A unified method for interpolation and de-noising of seismic records in the $f-k$ domain

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1 Introduction
   • Motivations
   • Overview

2 Theory
   • Identifying dominant dips in f-k domain
   • Building a mask function
   • Least-squares fitting

3 Examples
   • Synthetic data
   • Real data

4 Conclusions
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4 Conclusions
Utilizing information from full frequency band for de-noising or interpolation of any single frequency in the f-k domain.
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Interpolation methods

Signal processing based methods

- $f-x$ interpolation [Spitz, 1991]
- $f-k$ interpolation [Gulunay, 2003]
  - Using low frequencies to interpolate high frequencies.
- Multi-step autoregressive [Naghizadeh and Sacchi, 2007]
  - Combining minimum weighted norm interpolation (MWNI) [Liu and Sacchi, 2004] and $f-x$ interpolation.
- Sparse Fourier inversion [Zwartjes and Sacchi, 2007]
- Pyramid transform [Guitton and Claerbout, 2010]
- Fourier-Radial Adaptive Thresholding [Curry, 2009]
- Curvelet interpolation [Hennenfent and Herrmann, 2008], [Naghizadeh and Sacchi, 2010]
  - Using coarser scales of curvelets to interpolate finer and aliased scales of curvelets.
De-noising methods

Including but not limited to

- \textit{f-x} prediction filter [Canales, 1984]
- \textit{f-x} projection filter [Soubaras, 1994]
- Singular Value Decomposition [Trickett, 2003]
- Cadzow methods [Trickett and Burroughs, 2009] or Singular Spectrum Analysis [Oropeza and Sacchi, 2009]
- Empirical Mode Decomposition [Bekara and Van der Baan, 2009]
- \textit{f-k} velocity filtering
- \ldots
Principle of single frequency de-noising (I)
Principle of single frequency de-noising (II)

![Diagram showing t-x domain and f-k domain with marked frequencies.](image-url)
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$f$-$k$ spectra of linear events (I)
$f$-$k$ spectra of linear events (II)
$f$-$k$ spectra of linear events (III)
Angular summation in the \( f-k \) domain

Search for dominant energy dips

- \( d(t, x) \): Data in the \( t-x \) domain
- \( D(\omega, k) \): Data in the \( f-k \) domain
- \( 0 < \omega < 0.5 \): Normalized frequencies
- \( -0.5 < k < 0.5 \): Normalized wavenumbers
- \( p \): Slope of summation path in the \( f-k \) domain

\[
M(p) = \sum_{n=1}^{N_\omega} D(\omega_n, k = p.\omega_n - \left\lfloor \frac{p + 1}{2} \right\rfloor)
\]
Schematic representation of angular summation in the $f$-$k$ domain.
Thresholding for dominant energy dips

Identifying peak values

Locating peak values above a threshold value and marking them as dominant energy dips.
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A mask function for \( f-k \) domain

### From dominant dips to mask function

1. \( \rho_1, \rho_2, \ldots, \rho_L \) are the identified dominant dips.

2. Deploying rays of dominant dips in \( f-k \) domain. Initiating \( H \) matrix with zeros,

\[
H(\omega_n, k = \rho_j \omega_n - \left\lfloor \frac{\rho_j + 1}{2} \right\rfloor) = 1, \quad \{ n = 1, 2, \ldots, N_\omega, j = 1, 2, \ldots, L. \}
\]

3. Mask Widening to account for uncertainties. Convolving \( H \) with a 1D box car function, \( B(1, L_b) \),

\[
W(\omega, k) = H(\omega, k) \ast B,
\]
Mask function

Normalized wavenumber

Normalized frequency

f-k domain
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A stable and unique solution can be found by minimizing the following cost function [Tikhonov and Goncharsky, 1987]

\[ J = \| d - T F^H W D \|_2^2 + \mu^2 \| D \|_2^2. \]

- **d**: Data in \( t-x \) domain
- **D**: Data in \( f-k \) domain
- **T**: Sampling function
- **F**: Fourier transform
- **W**: Mask function
- **\( \mu \)**: Trade-off parameter
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Original data

![t-x domain](image)

- Distance (m)
  - 500, 1000, 1500, 2000

- Time (s)
  - 0.1, 0.2, 0.3, 0.4

- Normalized frequency
  - -0.4, -0.2, 0, 0.2, 0.4

- Normalized wavenumber
  - -0.4, -0.2, 0, 0.2, 0.4

![f-k domain](image)
Noisy data (SNR=1.0)

**t-x domain**

- Time (s): 0.1, 0.2, 0.3, 0.4
- Distance (m): 500, 1000, 1500, 2000

**f-k domain**

- Normalized frequency: -0.4, -0.2, 0, 0.2, 0.4
- Normalized wavenumber: -0.4, -0.2, 0, 0.2, 0.4
De-noised data using Canales $f$-$x$ method

Distance (m)

Time (s)

Normalized frequency

Normalized wavenumber

f-k domain

t-x domain
De-noised data using the proposed method

**t-x domain**

Distance (m)

Time (s)

**f-k domain**

Normalized wavenumber

Normalized frequency
The mask function used for de-noising
Irregularly sampled data
Interpolation of irregularly sampled data

![Graph showing t-x and f-k domains with distance and time axes.](image)
Data with gap

![Graph showing t-x and f-k domains with labeled axes and scales.](image)

- **t-x domain**
  - Time (s): 0.1, 0.2, 0.3, 0.4
  - Distance (m): 500, 1000, 1500, 2000

- **f-k domain**
  - Normalized wavenumber
  - Normalized frequency
Gap interpolation
Data which needs extrapolation
Dealiased data

**t-x domain**

- Time (s): 0.1, 0.2, 0.3, 0.4
- Distance (m): 500, 1000, 1500, 2000

**f-k domain**

- Normalized wavenumber: -0.4, -0.2, 0, 0.2, 0.4
- Normalized frequency: 0.1, 0.2, 0.3, 0.4
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Original data from the Gulf of Mexico
Interpolated data from the Gulf of Mexico

**t-x domain**

**f-k domain**
A time window of original data
A time window of interpolated data
Ground-roll elimination by proposed method

Noisy data

De-noised (proposed method)

De-noised (f-k filtering)
For linear seismic events information from any band of frequencies can be utilized to interpolate or de-noise any frequency.

The assumption of linear events needs to be fulfilled for the success of the proposed method. Therefore, proper spatial windowing is required for optimal performance.

The least-squares fitting of $f$-$k$ and $t$-$x$ domains prevents the appearances of artifacts akin to $f$-$x$ methods.

The thresholding criteria requires special care otherwise the algorithm can create artificial unrealistic events.

The Proposed method can be used for both random and coherent noise elimination.
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