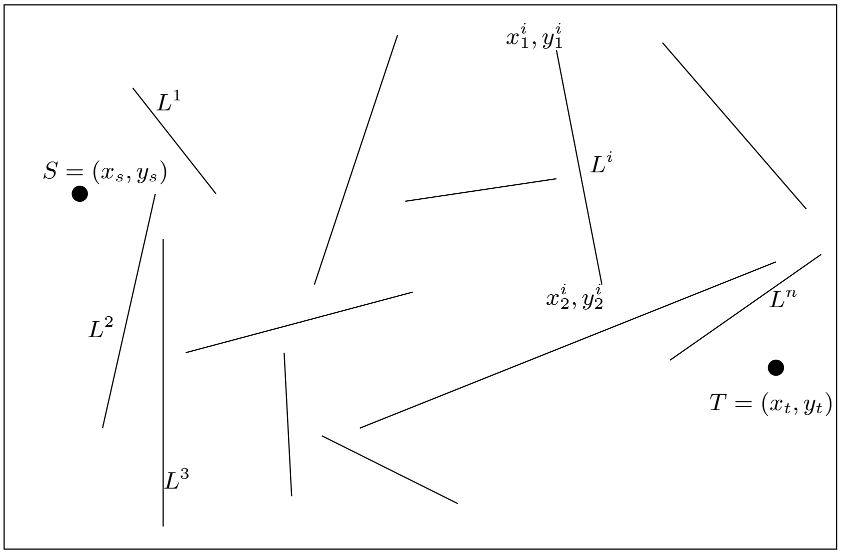
CMPUT204
Department of Computing Science
University of Alberta

Homework Assignment #5

Due: Dec 8, 2004

Question 1. (15 marks)
Consider an edge weighted graph G with weights $w[\cdot]$, and a minimum spanning tree of the graph, T. Suppose for some edge (u, v) you decrease $w[u, v]$ to be w_*. Describe a procedure (pseudocode is unnecessary) that would find the new MST with the updated weight. Your algorithm must run faster than recomputing the MST from scratch, i.e., $o(|E|log|V|)$. Give the running time of your procedure.

Question 2. (20 marks) A robot is in a room with many walls (obstacles) and wants to go from the start point $S = (x_s, y_s)$ to target point $T = (x_t, y_t)$. There are a total of n walls $L^1, \ldots, L^n$ and each wall L^i is a straight line segment which runs between points x_1^i, y_1^i and x_2^i, y_2^i (see the example in the figure). Assume that the robot is provided with the cordinates of the walls as well as the start point and the target point. Describe an efficient algorithm for the robot which helps it to go from the start point to the end point in the shortest amount of time (assuming that the robot moves at a fixed speed). You may also make use of a geometry procedure INTERSECT that can answer in constant time if two line segments (identified by their endpoints) intersect. Analyse the running time of your algorithm.



Question 3. (15 marks) For each of the following problems, first prove that it belongs to NP. Then state whether you think the problem is NP-complete. Justify your answer by either proving it to be NP-compelete or giving good reason to suspect that it is not. You can assume the facts that the following problems are NP-complete: SAT, 3SAT, CLIQUE, IND-SET, HAM-PATH, HAM-CYCLE. Remember that it is widely believed that P≠NP.

a. MIN-LEAF-SPANNING-TREE
Input: A graph G and an integer k.
Question: Does G have a spanning tree with at most k leaves?

b. 8-CLIQUE
Input: A graph G.
Question: Does G have a clique of size at least 8?

c. MANY-3SAT
Input: A boolean formula Φ in 3-CNF and an integer k.
Question: Is there a truth assignment to Φ such that at least k clauses are satisfied?