2.1 Rates of Change and Limits

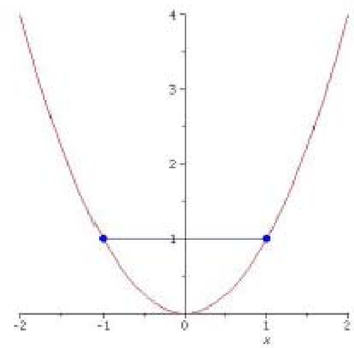
Def'n: The average rate of change of the function $y = f(x)$ over the interval $[x_1, x_2]$ is

$$\frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

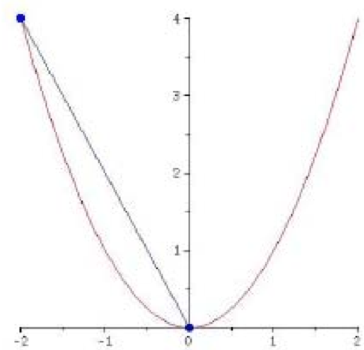
Ex2.1) Find the average rate of change of the function $y = x^2$ over the interval

(i) [-1, 1]
(ii) [-2, 0]
(iii) [-1, 2]

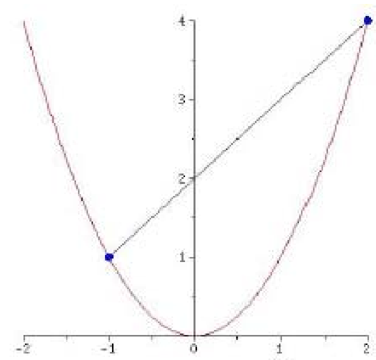
(i)



(ii)



(iii)



Remark: Average rate of change of $y = f(x)$ over this interval $[x_1, x_2]$ is the slope of the <u>secant line</u> through the points $(x_1, f(x_1))$ and $(x_2, f(x_2))$.

Ex2.2) The position of a car is given by the values in the following table:

t (seconds)	0	1	2	3	4	5
d (feet)	0	10	32	70	119	178

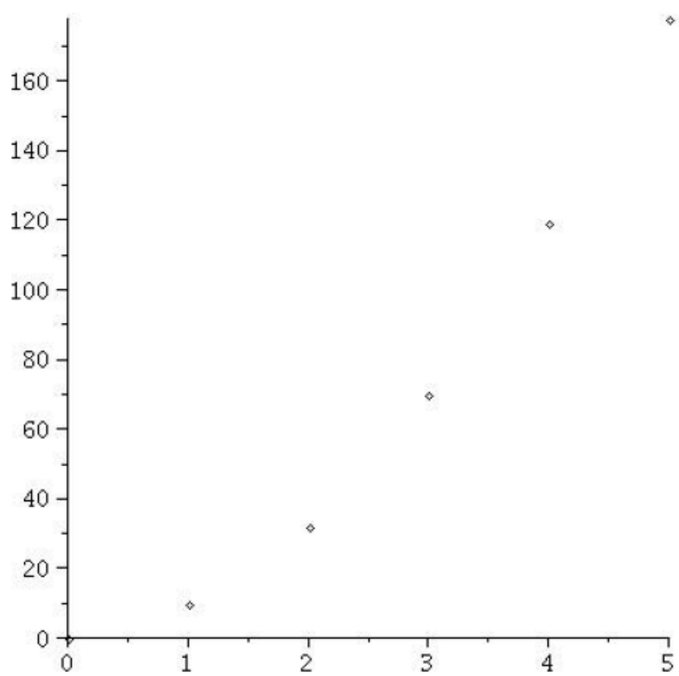
Find the average velocity for the time period between:

(i) $t = 2$ and $t = 5$
(ii) $t = 2$ and $t = 4$
(iii) $t = 2$ and $t = 3$

(i)

(ii)

(iii)



Ex2.3) If an arrow is shot upward on the moon with a velocity of 50 m/s, its height in meters after t seconds is given by $h(t) = 50t - t^2$.

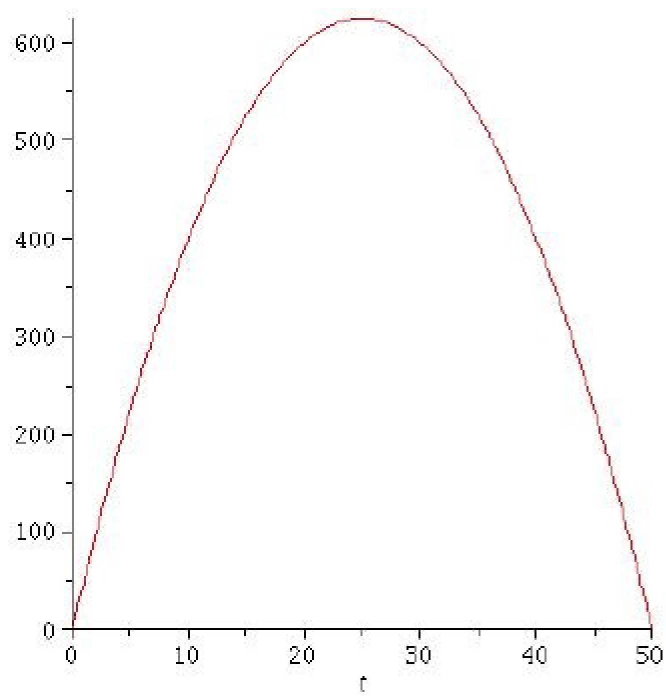
Find the average velocity over the interval:

(i) [1, 2]
(ii) [1, 1.1]
(iii) [1, 1.001]

(i)

(ii)

(iii)



We are getting closer and closer to finding the <u>instantaneous velocity</u> of the arrow at time $t = 1$. To calculate instantaneous rates of change, we need the concept of limits.