

Homework Assignment #4
Due: Nov 24, 2004

Question 1. (15 marks) The *square* of an undirected graph $G = (V, E)$ is a new graph $G^2 = (V, E^2)$ where $(u, v) \in E^2$ if and only if there exists a $w \in V$ and $(u, w) \in E$ and $(w, v) \in E$. Describe efficient algorithms for computing G^2 if the graph was represented with (i) adjacency lists, and (ii) adjacency matrix. You must also analyze the running time of your algorithm.

Question 2. (15 marks) A vertex s of a *directed* graph $G = (V, E)$ is called a *sink* if for every vertex $v \in (V - \{s\})$, $(v, s) \in E$ and $(s, v) \notin E$. In other words, every vertex has an edge to s and no edge from s . Write an algorithm that given a directed graph, finds a sink (or return that one does not exist) in only $O(|V|)$ time. The graph is given by its adjacency matrix A . Notice that a running time of $O(|V|)$ is remarkable given that the input to the procedure is $\Omega(|V|^2)$.

Question 3. (20 marks)

- a. (10 marks) Exercise 21.3-2 on page 509 of the textbook.
 - b. (10 marks) Exercise 21.3-3 on page 509 of the textbook.
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Question 4. (15 marks) Consider the disjoint-sets abstract data type, but modified so that each member element also has an associated key. MAKE-SET(x, k) creates a new element x with key k . FIND(x) returns the smallest key of any member in the same set as x . UNION remains unchanged. Write an efficient implementation of this modified version of disjoint sets. Compare its running time to that of the best implementation of basic disjoint sets.

Hint: Modify the pseudocode on page 508 to handle keys.

Question 5. (20 marks)

- a. (10 marks) Prove that every connected graph with $|V| > 2$ has at least two vertices such that if one or both were removed along with their incident edges, the resulting graph is connected.
- b. (10 marks) Describe an efficient algorithm to find two such vertices.

Hint: The algorithm should be based on the proof, or the proof should be based on the algorithm.