The Whale Population Problem
(section 2.4, problem 1)

Ecologists use the following model to represent the growth process of two competing species, x and y:

$$\begin{aligned} \frac{dx}{dt} &= r_1 x\left(1-\frac{x}{K_1}\right)-\alpha_1 xy \\ \frac{dy}{dt} &= r_2 y\left(1-\frac{y}{K_2}\right)-\alpha_2 xy \end{aligned}$$

The **variables** x and y represent the number of individuals in each population. The variable t represents time (in years).
The **model parameters** r_i represent the intrinsic growth rates of each species; K_i represent the maximum sustainable population in the absence of competition; and α_i represent the effects of competition.

For populations of blue whales and fin whales, which are competing with each other for the same resources, ecologists have determined the following parameter values:

	Blue Whale	Fin Whale
r	0.05	0.08
K	150,000	400,000
α	10^{-8}	10^{-8}

Determine the populations levels x and y that maximize the rate at which the total whale population is increasing.

Sensitivity Analysis for the Whale Population Problem

1. Perform a graphical sensitivity analysis, examining the sensitivity of the optimal population levels x and y to changes in the intrinsic growth rate for the blue whale population, r_B, set to 0.05 in the original model. Also do the corresponding quantitative sensitivity analysis.

2. Repeat for the model parameter K_B.

3. Assuming that $\alpha = \alpha_B = \alpha_F$, is it ever optimal for one of the whale species to go extinct? If so, under which conditions would this happen, and which of the two species would go extinct?