3.5 Implicit Differentiation

So far, all the functions seen have been explicitly given in terms of x.
For example, $y = 3x^2 - \sin x$ or $y = 14x\sqrt{x^2+3}$.

Now consider functions (or combinations of fcns) that express y implicitly in terms of x.
For example, $x^2 + y^2 = 1$ or $\sin(xy) = \frac{1}{2}$.
Note: the unit circle can be broken up into two functions that satisfy the vertical line test.

One can still calculate $\frac{dy}{dx}$ for a function where y is implicitly expressed in terms of x, but **it is important to always think of y as a function of x.**

Implicit Differentiation

1. Differentiate both sides of the equation with respect to x. Whenever taking the derivative of something with a y, multiply by y' (chain rule).
2. Put all terms with a y' on one side of the equation and all terms without a y' on the other side.
3. Factor out y' and divide by everything else to solve the equation for y'.

Ex3.4) 1. Find y' when $x^3 + y^3 = 18xy$.

2. Find y' when $x^3 - xy + y^3 = 1$.

3. Find y' at the point (1, 0) when $x^2 = \dfrac{x-y}{x+y}$.

4. Find $\dfrac{dy}{dx}$ when $e^y \sin x = x + xy$.

5. Given the circle $(x-1)^2 + (y-2)^2 = 4$, find the equation of the tangent line through the point $(2, \sqrt{3}+2)$.