

Supply and Demand

Consider the following economic model: Let P be the price of a single item on the market. Let Q be the quantity of the item available on the market. Both P and Q are functions of time. The following model has been proposed:

$$\begin{aligned}\frac{dP}{dt} &= aP \left(\frac{b}{Q} - P \right), \\ \frac{dQ}{dt} &= cQ (fP - Q),\end{aligned}$$

where a , b , c , and f are constants with the following values: $a = 1$, $b = 20,000$, $c = 1$, $f = 30$. Interpret and discuss the equations. Do they make sense?

Find the steady states of this system, and perform a graphical stability analysis in the phase plane. What can you conclude

about the stability of the nontrivial steady state, if anything?
Why or why not?

Give an economic interpretation of the nontrivial curve along which $dP/dt = 0$, and of the nontrivial curve along which $dQ/dt = 0$.

Perform a linear stability analysis of the nontrivial steady state.

What do you conclude? Sketch the nature of model solutions in the phase plane.