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Fast Generalized Fourier Interpolation of Nonstationary Seismic Records

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SUMMARY

We propose a fast and efficient method for the interpolation of nonstationary seismic data. The method uses the fast generalized Fourier transform FGFT to identify the space-wavenumber evolution of nonstationary spatial signals at each temporal frequency. The nonredundant nature of FGFT renders a big computational advantage to this interpolation method. A least-squares fitting scheme is used next to retrieve the optimal FGFT coefficients representative of the ideal interpolated data. For randomly sampled data on a regular grid, we seek a sparse representation of FGFT coefficients to retrieve the missing samples. A synthetic seismic data example was used to examine the performance of the method.
Introduction

The problems of seismic data reconstruction and interpolation have attained a special stature in the seismic data processing community in recent years. One of the key issues in judging the performance of reconstruction methods and their management of complex events lies in their response to data nonstationarity. In seismic data processing, nonstationarity means the frequency/wavenumber content of the signal varies in time/space. For instance, an absorptive medium causes nonstationarity in the time dimension by modifying the frequency content of a seismic pulse as a function of path length. Also, seismic sections which contain hyperbolic and parabolic (or any nonlinear) events produce nonstationary spatial signals in the f-x domain at a given frequency. Interpolation/reconstruction methods typically cope with nonstationary signals through spatial windowing. Inside sufficiently small spatial windows nonlinear seismic events appear linear or stationary. Hence, methods which assume stationarity such as those referenced above may be applied. Naghizadeh and Sacchi (2009) have proposed an alternative method providing a beyond-alias interpolation of nonstationary seismic data. It is essentially a modification of the f-x interpolation of Spitz (1991). In their method the windowing of the spatial axis is avoided through use of adaptive prediction filters. Although spatial windowing of data and the adaptive f-x interpolation method are capable of handling nonstationarity, they lead to computationally demanding algorithms.

On another note, there exists a range of transformations specifically designed to deal with identification and analysis of nonstationary behavior in signals that might be used as a basis for interpolation. One of these transforms is S-transform (Stockwell et al., 1996) which is a type of short-time Fourier transform in which the window size is frequency dependent. Larger windows are used for lower frequencies and smaller windows are used for higher frequencies. Consequently, the spectrum at a given frequency is estimated with a number of samples appropriate to that frequency. Recent work on the S-transform has led to a fast, non-redundant algorithm (Brown et al., 2010), renewing the possibility of developing an efficient and effective interpolation/reconstruction approach based on S-transform theory. In this paper, we develop such an approach and examine its behavior when applied to synthetic and field data sets. First, because the transform algorithm is new, we will describe a straightforward and intuitive frequency domain computation of the fast generalized Fourier transform (FGFT). We will then combine FGFT with a least-squares fitting principle to formulate our FGFT Interpolation method. Synthetic and field data examples are examined.

Fast Generalized Fourier Transform

The S-transform of a time signal \( g(t) \) is defined as follows (Stockwell et al., 1996):

\[
S(\tau, f) = \int_{-\infty}^{\infty} \frac{|f|}{\sqrt{2\pi}} e^{-\frac{(\tau-t)^2}{2}} e^{-i2\pi ft} dt,
\]

where \( \tau \) and \( f \) are the time and frequency coordinates, respectively. The term \( \frac{|f|}{\sqrt{2\pi}} e^{-\frac{(\tau-t)^2}{2}} \) is a Gaussian window which depends on time lag and frequency. The width of the Gaussian window decreases with increasing frequency. This results in finer frequency resolution for low frequencies and finer time resolution for high frequencies. The frequency-dependence of the resolution of the S-transform suggests that computational efficiency can be increased by adopting sampling criteria varying from high to low frequency (Brown et al., 2010). This results in a nonredundant transform called fast generalized Fourier transform (FGFT) which has better time resolution in the high frequencies and better frequency resolution in the low frequencies. The details of computing FGFT can be found in Brown et al. (2010) and Naghizadeh and Innanen (2011).

Figure 1a shows a chirp function. Figure 1b shows the FGFT of Figure 1a. Figure 1c illustrates the 2D array of FGFT coefficients after proper up-scaling, i.e., the time-frequency decomposition of the chirp.
Because the original chirp input is real, we only include the positive frequencies for this example. The FGFT evidently captures the nonstationary nature of the chirp function. The low frequencies predominate at the beginning of the signal and high frequencies predominate at the end. Figure 1d illustrates the adjoint FGFT acting on the FGFT coefficients of the chirp function. The adjoint FGFT recovers the original data within a small error level produced by the windowing step.

**FGFT interpolation of seismic data**

The interpolation problem is under-determined and hence to solve it we require some prior information. To provide this, let us consider the FGFT coefficients $g$ of the desired, fully sampled signal. These must be related to $d$ by

$$g = G d,$$  \hspace{1cm} (2)

where $G$ represents the forward FGFT operator. The adjoint FGFT operator $G^T$ can furthermore be used to express the desired interpolated data $d$ in terms of $g$ as follows

$$d \approx G^T W g,$$  \hspace{1cm} (3)

where we have introduced a diagonal weight function $W$ that preserves a subset of FGFT coefficients. The relationship between the observed data $d_{\text{obs}}$ (before interpolation) and FGFT coefficients is given by

$$d_{\text{obs}} \approx TG^T W g,$$  \hspace{1cm} (4)

where $T$ is the sampling function. Let us assume for the moment that the operator $W$ is known. The system of equations in equation 4 is under-determined (Menke, 1989) and therefore, it admits an infinite
number of solutions. A stable and unique solution can be found by minimizing the following cost function (Tikhonov and Goncharsky, 1987)

$$J = ||d_{obs} - TG^TWg||_2^2 + \mu^2||g||_2^2,$$

where $\mu$ is the trade-off parameter. We minimize the cost function $J$ using the method of conjugate gradients (Hestenes and Stiefel, 1952). The conjugate gradients method does not require the explicit knowledge of $G$ in matrix form. It requires the action of operators $G^T$ and $G$ on a vector in the coefficient and data spaces, respectively (Claerbout, 1992). The goal of the proposed algorithm is to find the coefficients $\hat{g}$ that minimize $J$, and use them to reconstruct the data via the adjoint FGFT operator $\hat{d} = G^T\hat{g}$.

For a randomly sampled signal one can impose a sparsity constraint to the FGFT coefficients. This corresponds to a weight function which diminishes small amplitude coefficients and preserves large ones (Sacchi et al., 1998). The large FGFT coefficients correspond with coherent features in a data set. By suppressing the small FGFT coefficients, the coherent data features will be reintroduced across regions where data was initially missing. We adapt an Iteratively Re-weighted Least Squares (Scales and Gersztenkorn, 1988) approach to achieve this purpose. The algorithm is summarized as follow

Initialization $W^0 = I$
For $k = 1, 2, 3 \ldots$

$$\hat{g}^k = \text{argmin}_g \{||d_{obs} - TG^TW^{k-1}g||_2^2 + \mu^2||g||_2^2\}$$

$$W^k = \text{diag}(\text{abs}(\hat{g}^k))$$
End

where diag(abs(.)) builds a diagonal matrix from the absolute values of a vector. The weight function was initiated with an identity matrix and was updated by the absolute value of FGFT coefficients after each least-squares fitting. The updating step of IRLS method serves to amplify the high amplitude FGFT coefficients and eliminate the low amplitude ones.

In order to interpolate seismic records, one needs to transform the data to the $f$-$x$ domain. Then, for irregularly sampled seismic records the FGFT interpolation is applied for each frequency independently using equation 6. However, for interpolation of regularly sampled seismic records it is necessary to deploy Spitz (1991) trick to utilized the FGFT of low frequencies for the interpolation of high frequencies.

**Seismic data example**

Figure 2a shows an original synthetic seismic section with three hyperbolic events composed of 202 traces. Figure 2b shows the section of missing data after randomly eliminating 50% of the traces. Figure 2c shows the FGFT reconstructed data after applying Equation 6 for each single frequency. Notice that the non-redundant nature of FGFT interpolation create small amplitude artifacts in the interpolated data. This means there is a trade-off between the resolution and speed of the interpolation method. Figures 2d, 2e, and 2f show the $f$-$k$ spectra of Figures 2a, 2b, and 2c, respectively.

**Conclusions**

The FGFT is a fast and efficient way of analyzing nonstationary signals and identifying their time-frequency evolution. We used FGFT inside a least-squares fitting algorithm to interpolate nonstationary seismic data. The method has the ability to cope with rapid and local changes of dip information in the seismic data. For randomly sampled data on a regular grid, we seek a sparse representation in the FGFT domain to recover the missing samples. The proposed method is very fast and less demanding on computational memory compared to the alternative methods. The extension of FGFT interpolation to
Figure 2 a) Original synthetic seismic section with 3 hyperbolic events. b) The seismic section with 50% randomly missing traces. c) Reconstructed data using the FGFT interpolation method. d)-f) are the f-k representations of (a)-(c).

multidimensional applications is a straightforward task because in principle, it involves simple multidimensional Fourier transforms.

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References


Tikhonov, A.N. and Goncharsky, A.V. [1987] Ill-posed problems in the natural sciences. MIR Publisher.