2.6 Limits Involving Infinity; Asymptotes of Graphs

Limits at Infinity (horizontal asymptotes)

One can take limits as $x \to \infty$ or $x \to -\infty$. At times, a function may approach a certain finite value as $x \to \pm\infty$.

Ex2.18) $f(x) = \tan^{-1} x$

Ex2.19) $f(x) = \ln x$

Ex2.20) $f(x) = \dfrac{1}{x}$

Fact: $\lim\limits_{x\to\infty} \dfrac{n}{x^p} = 0$ when $p > 0$.

$\lim\limits_{x\to-\infty} \dfrac{n}{x^p} = 0$ when $p > 0$.

Ex2.21) $\lim\limits_{x\to\infty} \dfrac{10}{x} =$ $\lim\limits_{x\to\infty} \dfrac{35}{x^2} =$ $\lim\limits_{x\to-\infty} \dfrac{1000}{x^{0.001}} =$

$\lim\limits_{x\to\infty} \dfrac{-6}{x^{1/2}} =$ $\lim\limits_{x\to-\infty} \dfrac{1000000}{x^5} =$

This fact helps calculate limits at infinity of rational functions. The strategy is to first divide by the highest power of x in the denominator, then use the above fact.

Ex2.22) 1. $\lim_{x\to\infty} \dfrac{2x^3+7}{x^3-x^2+x+7}$

2. $\lim_{x\to\infty} \dfrac{3x^3+2}{x^2-9}$

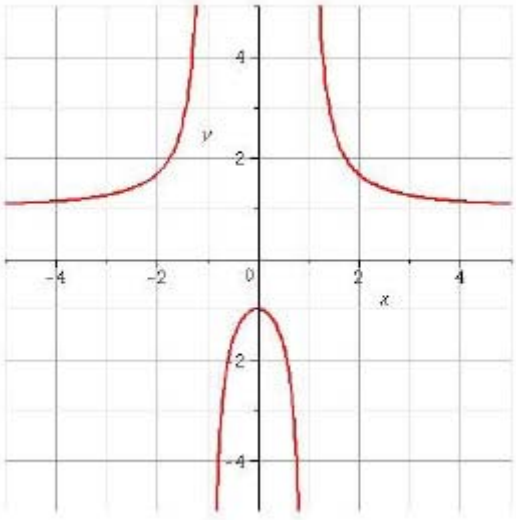
3. $\lim_{x\to-\infty} \dfrac{16x^4-12}{4+x^5}$

4. $\lim_{x\to-\infty} \dfrac{\sqrt[3]{x}-5x+3}{2x+x^{2/3}-4}$

Infinite Limits and Vertical Asymptotes

Already, there have been a few functions where a graph grows arbitrarily large (or small) as x approaches a value a.

Ex2.23) $f(x)=\dfrac{x^2+1}{x^2-1}$



If a function approaches either ∞ or $-\infty$ as x approaches a from either the left or the right (or both), then we say $f(x)$ has a <u>vertical asymptote at a</u>.

Ex2.24) $f(x)=\dfrac{1}{x}$

If there is a rational function that cannot be factored to find a limit at a, then there is a vertical asymptote at a.

Ex2.25) 1. $\lim\limits_{x\to 2^-}\dfrac{x^2+6}{x-2}$

2. $\lim_{x\to 0^-} \frac{5}{2x}$ $\lim_{x\to 0^+} \frac{5}{2x}$

3. $\lim_{x\to 3^-} \frac{1}{x-3}$ $\lim_{x\to 3^+} \frac{1}{x-3}$

4. $\lim_{x\to 0^-} \frac{1}{x^{2/3}}$ $\lim_{x\to 0^+} \frac{1}{x^{2/3}}$

$\lim_{x\to 0} \frac{1}{x^{2/3}}$

5. $\lim_{x\to 0^-} \frac{-1}{x^2(x+1)}$ $\lim_{x\to 0^+} \frac{-1}{x^2(x+1)}$

$\lim_{x\to 0} \frac{-1}{x^2(x+1)}$ $\lim_{x\to -1^-} \frac{-1}{x^2(x+1)}$

$\lim_{x\to -1^+} \frac{-1}{x^2(x+1)}$ $\lim_{x\to -1} \frac{-1}{x^2(x+1)}$

6. $\lim_{x\to -\frac{\pi}{2}^-} \sec x$ $\lim_{x\to -\frac{\pi}{2}^+} \sec x$

7. Let $f(x) = \frac{x^2-1}{2x+4}$

8. Let $f(x) = \dfrac{x}{x^2 - 1}$

9. Let $f(x) = \dfrac{x^2 - 3x + 2}{x^3 - 4x}$