5.2 – The Definite Integral

Recall, we have used an infinite sum to calculate the area under a curve, above the *x*-axis and between the values $x = a$ and $x = b$.

We use the notation $A = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k)\Delta x$ to denote this.

That is, $\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k)\Delta x$.

This is called the definite integral. It looks like the indefinite integral, but it also has the limits of integration *a* and *b*.

Def'n: A function is integrable on $[a, b]$ if $\lim_{n\to\infty} \sum_{k=1}^{n} f(x_k)\Delta x$ exists (is finite).

Theorem: A function that is continuous on $[a, b]$ is integrable on $[a, b]$.

Remark: Non-continuous functions may or may not be integrable.

If $f(x) \geq 0$ for all $x \in [a,b]$, then $\int_a^b f(x)dx$ gives the area under $f(x)$, above the *x*-axis and between $x = a$ and $x = b$.

If $f(x) < 0$ for all $x \in [a,b]$, then $\int_a^b f(x)dx$ calculates the "signed area".

Ex5.3) Calculate $\int_{1}^{2}(x+1)dx$.

Ex5.4) Calculate $\int_{-2}^{2}\sqrt{4-x^2}\,dx$.

Ex5.5) Calculate $\int_{-1}^{2}3x\,dx$.

Ex5.6) Calculate $\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}}\sin x\,dx$.

Properties of Definite Integrals:

1. $\int_b^a f(x)dx = -\int_a^b f(x)dx$

2. $\int_a^a f(x)dx = 0$

3. $\int_a^b kf(x)dx = k\int_a^b f(x)dx$, where k is any constant

4. $\int_a^b \left(f(x) \pm g(x) \right) dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$

5. $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$

Remark: $\int_a^b \left(f(x) \cdot g(x) \right) dx \neq \int_a^b f(x)dx \cdot \int_a^b g(x)dx$