4.1 – Maximum and Minimum Values

Def'n: Let $f(x)$ be a function with domain D. Then, $f(x)$ has a local maximum value on D at a point c if $f(x) \leq f(c)$ for all x in some open interval containing c.

Similarly, $f(x)$ has a local minimum value on D at point c if $f(x) \geq f(c)$ for all x in some open interval containing c.

Def'n: If the inequalities in the above definition are true for all x in the domain of f, then $f(c)$ is an absolute maximum/minimum value.

Def'n: A critical number of a function f is a value x within its domain where $f'(x) = 0$ or $f'(x)$ doesn't exist.

Fact: **Local** (and hence absolute) extrema can only occur at critical numbers or endpoints of the domain.

Remark: Critical points are not always local extrema.

Theorem: (Extreme Value Theorem)

If f is continuous on closed and bounded interval $[a, b]$, then f surely attains both an **absolute** max and an **absolute** min on $[a, b]$.

Remark: To find the absolute max and min of $f(x)$ on interval $[a, b]$:

1. Find all critical points of $f(x)$.
2. Evaluate $f(x)$ at each critical point and each endpoint.
3. The largest of all values in Step #2 is the absolute max; the smallest is the absolute min.

Ex4.1) Find the absolute max/min of $f(t) = \sqrt[3]{t}(8-t)$ on the interval [0, 8].

Remark: For a continuous function $f(x)$, **local** maximums occur when a function switches from increasing to decreasing. That is, when $f'(x)$ switches from positive to negative.

Similarly, local minimums occur when $f'(x)$ switches from negative to positive.

Ex4.2) Find the local max/min's of $f(x) = \dfrac{x^2}{x^2+3}$.

Ex4.3) Find the local max/min's of $f(x) = x\sqrt{x+3}$.