3.7 Rates of Change

Recall that for a function $f(x)$, $\Delta x = x_2 - x_1$ is the change of x over the interval $[x_1, x_2]$. Also, $\Delta y = f(x_2) - f(x_1)$ is the change in y. The difference quotient

$$\frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

calculates the average rate of change of y with respect to x over the interval $[x_1, x_2]$.

As $x_2 \to x_1$, we get the derivative

$$\frac{dy}{dx} = \lim_{x_2 \to x_1} \frac{\Delta y}{\Delta x}$$

which is the instantaneous rate of change of y with to respect to x at x_1, provided the limit exists. Alternatively, the instantaneous rate of change of f with respect to x at x_1 is the derivative

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x_1 + \Delta x) - f(x_1)}{\Delta x}$$

provided the limit exists.

Motion Along a Line: Displacement, Velocity, Speed, Acceleration, and Jerk

Suppose that an object is traveling back and forth along a straight line, and $s = f(t)$ calculates the position of the object along the line at time t. The displacement of the object over the time interval from t to $t + \Delta t$ is

$$\Delta s = f(t + \Delta t) - f(t)$$

and the average velocity of the object over the interval is

$$v_{ave} = \frac{\Delta s}{\Delta t} = \frac{f(t + \Delta t) - f(t)}{\Delta t}$$

At time t,

- the instantaneous velocity of the object is
  $$v(t) = \frac{ds}{dt} = \lim_{\Delta t \to 0} \frac{f(t + \Delta t) - f(t)}{\Delta t}$$
- the speed is $|v(t)| = \left|\frac{ds}{dt}\right|$
- the acceleration is $a(t) = \frac{dv}{dt} = \frac{d^2 s}{dt^2}$
- the jerk is $j(t) = \frac{da}{dt} = \frac{d^3 s}{dt^3}$

Ex3.6) Suppose that a particle moves back and forth along a line such that it has a position function $s = f(t) = t^3 - 9t^2 + 24t$, where t is measured in seconds and s in feet.

a) Find the velocity of the particle at time t.

b) What is the velocity at 1 and at 3 seconds?

c) When is the particle at rest?

d) When is the particle moving forward (in the positive direction)?

e) Draw a diagram to represent the motion of the particle.

f) Find the total distance traveled by the particle during the first 6 seconds.

g) What is the acceleration of the particle at time t, and at 3 seconds?

h) Graph the position, velocity, and acceleration functions for the first 6 seconds.

i) When is the particle speeding up? When is it slowing down?