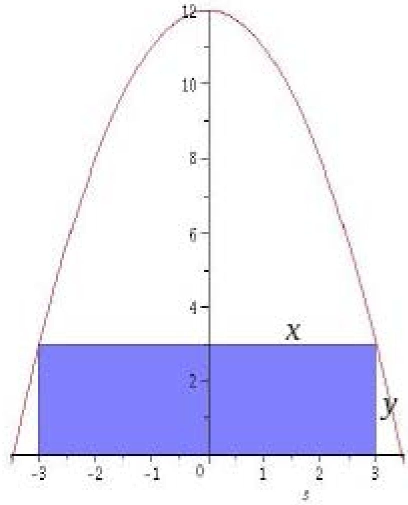
1.



Constraint: $y = 12 - x^2$

Maximize: $A = 2xy$

It follows that $A = 2x(12 - x^2) = 24x - 2x^3$

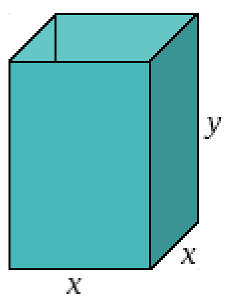
and $\frac{dA}{dx} = 24 - 6x^2 = 6(4 - x^2)$.

The critical points are $x = -2, 2$.

Plugging into constraint, $y = 8$.

Thus, the maximum area is $A = 32$ units^2.

2.



Constraint: $x^2 y = 32000$

Minimize: $A = x^2 + 4xy$

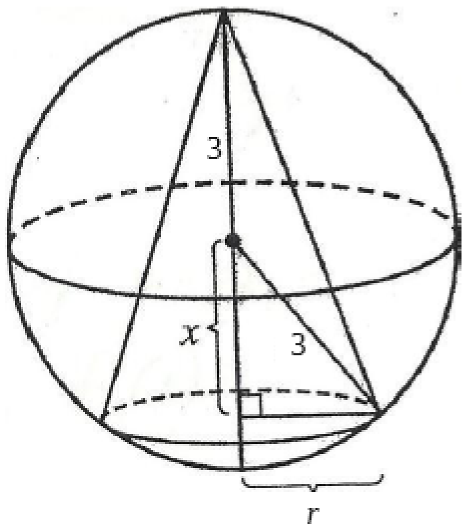
It follows that $A = x^2 + 4x\left(\frac{32000}{x^2}\right) = x^2 + \frac{128000}{x}$

and $\frac{dA}{dx} = 2x - \frac{128000}{x^2} = \frac{2(x^3 - 64000)}{x^2}$.

The critical points are $x = 0, 40$. Only 40 is valid.

Thus, the dimensions of the box are $40 \times 40 \times 20$.

3.



Constraint: $x^2 + y^2 = 9$

Maximize: $V = \frac{1}{3}\pi r^2(3+x)$

It follows that $V = \frac{1}{3}\pi(9-x^2)(3+x)$

$= \pi(9+3x-x^2-\frac{1}{3}x^3)$

$\frac{dV}{dx} = \pi(3-2x-x^2)$

$= \pi(3+x)(1-x)$

The only positive critical points is $x = 1$.

Thus, the max volume is $V = \frac{32\pi}{3}$ units^3.

4. Let the two numbers be x and y. Constraint: $x - y = 200$

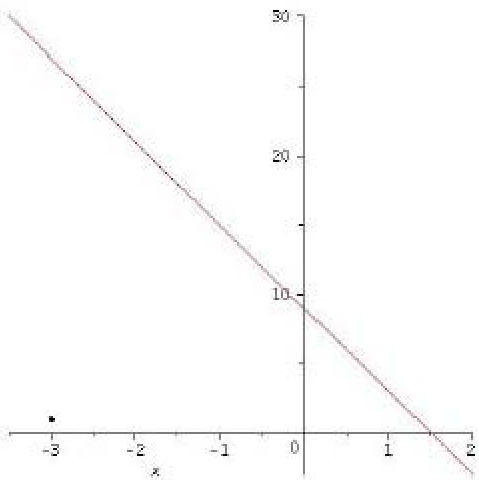
Minimize: $P = xy$

It follows that $P = (200+y)y = 200y + y^2$ and taking the derivative gives

$$\frac{dP}{dy} = 200 + 2y$$

The only critical point is $y = -100$. Plugging into constraint, $x = 100$.

5.



Constraint: $6x + y = 9$

Minimize: $d = \sqrt{(x+3)^2 + (y-1)^2}$

We can minimize d by minimizing
$d^* = (x+3)^2 + (y-1)^2$

It follows that $d^* = (x+3)^2 + (9-6x-1)^2$
$= (x+3)^2 + (8-6x)^2 = 37x^2 - 90x + 73$
and $\dfrac{dd^*}{dx} = 74x - 90$.

The only critical point is $x = \dfrac{45}{37}$. Plugging into constraint, $y = \dfrac{63}{37}$.

Thus, the point on the line $6x + y = 9$ closest to (–3,1) is $\left(\dfrac{45}{37}, \dfrac{63}{37}\right)$.