3.1 and 3.2 Differentiation Rules

1. Constant Rule:

$$\frac{d}{dx}(c) = 0$$

2. Power Rule:

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

3. Constant Multiple Rule:

$$\frac{d}{dx}\left[cf(x)\right] = c\frac{d}{dx}f(x) \qquad \text{OR} \qquad (cf)' = cf'$$

4. Sum/Difference Rule:

$$(f+g)' = f' + g' \qquad\qquad (f-g)' = f' - g'$$

5. Product Rule:

$$(fg)' = fg' + gf' \qquad \text{OR} \qquad \frac{d}{dx}\left[f(x)g(x)\right] = f(x)\frac{d}{dx}\left[g(x)\right] + g(x)\frac{d}{dx}\left[f(x)\right]$$

6. Quotient Rule:

$$\left(\frac{f}{g}\right)' = \frac{gf' - fg'}{g^2}$$

7. Natural Exponential Function:

$$\frac{d}{dx}(e^x) = e^x$$

Ex3.1) 1. $f(x)=x^2+x+8$

2. $f(x)=3x^7-7x^3+21x^2$

3. $y=\frac{x^3}{3}+\frac{x^2}{2}+\frac{x}{4}$

4. $s=-2t^{-1}+\frac{4}{t^2}$

5. $y=4-2x-x^{-3}$

6. $r=\frac{12}{\theta}-\frac{4}{\theta^3}+\frac{1}{\theta^4}$

7. $y=(x-1)(x^2+x+1)$

8. $y=\left(x+\frac{1}{x}\right)\left(x-\frac{1}{x}+1\right)$

9. $z = \dfrac{2x+1}{x^2-1}$

10. $f(x) = \dfrac{e^x}{1-e^x}$

11. $w = (2x-7)^{-1}e^x$

12. $u = \dfrac{5x+1}{2\sqrt{x}}$

13. $r = 2\left(\dfrac{1}{\sqrt{\theta}} + \sqrt{\theta}\right)$

14. $y = \dfrac{(x+1)(x+2)}{(x-1)(x-2)}$

15. $y = \dfrac{x^5}{120} + 12e^x$

16. $s = \dfrac{t^2+5t-1}{e^t t^2}$

Higher Derivatives

If $f(x)$ is a differentiable function, then $f'(x)$ is also a function. If $f'(x)$ is also differentiable, then one can differentiate $f'(x)$ to get $\frac{d}{dx} f'(x) = f''(x)$.

The function $f''(x)$ is called the <u>second derivative</u> of $f(x)$.

Notation: $f''(x) = y'' = \frac{d^2 f}{dx^2} = \frac{d}{dx}\left(\frac{df}{dx}\right)$

If $f''(x)$ is differentiable, then one can calculate $f'''(x)$.

If $f'''(x)$ is differentiable, then one can calculate $f^{(4)}(x)$.

If $f^{(n-1)}(x)$ is differentiable, then one can calculate $f^{(n)}(x)$.

Continue with 3.2 ODDS.

The Derivative as a Rate of Change

If $s(t)$ gives an object's <u>position</u> after t seconds, then

- $s'(t) = v(t)$ gives the object's (instantaneous) <u>velocity</u> after t seconds.
- $s''(t) = v'(t) = a(t)$ gives the object's <u>acceleration</u> after t seconds.

Remark: Velocity has a direction, which is indicated by '+' or '–'. <u>Speed</u> has no direction. That is, speed = | velocity |.