CMPUT 204 (B1)
Final Examination: Version A
April 16, 2004

Department of Computing Science
University of Alberta

Instructor: M. Bowling

Time: 180 Minutes

Last Name: _____

First Name: _____

- Put your name above (no ID). Print clearly.
- There are 7 problems, in 7 pages (70 marks in total).
- Closed Book: Only Pen, Pencil & Eraser allowed.
- Use space below the questions to write your answers.

Do NOT write on this page, for instructor's use only.

total marks	question	your marks
10	1	
10	2	
10	3	
10	4	
10	5	
10	6	
10	7	
70		

Last Name:_____ **ID#:**_____

1. [**7 + 3 marks**] We can find the largest element in an array of size n with $n-1$ array comparisons. We could then find the second largest element with another $n-2$ comparisons, resulting in a total of $2n-3$ array comparisons.

   (a) Use divide-and-conquer to design an algorithm that finds the two largest elements using at most $\frac{3}{2}n$ array comparisons. You may describe your algorithm in English or pseudocode.

   (b) Write a recurrence for your algorithm (assuming n is a power of 2) and solve the recurrence.

Last Name:_____ **ID#:**_____

2. [**2 + 2 + 6 marks**] *This problem should seem familiar.* We often assume in our sorting analyses that the keys are all distinct. What if many of the keys to be sorted are duplicates? In the extreme, suppose the array to sort contains only keys "0" and "1".

(a) In theta notation, what is the worst-case number of key comparisons for InsertionSort with this class of inputs? Describe a worst-case input.

(b) In theta notation, what is the worst-case number of key comparisons for QuickSort with this class of inputs? Describe a worst-case input.

(c) Design an efficient algorithm for sorting for this class of inputs. You may describe your algorithm in English or pseudocode.

3. [**3 + 7 marks**] Consider the 5-coloring decision problem. It takes simple undirected graphs as instances and accepts the graph if there exists an assignment, $c(v) \in \{1, 2, 3, 4, 5\}$, of vertices to five colors such that, if $(x, y) \in E$ then $c(x) \neq c(y)$.

(a) Show that 5-coloring belongs to NP.

(b) Prove that 5-coloring is in NP-complete using a reduction from 3-coloring. The reduction must be explicit, and it must be shown to be polynomial time.

4. [**5 + 5 marks**] Consider the spanning trees generated by BFS and DFS traversals in a connected undirected simple graph.

(a) Construct a graph with the fewest possible vertices such that DFS and BFS traversals must result in *different* spanning trees.

(b) Construct a graph with the fewest possible vertices such that DFS and BFS traversals must result in the *same* spanning trees.

Last Name:_____ **ID#:**_____

5. [**10 marks**] Consider (as in question 4) the spanning trees generated by BFS and DFS traversals in a connected undirected simple graph. Prove or disprove: DFS and BFS result in *different* spanning trees if and only if the graph has a cycle.

6. [**5 + 5 marks**] Consider Dijkstra's algorithm for finding single-source shortest paths. We discussed in class that the algorithm assumes all weights are positive. Consider allowing negative edge weights as long as the graph does not contain negative-weight cycle (i.e., a cyle whose edge weights sum to a negative value).

(a) Draw an undirected edge-weighted graph such that Dijkstra's algorithm will output an incorrect distance for at least one of the vertices. Clearly label the source vertex and the vertex with an incorrectly reported distance. *Hint: You'll want to make this graph small; see part (b).*

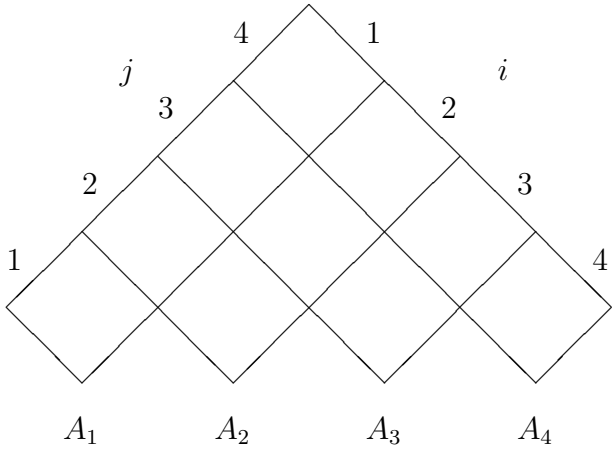
(b) Trace the execution of Dijkstra's algorithm on this graph. For each iteration show the vertex that is de-queued and its reported distance.

Last Name:_____ **ID#:**_____

7. [**7 + 3 marks**] Recall the dynamic programming algorithm for computing an optimal order for multiplying a chain of matrices together.

(a) The algorithm involved computing entries in a two dimensional table. Show the results of this table below after running the algorithm with the given array dimensions.

$A_1 : 7 \times 3 \qquad A_2 : 3 \times 4 \qquad A_3 : 4 \times 9 \qquad A_4 : 9 \times 2$



(b) Parenthesize the expression below according to the computed optimal ordering of matrix multiplies.

$A_1 \quad A_2 \quad A_3 \quad A_4$