on studying the action with polynomial functions of the scalar curvature ($f(R)$-truncation), recently computed up to order 71 [2]. Similarly, researchers have studied effective actions involving functions of the Riemann tensor and the Ricci tensors. Notably, in all of these scenarios a non-Gaussian fixed point was identified, consistent with the Weinberg criterion, which offers a strong expectation that the full theory should possess a suitable fixed point.

4 Applications of Asymptotic Safety

4.1 Couplings with Matter Fields

The analysis of pure gravity via the Einstein-Hilbert truncation strongly suggests the existence of a fixed point, but it is natural to question how gravitational couplings with matter fields impact AS. Most studies on incorporating matter into AS use a minimal coupling, where it is assumed that matter can only interact with gravity through mass and energy. This introduces the minimum number of terms into the action.

In [6], Eichhorn showed that the inclusion of matter in the action generates new non-vanishing couplings, which contribute to the RG flow. For a scalar field coupled with gravity, the metric induced momentum-dependent self-interactions, and along with matter self-interactions they remove a previously existing Gaussian fixed point corresponding a coupling constant independent of momentum [3]. This means that including quantum gravity changes the dynamics and properties of matter systems. It was also noted that this could fundamentally change matter couplings, possibly from marginal to irrelevant.

With respect to the Standard Model, there is evidence that gravity remains asymptotically safe even when coupled to observed matter fields [14]. More interestingly, it was very recently shown that the Standard Model couplings become asymptotically free due to asymptotically safe gravity. Importantly, this allows AS to predict the quartic coupling as a calculable quantity for the Higgs field, in the infrared limit. That is, one can use Asymptotic Safety to calculate the mass of the Higgs boson. This calculation was completed by [22], and their results predicted a mass of 126 GeV with minimal uncertainty. This theoretical result for the Higgs mass is in excellent agreement with the experimentally observed value. This highlights the predictive power of Asymptotic Safety. Moreover, the theory predicts IR values for Abelian gauge couplings and Yukawa couplings, but there is significant uncertainty surrounding these values, which is uncertain if the predictions are consistent with observations.

4.2 Fine Structure Constant

It is known that QED has a Landau pole for the fine structure constant α. However, if QED is coupled to quantum gravity in AS, this is surprisingly not the case, as shown in [9]. Examining the RG flow for a simple truncation of the effective average action (coupled with QED) shows the existence of two non-trivial fixed points. One of them suggests that the fixed point value of the fine structure constant is zero, that is, electromagnetic interactions play no role and the system reduces to pure gravity. In this scenario, the infrared value for the fine structure would have to be determined from experiment (i.e. it is a free parameter in the theory). The other nontrivial fixed point is much more interesting. In this case, the value of α is non-zero at the fixed point, which implies that one can predict the low-energy value of α entirely from the theory in terms of the electron mass. That is, if one simultaneously considers QED and Quantum Einstein Gravity (QEG), there is no Landau pole, and the theory is well-behaved in the ultraviolet limit.

5 Controversies and Criticisms

One of the key principles in all quantum theories is the requirement that the probabilities sum to one. However, it is unclear whether Asymptotic Safety satisfies this criterion. Therefore, AS suffers from a lack of unitarity, directly due to the higher-derivative terms in the action. This is a general result, present in classical theories according to Ostrogradsky's theorem [12]. It essentially states that Hamiltonians with higher order time derivatives feature an instability such that the Hamiltonian is unbounded. In quantum systems, this instability manifests itself as ghost states that violate probability conservation [2, 5].

Another issue has to do with the Lorentzian nature of gravity. Currently (and thus far in this report), all results shown and conclusions obtained were based on a functional integral defined on an *Euclidean* metric (i.e. $\int d^dx$). This had the benefit that the momentum squared (k^2) was positive semi-definite, so it made sense to define a direction on the RG flow. It was easy to implement the Wilsonian RG approach of integrating out modes with higher momenta and successively moving to lower values of k. While it is common practice to work in a Euclidean signature via a Wick rotation, the presence of gravity makes it challenging to do so for Asymptotic Safety [2, 5]. This remains one of the open questions for Asymptotic Safety.

6 Conclusion

In conclusion, this report presents a general overview of Asymptotic Safety and its possible implications for quantum gravity. Section 2 began with a discussion of renormalization as a way to shift between energy scales via integrating out higher degrees of freedom. This then leads into the idea of a Renormalization Group, where one observes that successive shifts between energy scales are additive in the sense of a group (i.e. preserve the form of the functional integral). Using this, it is easy to see that the effects of renormalization can be absorbed into the coupling constants, and to study how the system evolves as a function of energy, it is sufficient to consider how the coupling constants flow. This is the essential idea behind the Renormalization Group flow. The details of the RG flow are characterized by β-functions, which lead to a set of differential equations to be qualitatively studied via stability analysis. A brief example of QED and QCD is given to illustrate how flows can vary.

With the important tools and background at hand, focus is turned to studying qualitatively the RG flows for the Einstein-Hilbert truncation in Section 3. It was discovered that there exist two fixed points, one Gaussian and the other non-Gaussian. The Gaussian fixed point was shown to be IR-attractive, that is for low energies the gravitational and cosmological constants tended to zero. For the non-Gaussian fixed point, however, analysis revealed it as UV-attractive, so at arbitrarily high energies gravity appears asymptotically safe based on the Einstein-Hilbert truncation. This result is reinforced by a extensive later calculations for generalized versions of the Einstein-Hilbert truncation.

Section 4 more broadly examined the physical applications of Asymptotic Safety in the context of coupling to matter fields, the Higgs mass, and the fine-structure constant. For coupling to matter fields, calculations show that including quantum gravity interactions modifies the fixed-point behavior and affects dynamics of matter fields. Within the Standard Model, Asymptotic Safety showcases its predictive power in providing an accurate theoretical prediction for the Higgs mass. It also offers predictions for other couplings, but due to uncertainties it is unclear whether the results are in good agreement. In the latter case, AS suggests that if one considers both QED and QEG then the Landau pole may not pose a problem at all in the UV limit. Finally, we also considered some of the issues surrounding AS, such as the emergence of ghost states violating unitarity due to higher order derivatives and the difficulty in performing calculations for AS in the Lorentzian metric as opposed to the Euclidean metric.

Overall, AS is an interesting theory for quantum gravity in the sense that it works within the existing framework of effective field theory and quantum field theory without the introduction of exotic assumptions (extra dimensions from string theory, supersymmetry..etc). Present results from the Einstein-Hilbert action strongly suggest that gravity is asymptotically safe, such gravity is likely to be non-perturbatively renormalizable.

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