The impact of dispersion on porous media gravity currents propagating over an interbed layer

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\textsuperscript{2} de Anna et al., Geophysics Res. Lett., 41(2014)

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1-Introduction:

- Buoyancy-driven flow in porous media is seen in lots of industrial projects such as:
  - CO\textsubscript{2}/acid-gas sequestration
  - H\textsubscript{2} storage
  - Analysis of groundwater contamination

**CO\textsubscript{2} sequestration**

- The flow of the injected buoyant fluid into the dense ambient leads to the formation of gravity currents, where the flow is approximately parallel to the top boundary.

**Underground storage of H\textsubscript{2}**

- Multi-layered strata
- Injection
- Withdrawal

Feldmann et al., 2016
1-Introduction:

• We focus on dispersive effects by which gravity current fluid mixes with ambient fluid. To do so, we consider miscible fluids, a situation applicable to underground hydrogen storage (UHS) in depleted reservoirs.

• The reservoir consists of layers with different permeabilities i.e., \( \text{H}_2 \) storage that has been studied numerically by Feldmann et al. (2016).

• In this case, the gravity current experiences distributed drainage throughout the upper boundary. As we will see, this drainage can lead to a greater degree of dispersion.

• For analytical convenience and consistent with previous studies, we assume the gravity current density is larger than that of the surrounding ambient.

• As a result, the gravity current is “upside-down” relative to those expected in real UHS facilities.
2-Theoretical model: governing equations

- We separate bulk and dispersed interfaces and thereby introduce two entrainment velocities:

\[ w_{e1} = \varepsilon_1 \tilde{u}_1 \]
\[ w_{e2} = \varepsilon_2 \tilde{u}_2 \]

\( \varepsilon_1, \varepsilon_2 \): Entrainment coefficients. We assume \( \varepsilon_1 = \varepsilon_2 = \varepsilon \)

\( k \): permeability

\( k_b \): interbed permeability

**Mass conservation for fluid in the bulk phase**

\[ \varphi \frac{\partial h_1}{\partial t} + \frac{\partial}{\partial x} (\tilde{u}_1 h_1) = -w_{e1} - w_{d1} \]

**Mass conservation for fluid in the dispersed phase**

\[ \varphi \frac{\partial h_2}{\partial t} + \frac{\partial}{\partial x} [\tilde{u}_2 (h_2 - h_1)] = -\frac{\partial}{\partial x} (\tilde{u}_1 h_1) + w_{e2} - w_{d1} - w_{d2} \]

**Buoyancy conservation in the dispersed phase**

\[ \varphi \frac{\partial b_2}{\partial t} + \frac{\partial}{\partial x} (\tilde{u}_2 [b_2]) = w_{e1} c_s - w_{d2} \tilde{c}_2 \]

\[ b_2 = \tilde{c}_2 (h_2 - h_1) \]

The value of \( w_{d1} \) and \( w_{d2} \) is influenced by the degree of mixing occurring in the lower layer, predicting the details of this mixing is a complicated task.
2-Theoretical model: drainage velocity

No mixing

\[
\frac{\partial \ell}{\partial t} = \begin{cases} 
0 & 0 \leq x < x_{Nb} \\
\frac{w_{d1}}{Cw_{d2}} & x_{Nb} \leq x \leq x_d
\end{cases}
\]

\[
w_{d1} = \begin{cases} 
k_b g'_s \cos \theta \left( \frac{c_s h_1 + b_2}{c_s \xi} + 1 \right) & \ell < \xi \\
\frac{c_s h_1 + b_2 + c_s \ell}{(1 - K)c_s \xi + K c_s \ell} & \ell \geq \xi
\end{cases}
\]

\[
w_{d2} = \begin{cases} 
0 & 0 \leq x < x_{Nb} \\
k_b g'_s \cos \theta \left( \frac{h_2}{\xi} + 1 \right) & x_{Nb} \leq x \leq x_d
\end{cases}
\]

Perfect mixing

\[
K = \frac{k_b}{k} \\
g'_s = g \beta c_s : \text{source reduced gravity}
\]

\[
\nu : \text{viscosity}
\]

\[
C = \frac{c_2}{c_s}
\]

\[
w_{d1} = \begin{cases} 
k_b g'_s \cos \theta \left( \frac{c_s h_1 + b_2}{c_s \xi} + 1 \right) & 0 \leq x < x_{Nb} \\
0 & x_{Nb} \leq x \leq x_d
\end{cases}
\]

\[
w_{d2} = \begin{cases} 
0 & 0 \leq x < x_{Nb} \\
k_b g'_s \cos \theta \left( \frac{h_2}{\xi} + 1 \right) & x_{Nb} \leq x \leq x_d
\end{cases}
\]
3-Numerical investigation: COMSOL set-up

• COMSOL simulations are performed to assess our mathematical model in various scenarios and to estimate the entrainment coefficient, $\epsilon$, in the theoretical model.

• Simulations are conducted in a rectangular box filled with a porous medium saturated with water having a density $\rho_0 = 0.998\ \text{gcm}^{-3}$.

• Salt water is discharged at a constant rate from a source.

By considering the Boussinesq approximation, mass conservation, Darcy’s equation, and the solute transport equation, we write:

\[
\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0 \\
\frac{1}{\rho_0} \frac{\partial P}{\partial x} + \frac{v}{k} u = 0 \\
\frac{1}{\rho_0} \frac{\partial P}{\partial z} + \frac{v}{k} w = \frac{\rho}{\rho_0} g
\]

\[
\phi \frac{\partial c}{\partial t} + u \frac{\partial c}{\partial x} + w \frac{\partial c}{\partial z} = \frac{\partial}{\partial x} \left( D_{xx} \frac{\partial c}{\partial x} + D_{xz} \frac{\partial c}{\partial z} \right) + \frac{\partial}{\partial z} \left( D_{zz} \frac{\partial c}{\partial z} + D_{xz} \frac{\partial c}{\partial x} \right) \\
\rho = \rho_0 (1 + \beta c)
\]

$\rho$: Density  \quad $P$: Pressure  \\
$u$: Horizontal velocity  \quad $w$: Vertical velocity  \\
$\beta$: Solute contraction coefficient  \\
$c$: Solute concentration  \\
$D$: Dispersion and diffusion coefficient
4-Results:

- In the no mixing model, the gravity current extends beyond its steady-state value when \( \ell \) (the drained fluid depth) is similar in magnitude to \( \bar{h} \) (the gravity current height). As the depth of drained fluid in the lower layer increases, the current gradually retracts.

- No mixing model gives reasonable predictions up to the point of retraction. Thereafter, more favorable agreement with numerical data is observed in the perfect mixing model.

- The severity of the retraction in the no mixing model is because of the inability to predict the instabilities that develop in the lower layer.

Here, \( \theta = 0^\circ \) and

\[
K_{\text{eff}} = K \left(1 + \frac{1}{\bar{\zeta_5}}\right) = 0.01
\]
4-Result: Effects of $K_{\text{eff}}$ on dispersion

We can quantify the amount of dispersion by defining buoyancy (per unit box width) $B$ in the bulk and dispersed phases as:

$$B_{\text{bulk}}^*: \text{buoyancy in the bulk phase} \quad \text{and} \quad B_{\text{disp}}^*: \text{buoyancy in the dispersed phase}$$

So, the fraction of buoyancy in the dispersed phase is

$$\bar{B}_{\text{disp}}^* = \frac{B_{\text{disp}}^*}{B_{\text{disp}}^* + B_{\text{bulk}}^*}$$

Buoyancy fraction increase with $K_{\text{eff}}$ (effective permeability).

- **Increase with $K_{\text{eff}} = \frac{k_b}{k} \left( 1 + \frac{1}{\xi^*} \right)$**: drainage is more robust for larger $K_{\text{eff}}$. Robust drainage retards the elongation of the bulk phase. The effect of drainage on the dispersed phase is mild. Therefore, the dispersed buoyancy fraction increases.
5- Field Application:

• Our assumptions in driving theoretical and numerical models:
  • Incompressible flow  
  • Constant pressure and temperature  
  • Dynamic viscosity, $\mu$, is independent of concentration.
  • Ignore diffusion

• In light of these assumptions, it is unclear the degree to which our model properly mimics a real underground hydrogen storage (UHS) flow. Let's investigate!

• Reservoir simulation software, CMG and Open-Go-Sim, are used to examine our model predictions in an actual UHS project. To do so, we consider a domain with size $1000 \times 500 \times 150$ m, permeability 100 mD and porosity 0.2. H$_2$ is injected with mass flow rate $2 \times 10^5$ Sm$^3$/s from a line source.

- The bottom part of domain is saturated with water. The gas-water contact is 10 m above the bottom boundary.

Cushion gas:
- CH$_4$ 93.9%
- N$_2$ 4.55%
- CO$_2$ 1.55%
5- Field Application:

- The perfect mixing theoretical solutions (red lines) are compared with Open-Go-Sim and CMG results after 6 months of H₂ injection.

- There is generally good agreement between the theoretical results and numerical simulations. Note that the numerical results overpredict dispersion due to numerical dispersion, though we try to reduce this effect by using fine grid and higher order solvers.

\[ \theta = 0^\circ \]
\[ t = 180 \text{ day} \]
\[ P_{\text{initial}} = 100 \text{ bar} \]
\[ P_{\text{Bhl}} = 150 \text{ bar} \]
5- Field Application:

- Numerical simulations also predict that the degree of dispersion increases with $K_{\text{eff}}$.
- Although CMG predicts dispersion a bit more than OpenGoSim, the predictions are almost similar.
- The highlighted parts of figures show times where the bulk phase either stops elongating or experiences a retraction.
5- Field Application:

- When the reservoir is confined, the layer depth is reduced to 50 m and the theoretical model does not work well.
- As highlighted by Bharath et al. (2020), the secondary gravity current in the upper layer pulls the main gravity current, which is not considered in the theoretical model. Also the accumulated fluid in the upper layer causes a overall decrease in the drainage velocity.
- The shear force between ambient fluid in the lower layer and the gravity current slows down its elongation.

\[ \theta = 0^\circ, t = 180 \text{ day} \]

The accumulation of the drained fluid in the upper layer has a larger influence than the shear force of the lower boundary. Therefore, the numerical gravity current is ahead of the theoretical prediction.
5- Field Application:

• The theoretical model is unable to predict the flow evolution in anticlines because the inclination angle varies with position in anticline reservoirs.

• OpenGoSim and CMG simulations allow us to explore phenomena beyond the scope of the theoretical model e.g. curved interbed layer in anticline reservoirs.

• Notwithstanding concerns on the theoretical model, it is necessary to reiterate that it can generate reasonably accurate estimates of hydrogen evolution in a small fraction of the time needed to run numerical simulations e.g. using CMG.
6-Conclusions:

➢ An analytical model is developed for a porous media gravity current consisting of a bulk phase and a dispersed phase.

➢ A complementary COMSOL numerical model is developed to specify the entrainment coefficient and to validate our theoretical models hydrodynamically.

➢ The outcomes of the theoretical model agree with underground hydrogen storage software simulation. Unless any of the assumptions such as unconfined domain in the theoretical model is violated.

➢ Therefore we present a simple hydrodynamic model that describes the evolution of hydrogen flow in a UHS facility reasonably well, without having to deal with thermodynamics, pressure, etc.


References:


6- IPCC, 2005: IPCC special report on carbon dioxide capture and storage.