2.3 Calculating Limits Using the Limit Laws

Theorem:

(i) The limit of a sum is the sum of the limits.

$$\lim_{x\to a}\left[f(x)+g(x)\right]=\lim_{x\to a}f(x)+\lim_{x\to a}g(x)$$

(ii) The limit of a difference is the difference of the limits.

$$\lim_{x\to a}\left[f(x)-g(x)\right]=\lim_{x\to a}f(x)-\lim_{x\to a}g(x)$$

(iii) The limit of a product is the product of the limits.

$$\lim_{x\to a}\left[f(x)g(x)\right]=\lim_{x\to a}f(x)\bullet\lim_{x\to a}g(x)$$

(iv) The limit of a quotient is the quotient of the limits.

$$\lim_{x\to a}\left[\frac{f(x)}{g(x)}\right]=\frac{\lim_{x\to a}f(x)}{\lim_{x\to a}g(x)} \qquad \text{if } \lim_{x\to a}g(x)\neq 0$$

(v) The limit of a constant multiple is the constant multiple of the limit.

$$\lim_{x\to a}\left[cf(x)\right]=c\lim_{x\to a}f(x)$$

(vi) The limit of a power is the power of the limit.

$$\lim_{x\to a}\left[f(x)\right]^n=\left[\lim_{x\to a}f(x)\right]^n \qquad \text{where } n \text{ is a positive integer}$$

(vii) The limit of a constant function is the constant.

$$\lim_{x\to a}c=c$$

Ex2.9) $\lim_{x \to 3} \left(3x^2 - 4x\right)$

Remark: If no problems arise when substituting *a* into a continuous function *f*(*x*), then $\lim_{x \to a} f(x) = f(a)$. (Direct Substitution Property)

Yet if a problem does arise (zero in the denominator), then it may be necessary to manipulate *f*(*x*) to determine whether the limit exists.

Ex2.10) 1. $\lim_{x \to -3} \dfrac{x+3}{x^2 + 4x + 3}$

2. $\lim_{x \to 0} \dfrac{5x^3 + 8x^2}{3x^4 - 16x^2}$

3. $\lim_{v \to 2} \frac{v^3 - 8}{v^4 - 16}$

4. $\lim_{x \to 4} \frac{4x - x^2}{2 - \sqrt{x}}$

5. $\lim_{x \to -1} \frac{\sqrt{x^2 + 8} - 3}{x + 1}$

6. $\lim_{h \to 0} \frac{\sqrt{5h + 4} - 2}{h}$

7. $\lim_{x \to 1} \frac{\sqrt{x} - x^2}{1 - \sqrt{x}}$

8. $\lim_{x \to 2} \frac{|x-2|}{x-2}$

Theorem: The Squeeze (Sandwich) Theorem

Suppose that $g(x) \le f(x) \le h(x)$ for all x in some open interval containing a.
Suppose also that $\lim_{x \to a} g(x) = \lim_{x \to a} h(x) = L$.

Then, $\lim_{x \to a} f(x) = L$.

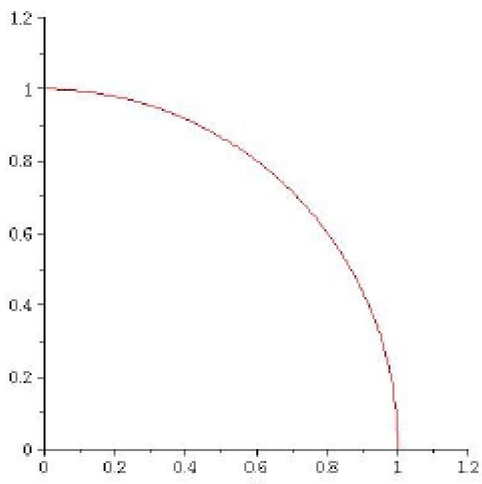
Ex2.11) $\lim_{x \to 0} x^2 \sin \frac{1}{x}$

Ex2.12) $\lim_{x \to 0} \sqrt{x^3 + x^2} \sin \frac{\pi}{x}$

Limits Involving $\frac{\sin \theta}{\theta}$ or $\frac{\theta}{\sin \theta}$

Fact: $\lim_{\theta \to 0} \frac{\sin \theta}{\theta} = \lim_{\theta \to 0} \frac{\theta}{\sin \theta} = 1$

Proof:



Ex2.13) 1. $\lim_{t\to 0}\frac{\sin 5t}{3t}$

2. $\lim_{x\to 0}\frac{\sin\left(4x^2-6x\right)}{4x^2-6x}$

3. $\lim_{t\to 0}\frac{\sin 5t}{\sin 4t}$

4. $\lim_{t\to 0}\frac{\sin 3t\cot 5t}{t\cot 4t}$