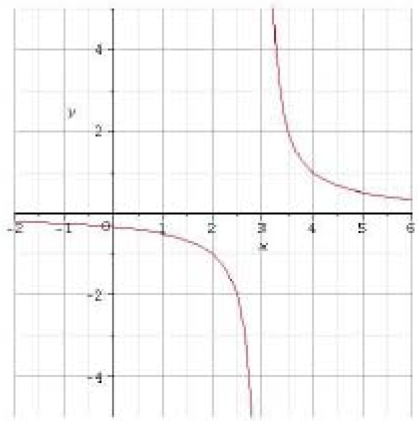
4.3 – First Derivative Test

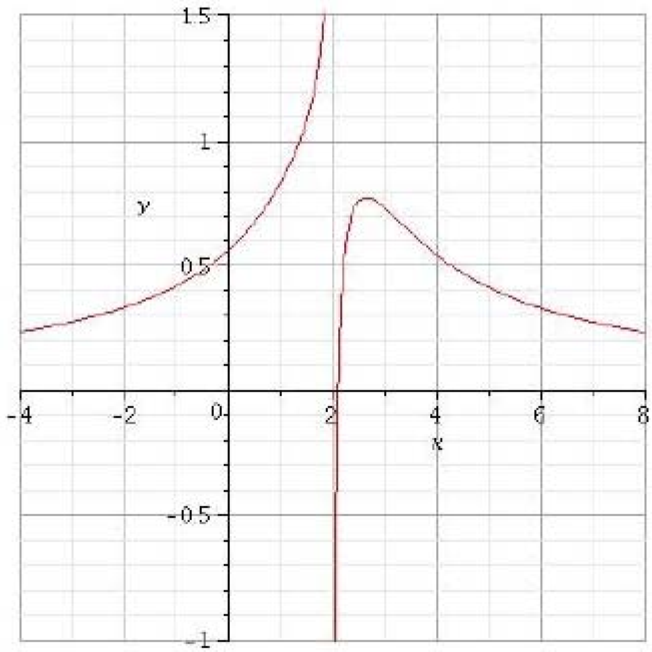
Def'n: A function that is rising to the right is said to be increasing. A function that is falling to the right is said to be decreasing.



Remark: We have seen that a function $f(x)$ is increasing when $f'(x) > 0$, and it is decreasing when $f'(x) < 0$. This is called the First Derivative Test. It is useful when graphing functions and locating local max/mins.

Ex4.6) On what intervals is $f(x) = 2x^3 - 18x$ increasing? Decreasing?

Def'n: If a graph lies above its tangent line, the graph is concave up; if it lies below, the graph is concave down.



Second Derivative Test:

If $f''(x) > 0$, then *f*(*x*) is concave up.

If $f''(x) < 0$, then *f*(*x*) is concave down.

Def'n: A point where the graph of *f*(*x*) has a tangent line and changes from concave up (down) to concave down (up) is called an inflection point.

Ex4.7) Given $f(x) = \dfrac{x^2 - 3}{x^2 + 2}$, find:

(a) The intervals of increase/decrease.

(b) The local max/min's.

(c) The intervals of concavity.

(d) The inflection points.

4.5 – Curve Sketching

When sketching curves, gather information about the curve by:

Domain: First determine the set of values of x for which $f(x)$ is defined.

Intercepts: The y-intercept, or $f(0)$, intersects the y-axis. To find the x-intercept(s), set $y = 0$ and solve for x (if done easily).

Symmetry: For all x in D (the domain), see if the following apply.

(i) If $f(-x) = f(x)$, then f is an even function and can be reflected across y-axis.
(ii) If $f(-x) = -f(x)$, then f is odd and can be rotated 180º about the origin.
(iii) If $f(x + p) = f(x)$, then f is periodic with p a positive constant representing the period of f. For example, $\sin x$ has a period of 2π and $\tan x$ has a period of π.

Asymptotes: (i) Horizontal Asymptotes:

Find if $\lim_{x\to\infty} f(x) = L$ or $\lim_{x\to-\infty} f(x) = L$ so that $y = L$ is an H.A.
If either limit = $\pm\infty$, there is no H.A., but info still useful.

(ii) Vertical Asymptotes:

If any of $\lim_{x\to a^-} f(x) = \pm\infty$, $\lim_{x\to a^+} f(x) = \pm\infty$, then $x = a$ is a V.A.
For rational expressions, equate denominator to zero after canceling common factors. (These may be points of discontinuity.)

Intervals of Increase/Decrease: Compute $f'(x)$ and find the critical numbers of f (when $f'(c) = 0$ and $f'(c)$ DNE). Use the numbers to create intervals and determine for each interval if $f'(x)$ is positive (increasing f) or negative (decreasing f).

Local Max/Min Value(s): Use intervals to determine if a critical number is a local maximum (increase, then decrease) or local minimum (decrease, then increase). Or, if $f'(c) = 0$ and $f''(c) \neq 0$, $f''(c) > 0 \rightarrow$ max and $f''(c) < 0 \rightarrow$ min

Concavity/Points of Inflection: Compute $f''(x)$ and find the inflection points where $f''(x) = 0$. Use these points to create intervals and determine for each interval if $f''(x)$ is positive (concave up) or negative (concave down).

Ex4.8) Sketch the graph of $f(x) = x^{2/3}(x-5)$.

Ex4.9) Sketch the graph of $f(x) = \dfrac{x^3}{x^2 - 4}$.