2.2 The Limit of a Function

Def'n: Suppose a function *f*(*x*) is defined on all points arbitrarily close to a value *a*. If *f*(*x*) approaches a value *L* as *x* approaches *a* from both the left and the right, then *L* is the limit of *f* as *x* approaches *a*, or

$$\lim_{x \to a} f(x) = L$$

Remark: *f*(*x*) must approach the same value *L* as *x* approaches *a* from both sides. Otherwise, the limit of *f*(*x*) does not exist as *x* approaches *a*.

Remark: The value of *f*(*a*) does not affect the limit.

Ex2.4)

$$f(x) = \begin{cases} x+2 & , \quad x < -2 \\ -x-2 & , \quad -2 < x \le -1 \\ -1 & , \quad -1 < x < 0 \\ 0 & , \quad x = 0 \\ 1 & , \quad x > 0 \end{cases}$$

$$\lim_{x \to -2} f(x) =$$

$$\lim_{x \to -1} f(x) =$$

$$\lim_{x \to 0} f(x) =$$

Extending the concepts to one-sided limits is possible, but consider what value $f(x)$ approaches as x approaches a from either the left or the right.

Denote the limit of $f(x)$ as x approaches a from the left by $\lim_{x\to a^-} f(x)$.

Denote the limit of $f(x)$ as x approaches a from the right by $\lim_{x\to a^+} f(x)$.

Remark: 1. The value of $f(a)$ does not affect either of these limits.

2. In order for $\lim_{x\to a} f(x)$ to exist, it must be true that $\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x)$.

Ex2.5) $f(x) = \begin{cases} x^2 & , \quad -1 \le x < 0 \\ x+1 & , \quad x \ge 0 \end{cases}$

Ex2.6) $\lim_{x\to 1} \frac{x^2 - 1}{|x-1|}$

Ex2.7) $\lim_{t \to 0^+} \ln t$

Ex2.8) $f(x) = \dfrac{x^2 + 1}{x^2 - 1}$