CMPUT204
Department of Computing Science
University of Alberta

Homework Assignment #4
Due: Nov 24, 2004

Question 1. (15 marks) The *square* of an undirected graph $G = (V, E)$ is a new graph $G^2 = (V, E^2)$ where $(u, v) \in E^2$ if and only if there exists a $w \in V$ and $(u, w) \in E$ and $(w, v) \in E$. Describe efficient algorithms for computing G^2 if the graph was represented with (i) adjacency lists, and (ii) and adjacency matrix. You must also analyze the running time of your algorithm.

Question 2. (15 marks) A vertex s of a *directed* graph $G = (V, E)$ is called a *sink* if for every vertex $v \in (V - \{s\})$, $(v, s) \in E$ and $(s, v) \notin E$. In other words, every vertex has an edge to s and no edge from s. Write an algorithm that given a directed graph, finds a sink (or return that one does not exist) in only $O(|V|)$ time. The graph is given by its adjacency matrix A. Notice that a running time of $O(|V|)$ is remarkable given that the input to the procedure is $\Omega(|V|^2)$.

Question 3. (20 marks)
a. (10 marks) Exercise 21.3-2 on page 509 of the textbook.
b. (10 marks) Exercise 21.3-3 on page 509 of the textbook.

Question 4. (15 marks) Consider the disjoint-sets abstract data type, but modified so that each member element also has an associated key. MAKE-SET(x, k) creates a new element x with key k. FIND(x) returns the smallest key of any member in the same set as x. UNION remains unchanged. Write an efficient implementation of this modified version of disjoint sets. Compare its running time to that of the best implementation of basic disjoint sets.

Hint: Modify the pseudocode on page 508 to handle keys.

Question 5. (20 marks)
a. (10 marks) Prove that every connected graph with $|V| > 2$ has at least two vertices such that if one or both were removed along with their incident edges, the resulting graph is connected.
b. (10 marks) Describe an efficient algorithm to find two such vertices.

Hint: The algorithm should be based on the proof, or the proof should be based on the algorithm.