Gravity current flow in two-layer stratified media

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Gravity currents in the atmosphere

- Gravity currents are primarily horizontal flows driven by density differences and are ubiquitous features of many industrial and environmental processes.

- Atmospheric manifestations include dust storms ("haboobs"), thunderstorm outflows and "microbursts"; all represent a significant threat to aircraft (Linden & Simpson, 1985).

Photo credits:
http://dailyshot.homestead.com
http://www.stormeyes.org/tornado/SkyPixhaboob.htm
http://www.damtp.cam.ac.uk/user/fdl/people/jes14

Phoenix sandstorm (July 5, 2011)

Thunderstorm outflow

Wet microburst
Gravity currents in marine environments

- Marine manifestations include river plumes (Nash & Moum 2005), which influence coastal ecology, pollution transport, etc.

Photo credits:
http://www.glerl.noaa.gov
http://fvcom.smast.umassd.edu

Grand River plume (Grand Haven, MI)

Changjiang River plume (East China Sea)
Introduction

A flow this ubiquitous merits detailed attention

Q? If gravity currents are driven by density differences, how can we relate the gravity current front speeds to these differences?

Q? Does the front speed also depend on geometric parameters such as the gravity current height?

Will address these questions using the following assumptions:

• High-Re flow (ignore viscous dissipation)

• Rectilinear geometry, channel of finite height (channel of infinite height is a straightforward extension)

• Non-rotating reference frame

• Density difference due to difference of composition, temperature or dissolved salt, not (sedimenting) particles

Front speed is then constant at least for early times
Benjamin’s 1968 theory

- The flow of a high-Re gravity current was examined theoretically by T. Brooke Benjamin in 1968* who applied a Galilean change of reference frame (gravity current front stationary, ambient in motion)

\[
\begin{align*}
U H &= u(H - h) \\
\Rightarrow \quad u &= \frac{U H}{H - h}
\end{align*}
\]

One eqn. but three unknowns: \(U, u, h\)

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*T.B. Benjamin, *J. Fluid Mech.*, 31 (1968) -- Cited > 600 times
No external forces acting on the flow, therefore “flow force” is conserved, i.e.

$$\int (p + \rho v^2) \, dz = \text{constant}$$

$v$ - horizontal velocity

Apply this result far up- and downstream where mixing is negligible and pressure, $p$, is hydrostatic

$$\int^D_A (p + \rho v^2) \, dz = \int^C_B (p + \rho v^2) \, dz$$
Benjamin’s 1968 theory

\[ \int_{A}^{D} (p + \rho v^2) \, dz = \int_{B}^{C} (p + \rho v^2) \, dz \quad \text{yields} \]

\[ \text{Fr}^2 = \frac{U^2}{g'H} = \frac{h(H-h)(2H-h)}{H^2(H+h)} \]

\( \text{Fr} \) - Froude number (non-dim. front speed)

\[ g' = g \left( \frac{\rho_0 - \rho_1}{\rho_1} \right) \] - reduced gravity
Benjamin’s 1968 theory

- Solutions with $h/H > 0.5$ have negative dissipation and are therefore unphysical.
- Realized value of $h/H$ depends on initial condition.
- If gravity current fluid initially spans the entire channel depth, $h/H = 0.5$.
Density-stratified ambient

- Benjamin’s 1968 theory assumes a uniform ambient, but this is inaccurate in most environmental and many industrial contexts.

- Density stratification introduces a myriad of new complications: dynamic coupling may arise between the gravity current front and internal or interfacial waves.

- When the ambient is stratified, gravity current may still propagate along lower (or upper) boundary, or it may propagate as an intrusion inside the stratified fluid.
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Gravity current flow in a two-layer ambient
Boundary gravity current

Effluent discharge in marine environments

(a)

[Diagram showing cold lower layer, warm upper layer, and internal wave]

(b)

[Diagram showing discharged effluent, warm upper layer, and cold lower layer]

Density-stratified ambient

Regarding the flow of a gravity current in the context of pollution dispersion leads to the following questions:

Q? How quickly does effluent travel downstream?

Q? How is this motion influenced by the interfacial wave that may propagate ahead of the gravity current front?

Satisfactorily addressing these (and other) questions requires a judicious combination of theory and experiment (laboratory and numerical)
Boundary gravity current

Theoretical model: apply usual change of reference frame (i.e. gravity current front stationary)

Photo credit: Alan W. Tan
Boundary gravity current - theory

Geometry:
\[ h_0 + h_1 + h_2 = h_1' + h_2' \]
\[ h_0 + h_1 = h_1' + \eta \]

Mass balance:
\[ u_i h_i = U h_i' \quad i = 1, 2 \]

Flow force balance:
\[ \int_A^D (p + \rho v^2) \, dz = \int_B^C (p + \rho v^2) \, dz \]
\[
\frac{1}{2} U^2 H + \frac{1}{2} g'_{02} H^2 = g'_{12} \left[ h_1' (H - \frac{1}{2} h_1') + \frac{1}{2} h_2^2 \right] + \frac{1}{2} g'_{01} (h_1 + h_2)^2 + U^2 \left( \frac{h_1'^2}{h_1} + \frac{h_2'^2}{h_2} \right)
\]

One equation in three unknowns: \( U, h_1, h_2 \)  

Q: How do we achieve closure?
Bernoulli’s equation (layer 1):
\[ \frac{1}{2} U^2 = g'_{01} \frac{h_1^2}{h_1'} (H - h_1 - h_2) \]

Bernoulli’s equation (layer 2):
\[ \frac{1}{2} U^2 = \frac{h_2^2}{h_2'} \left[ g'_{02} (H - h_1 - h_2) - g'_{12} (h_1' - h_1) \right] \]

Applying both equations leads to unphysical multiplicities, however...

Boundary gravity current - theory

- We choose to apply Bernoulli’s equation once then relate the amplitude $\eta$ of the interfacial disturbance to the other parameters of the problem.

When the gravity current fluid initially spans the entire channel depth, parameterization is easy!

$$\eta = \frac{1}{2}(H - h_1')$$
Boundary gravity current - theory

- We choose to apply Bernoulli’s equation once then relate the amplitude $\eta$ of the interfacial disturbance to the other parameters of the problem.

When the gravity current fluid initially spans the entire channel depth, parameterization is easy!

\[ \eta = \frac{1}{2} (H - h_1') \]

Closed symbols: lab expts.
Open symbols: numerics
Experiments

- Experiments run in a 2.3 m long tank using salt water of various densities

Experiments

- One of the challenges of running laboratory experiments is that it is difficult to minimize the thickness of the ambient interface.

- Fortunately, this thickness has a very minor impact on the front speed c.f. Faust & Plate (1984)
Numerical simulations

- 2D simulations (mixed spectral-FD) use Diablo (http://numerical-renaissance.com/), which has been applied in numerous related studies e.g. Taylor 2008, Bolster et al. 2008, Flynn et al. 2008

Comparison (theory vs. measurement)

\[ \text{Fr} = \frac{U}{\sqrt{g'_{02}H}} \]

\[ g'_{02} = g \left( \frac{\rho_0 - \rho_2}{\rho_0} \right) \quad \text{etc.} \]
Comparison (theory vs. measurement)

- Generally positive agreement, but...
- Analytical solution “peters out” at \( g'_{12}/g'_{02} = 0.75 \)

Q? Why is this?

Boundary gravity current - theory

Gravity current front speed

Long wave/bore speed

Supercritical: \( \frac{g'_{12}}{g'_{02}} < 0.75 \)

Subcritical: \( \frac{g'_{12}}{g'_{02}} > 0.75 \)
Boundary gravity current - theory

When \( \frac{g'_{12}}{g'_{02}} \approx 0.75 \) there is a qualitative change of behavior, i.e. gravity current goes from being supercritical to subcritical.

Supercritical: \( \frac{g'_{12}}{g'_{02}} < 0.75 \)

Subcritical: \( \frac{g'_{12}}{g'_{02}} > 0.75 \)
Super- vs. subcritical

Gravity current able to travel for long distances at constant speed

Supercritical: \( g'_{12}/g'_{02} < 0.75 \)

Gravity current quickly overtaken by interfacial wave, which leads to sudden deceleration

Subcritical: \( g'_{12}/g'_{02} > 0.75 \)
Supercritical: $\frac{g'_{12}}{g'_{02}} < 0.75$

Subcritical: $\frac{g'_{12}}{g'_{02}} > 0.75$

Gravity current able to travel for long distances at constant speed

Gravity current quickly overtaken by interfacial wave, which leads to sudden deceleration

How quick is “quick?”
Super- vs. subcritical

Horizontal distance (normalized by lock length) where gravity current front first begins to decelerate

- When lower layer is thin and $g'_{12}/g'_{02}$ is small, front will travel at constant speed for a long time (c.f. Sutherland & Nault 2004)
- Not so when lower layer is thick and/or $g'_{12}/g'_{02}$ is large
**Lock condition**

- When lower layer is thin and $g'_{12}/g'_{02}$ is small, front will travel at constant speed for a long time.
- Not so when lower layer is thick or $g'_{12}/g'_{02}$ is large.

These (generic) statements are independent of the initial (i.e. lock) condition, but not so the quantitative details of our previous parameterization.

\[
\eta = \frac{1}{2}(H - h'_1)
\]

Equation applies only when the gravity current fluid initially spans the entire channel depth (full depth lock release).
Partial depth lock release

Two alternatives:

- Generalize previous parameterization (hard problem*)
- Apply a shallow-water model

*Tan M.Sc. thesis (U. Alberta) 2010
Partial depth lock release

Two alternatives:

• Generalize previous parameterization (hard problem*)

• Apply a shallow-water model

  Assumptions:
  • Away from the front, pressures are hydrostatic
  • The return (i.e. right to left) flow in either ambient layer is neglected

Shallow water models do not faithfully reproduce the details of the gravity current shape, but they have an impressive record of predicting the front speed (Ugarish 2009). Do they work well here? Yes!

*Tan M.Sc. thesis (U. Alberta) 2010
Shallow water model (results)

Normalized front speed vs. normalized lower layer depth for different lock conditions and ambient stratification

Flynn, Ungarish & Tan, Phys. Fluids, 24 (2012)
Shallow water model (results)

Normalized front speed vs. normalized lower layer depth for different lock conditions and ambient stratification

Column 1:
Deepest ambient (i.e. lock fluid spans 1/4 the channel depth)

Column 4:
Shallowest ambient (i.e. lock fluid spans entire channel depth)

Previous formulation can readily (and more or less successfully) be extended to an axisymmetric geometry.

Open circles: Laboratory experiments
Closed circles: Shallow water simulations
Line: Shallow water theory (assumes constant front speed)

Investigation motivated by a desire to improve the understanding of the dynamics of discharge and ventilation flows in marine environments.

Q? Have we been successful in this respect?
Outlook/conclusions

Q? Have we been successful in this respect?
Ans. Yes...

• Have a deeper understanding of the interplay between the gravity current and the interfacial waves or disturbances that it may excite when the ambient is stratified

• Have developed well-corroborated analytical models for boundary gravity currents that consider different ambient and initial conditions

... but more work remains to be completed.

• Gravity currents are assumed to be compositional, so cannot readily describe flow physics of e.g. river plumes, which carry suspended sediment

• Have considered an idealized 2D geometry with a flat bottom boundary (see talk by Mitch Nicholson later in this session)

• Have ignored “long time” behavior where deceleration of the front must be considered
Selected publications


