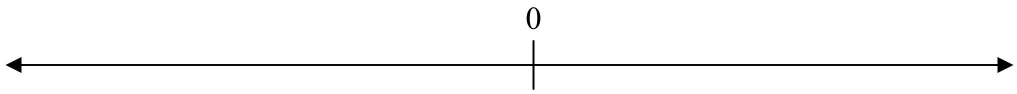
0.1 (Appendix A in text) Numbers, Inequalities, and Absolute Values

Def'n: The real numbers are numbers that can be expressed as decimals.

Ex0.1)

The real numbers can be represented as points on the real number line. (The set of real numbers has no "breaks".)



The set of all real numbers is denoted by $\mathbb{R}$.

One useful property of the real numbers is that they are ordered. That is, for any two real numbers a and b, exactly one of the following is true:

(i) $a < b$

(ii) $a > b$

(iii) $a = b$

Def'n: A set is a collection of objects called elements.

If S is a set, $a \in S$ means that "a is an element of S"; similarly, $a \notin S$ means "a is NOT an element of S".

Sets of numbers:

- The real numbers: $\mathbb{R}$ → All rational and irrational numbers
- The integers: $\mathbb{Z}$ → The whole numbers, negative whole numbers, and zero.
- The natural numbers: $\mathbb{N}$ → The whole numbers from 1 upwards. (Usually.)
- The rational numbers: $\mathbb{Q}$ → Number seen as $\frac{m}{n}$, where $m, n \in \mathbb{Z}$ and $n \neq 0$.
- The irrational numbers: $\mathbb{Q}^C$ → Any number that is NOT a rational number.

In <u>set notation</u>, curly braces "{ }" refer to a set. The vertical line "|" means "with the property that". Thus,

$A = \{x \mid x \in \mathbb{R}\}$ means "A is the set of x such that x is any real number".

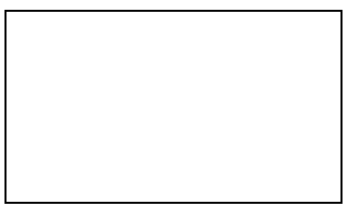
Ex0.2) $B = \{n\pi \mid n \in \mathbb{N}\} =$

$C = \left\{ \frac{n\pi}{4} \mid n \in \mathbb{Z} \right\} =$

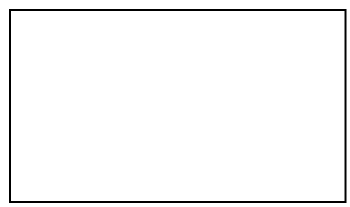
$D = \{x \in \mathbb{R} \mid 0 \le x \le 1\} =$

Def'n: If S and T are sets, then $S \cup T$ is their <u>union</u>, which consists of all elements in either S or T (or both).

The <u>intersection</u> of S and T, denoted by $S \cap T$, is the set of all elements in both S and T.



$S \cup T$



$S \cap T$

The <u>empty set</u>, denoted by $\varnothing$, is the set with no elements.

Ex0.3) 1. Let $S = \{1, 2, 3, 4, 5\}$ and $T = \{3, 4, 5, 6\}$.

Then,

2. Let $S = \{ x \mid x > 4 \}$ and $T = \{x \mid x \le 7 \}$.

Then,

3. $\mathbb{Q} \cup \mathbb{Q}^C =$ $\mathbb{Q} \cap \mathbb{Q}^C =$

Def'n: A subset of the real numbers that has no "breaks" is called an interval.

Ex0.4) $(a, b) = \{ x \mid a < x < b \}$ → open interval

$[a, b] = \{ x \mid a \leq x \leq b \}$ → closed interval

Remarks:

1. When plotting intervals on the real number line, use an open circle to indicate that the endpoint is not included and a closed circle to indicate that it is included.

   Ex0.5) $-1 < x \leq 2$

2. In interval notation, use round brackets to indicate the endpoint is not included and square brackets to indicate they are. Infinities are always given round brackets.

   Ex0.6) $-7 < x \leq 4$ $x > 3$ $\mathbb{R}$

When solving inequalities, use the same methods as solving equations, but remember **when multiplying (or dividing) both sides of an inequality by a negative number, the sign switches direction**.

Ex0.7) Solve the inequality $3(2 - x) > 2(4 + x)$ and show the solution…

(i) in set notation.
(ii) on the number line.
(iii) in interval notation.

(i)

(ii)

(iii)

Def'n: The <u>absolute value</u> of a number x, denoted by $|x|$, is defined by

$$|x| = x \qquad \text{if } x \geq 0$$
$$|x| = -x \qquad \text{if } x < 0$$

Ex0.8) $|5| = 5$ $\qquad |-5| = 5$ $\qquad |3\pi - 10| = 10 - 3\pi$

Properties of Absolute Value:

1. $|ab| = |a| \times |b|$
2. $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$ (In general, $b \neq 0$)
3. $|a^n| = |a|^n$

Further Properties of Absolute Value:

4. $\sqrt{a^2} = |a|$
5. Suppose $a > 0$. Then, $|x| = a$ if and only if $x = \pm a$.
6. Suppose $a > 0$. Then, $|x| < a$ if and only if $-a < x < a$.
7. Suppose $a > 0$. Then, $|x| > a$ if and only if $x > a$ OR $x < -a$.
8. Triangle Inequality: $|a + b| \leq |a| + |b|$

Ex0.9) 1. $|x - 3| = 7$

2. $\left|\frac{3}{2}x - 1\right| \leq 2$

Continue with Appendix A, Odd Exercises.