Lapis 2019 - M D Sacchi

Assignment:	Lapis 2019
Due:	May 13, 2019
Topic:	VSP inverse problem
Help:	msacchi@ualberta.ca

Provide codes and figures as your answers

Problem

Figure 1 displays a Vertical Seismic Profile (VSP). The detectors in the borehole record waves excited by a source at $z = 0$. The source is close to the well. Therefore, you can consider vertical propagation. We will assume that the earth can be discretized in terms of M layers of equal thickness and we will assign to each layer a velocity. Under this assumption one represent the vertical distribution of velocities in the subsurface as a vector of size $M \times 1$

$$\mathbf{m} = [s_1, s_2, s_3, \ldots, v_M]^T$$

where $s_i = \frac{1}{v_i}$ is the slowness of layer i.

Consider N detectors placed in the borehole at depth z_i, $i = 1, \ldots N$. We also assume that each detector has recorded first-arrival travel times of waves excited by the source t_i, $i = 1, \ldots N$. Our data vector is given by the travel-times $\mathbf{d} = [t_1, t_2 \ldots t_n]^T$, our vector of model parameters is given by the reciprocal of the velocity (slowness) $\mathbf{m} = [s_1, s_2, s_3, \ldots s_M]^T$.

a Write a function to compute the tomographic matrix $\mathbf{G}$

$$\mathbf{d} = \mathbf{G}\mathbf{m}$$

b Read the ascii data file `data_vsp.asc` containing depth of detector and travel-time. The first column of the table is the depth of the receiver in km. The second column is the traveltime in secs. Traveltimes provided in the file were contaminated by noise with standard error $\sigma = 0.0018$s.

c Write a code to estimate the velocity structure by inverting the travel-times by minimizing the cost

$$J = \|\mathbf{G}\mathbf{m} - \mathbf{d}\|_2^2 + \mu^2 \|\mathbf{D}(\mathbf{m} - \mathbf{m}_0)\|_2^2$$

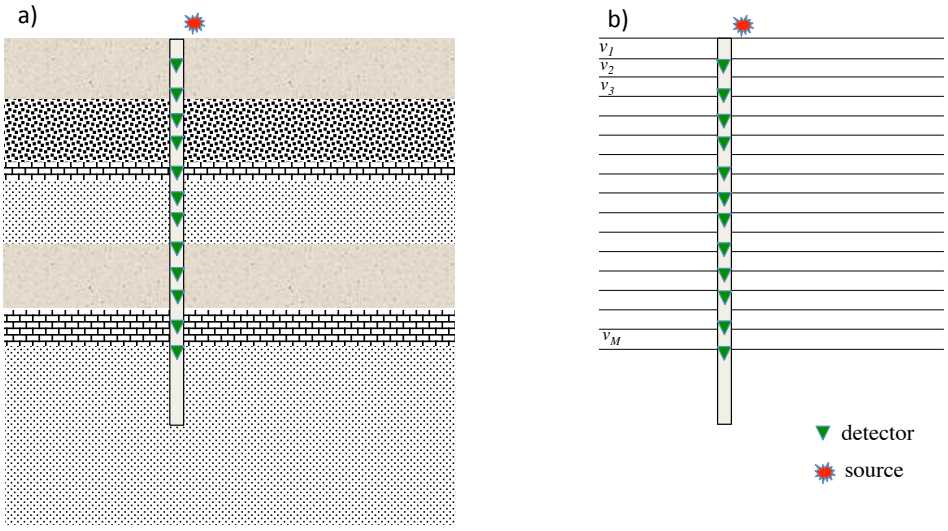

Figure 1: a) Subsurface model. b) Model of layers of equal thickness that is used to represent the subsurface. Detectors along the borehole are used to record waves generated by source at $z = 0$ close to the borehole. This configuration is often called zero-offset VSP.

Explore solutions with different values of trade-off parameter. I recommend that for $\mathbf{m}_0$ you adopt a constant slowness of $1/3\,s/km$ (a constant velocity of $3km/s$).

d Compute misfit versus μ and try to pick μ via the χ^2 test. Show the solution for the optimum value of μ. The operator $\mathbf{D}$ is the matrix of first order derivatives (read the paper by Lizarralde and Swift)

e Plot a trade-off curve (Model norm vs Misfit) for different values of μ. Adopt a logarithmic μ scale.