5.5 – Substitution Rule (Indefinite Integrals)

Suppose we want to calculate $\int f(x)dx$ but it is not clear what the antiderivative of $f(x)$ is. At times, we can use the substitution $u = g(x)$ to help us so that

$$\int f(g(x))g'(x)dx = \int f(u)du,$$

where $g(x)$ is differentiable and whose range is an interval I such that f is continuous on I.

Remember to replace $u = g(x)$ after you have found the antiderivative.

Ex5.9) 1. $\int 3\sqrt{4+3x}dx$

2. $\int 2x\sqrt{x^2-\pi}dx$

3. $\int (2x+1)^7 dx$

4. $\int \frac{3}{(2-x)^2}\, dx$

5. $\int \frac{4t}{\sqrt{2t^2+1}}\, dt$

6. $\int \sin(8z-5)\, dz$

7. $\int \tan^2 x \sec^2 x\, dx$

8. $\int \frac{6\cos t}{(2+\sin t)^3}\, dt$

9. $\int 3x^5\sqrt{x^3+1}\, dx$

Substitution Rule (Definite Integrals)

When using substitution rule with definite integrals, we don't have to substitute $g(x)$ back in for u as long as we change the limits of integration.

Old limits	New limits
a	$g(a)$
b	$g(b)$

Ex5.10) 1. $\int_0^{\pi} 3\cos^2 x \sin x \, dx$

2. $\int_0^{\sqrt{7}} t(t^2+1)^{1/3} \, dt$

3. $\int_0^1 \frac{x^3}{\sqrt{x^4+9}} \, dx$

4. $\int_{0}^{\sqrt{3}} x^3\sqrt{x^2+1}\, dx$

5. $\int_{0}^{1} r\sqrt{1-r^2}\, dr$

6. $\int_{0}^{1} \frac{x^2}{\sqrt{1-x}}\, dx$

7. $\int_{1}^{e} \frac{\ln x}{6x}\, dx$

8. $\int_{0}^{\ln \sqrt{e}} e^{2x} \, dx$