Mass and force in Imperial units

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On Earth, 1 lbm exerts a force of 1 lbf. Thus

$$\begin{aligned} 1\,\text{lbf} &= 1\,\text{lbm} \cdot g \\ &= 1\,\text{lbm} \cdot 32.2\,\frac{\text{ft}}{\text{s}^2} \\ &= 32.2\,\frac{\text{lbm} \cdot \text{ft}}{\text{s}^2} \end{aligned}$$

Masses may also be expressed in units of slugs where

$$\begin{aligned} 1\,\text{slug} &= 1\,\text{lbf} \cdot \frac{\text{s}^2}{\text{ft}} \\ &= 32.2\,\frac{\text{lbm} \cdot \text{ft}}{\text{s}^2} \cdot \frac{\text{s}^2}{\text{ft}} \\ &= 32.2\,\text{lbm} \end{aligned}$$

The slug is defined so that an object having a mass of 1 slug exerts a force of 32.2 lbf. Alternatively, 1 lbf is the force required to accelerate 1 slug of mass at $1\,\text{ft}/\text{s}^2$. Analogously 1 N is the force required to accelerate 1 kg of mass at $1\,\text{m}/\text{s}^2$. To accelerate a 1 kg mass at $9.8\,\text{m}/\text{s}^2$ would require 9.8 N so, on Earth, the weight of a 1 kg object is 9.8 N.

Suppose that you're solving a problem where the mass is given as 800 lbm. What is the associated force?

$$\begin{aligned} \text{mass} &= 800\,\text{lbm} \\ &= 800\,\text{lbm} \cdot \frac{1\,\text{slug}}{32.2\,\text{lbm}} \\ &= \frac{800}{32.2}\,\text{slug} \end{aligned}$$

$$\begin{aligned} \text{force} &= \text{mass} \cdot g \\ &= \frac{800}{32.2}\,\text{slug} \cdot 32.2\,\text{ft}/\text{s}^2 \\ &= 800\,\text{slug}\,\text{ft}/\text{s}^2 \end{aligned}$$

Recall, however, that $1\,\text{slug}\,\text{ft}/\text{s}^2$ equals 1 lbf. The force is therefore 800 lbf. Okay, but what about the conversion factor that says that $1\,\text{lbf} = 32.2\,\text{lbm}\,\text{ft}/\text{s}^2$? Doesn't this suggest that an object with a mass of 800 lbm has a gravitational force of 800×32.2 lbf? No! The above conversion factor is just that, a conversion factor, not a formula for computing force from mass. In short, an object with a mass of 1 lbm exerts a force of 1 lbf. Likewise, an object with a mass of 800 lbm exerts a force of 800 lbf.