2.7 Derivatives

Given the graph of a function $f(x)$, calculate the slope of the secant line through two points $(a, f(a))$ and $(b, f(b))$ by

$$m = \frac{f(b) - f(a)}{b - a}$$

As b gets closer to a, the secant line gets closer to the tangent line of $f(x)$ at a.

When b finally reaches a, one has the tangent line to $f(x)$ at a.

Thus, the slope of the tangent line at a is

$$m = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

Ex2.26) Find the slope of the tangent line to $f(x) = x^2 + 1$ at the point $x = 1$.

It is possible to state the slope of the secant line alternatively by using $h = b - a$.

$$m = \frac{f(a+h) - f(a)}{h}$$

Thus, the slope of the tangent line to $f(x)$ at a is

$$m = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

Ex2.27) Calculate the equation of the tangent line of $f(x) = x - 2x^2$ through the point (–1, –3).

Def'n: The slope of the tangent line to $f(x)$ at a point a is also called the <u>derivative of $f(x)$ at a</u>, denoted $f'(a)$.

That is, the derivative of $f(x)$ at a is:

- The slope of the tangent line at a.
- $\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$
- $\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$
- The instantaneous rate of change of f at a.

Ex2.28) If a ball is thrown into the air with a velocity of 40 ft/s, its height (in feet) after t seconds is given by $h(t) = 40t - 16t^2$.

(i) Find the velocity when $t = 2$.
(ii) At what time does the ball reach its maximum height?
(iii) What is the velocity of the ball when it hits the ground?