5.3 – The Fundamental Theorem of Calculus

Theorem: The Fundamental Theorem of Calculus

(I) If $F(x) = \int_a^x f(t)dt$, then $F(x)$ is differentiable. Furthermore,

$$F'(x) = \frac{d}{dx}\int_a^x f(t)dt = f(x)$$

(II) If $F(x)$ is any antiderivative of $f(x)$, then $\int_a^b f(x)dx = F(b) - F(a)$.

Remarks: 1. (I) says that integration and differentiation are "inverse processes".

2. (II) gives a slick way of calculating definite integrals.

Ex5.7) (Part I)

1. $\frac{d}{dx}\int_{17}^{x} \sin t dt$

2. $\frac{d}{dx}\int_{x}^{\pi} (t^2 + 3)dt$

3. $\frac{d}{dx}\int_{3}^{x^3} (\cos t + t)dt$

4. $\frac{d}{dx}\int_{x^2}^{\sin x} (4t^2 - 9)dt$

Ex5.8) (Part II)

1. $\int_{-1}^{3} x^5 dx$

2. $\int_{1}^{4} \left(x^2 - 3x^3 + 5x^{1/2} \right) dx$

3. $\int_{0}^{\pi} \cos(2\theta) d\theta$

4. $\int_{1}^{4} \frac{t + \sqrt{t}}{t^3} dt$

5. $\int_{0}^{1} \left(x^e + e^x \right) dx$

6. $\int_{2}^{5} \frac{23}{x} dx$

7. Using Riemann sums, calculate $\int_2^4 (x^2 - 2)dx$.