

Mixing Problems

A typical mixing problem involves a tank of fixed capacity filled with a thoroughly mixed solution of some substance, such as salt.

A solution of a given concentration enters the tank at a fixed rate and the mixture, thoroughly stirred, leaves at a fixed rate.

If $y(t)$ denotes the *amount* of the substance in the tank at time t , then $y'(t)$ is the rate at which the substance is being added minus the rate at which it is being removed.

We can use this type of reasoning to model a variety of phenomena: chemical reactions, discharge of pollutants into a lake, injection of a drug into the bloodstream, etc.

The Tank Problem

Consider a tank that contains 20 kg of salt dissolved in 5000 L of water.

Brine that contains 0.03 kg of salt per L of water enters the tank at a rate of 25 L/min.

The solution is kept thoroughly mixed and drains from the tank at the same rate.

Develop a differential equation for the *amount* of salt in the tank.

How much salt remains in the tank after half an hour? After many hours?

The Modified Tank Problem

Consider the same tank. Initially, it contains 5000 L of water.

Brine that contains 0.03 kg of salt per L of water enters the tank at a rate of 25 L/min, but now it drains at a rate of 30 L/min.

The solution is kept thoroughly mixed at all times.

Develop a differential equation for the *amount* of salt in the tank.