V041

**fx Gabor Seismic Data Reconstruction**

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**SUMMARY**

We introduce an fx Gabor reconstruction algorithm that can regularize seismic data in the presence of strong variations of dip. The available data in the fx domain are modeled via a Gabor discrete expansion. The coefficients of the Gabor expansion are estimated by an inversion scheme that uses a sparsity constraint. Synthetic and real data examples are used to test the algorithm.
Introduction

Seismic data regularization is an active area of research. Several methods have been proposed to interpolate and regularize seismic data. The regularization problem is often posed in the f-x (frequency-space) domain and solved by implementing spatial interpolation at each monochromatic frequency \( f \). Methods in this category are f-x prediction filter interpolation (Spitz, 1991), Band-limited Fourier Interpolation (Duijndam et al., 1999), Minimum Weighted Norm Interpolation (Liu and Sacchi, 2004; Trad et al., 2005), and Multi-Step Autoregressive Reconstruction (Naghizadeh and Sacchi, 2007). In general, f-x methods are optimal for modeling multi-dimensional plane waves. In fact, f-x prediction filter and Fourier-based interpolations provide accurate results when they are applied to spatial signals that can be modeled by a few dominant wavenumbers. It is often the case that spatial windowing is required to validate the aforementioned signal model. An alternative procedure is to use f-x adaptive filters to cope with spatially variable dips (Naghizadeh and Sacchi, 2009). Unfortunately, adaptive f-x filtering is implemented via a recursive least-squares algorithm that requires evenly sampled input data.

The Gabor expansion of a discrete signal (Bastiaans, and Geilen, 1996; Li and Ulrych 1996), on the other hand, can be used to model signals with a time-varying frequency structure or a space-varying wavenumber structure. In essence, a discrete signal is represented via a superposition of shifted and modulated versions of an elementary Gaussian signal. We propose to retrieve the coefficients of the Gabor expansion from the available spatial data and used them to synthesize data at missing spatial locations. The procedure described in this article is similar to the Fourier-based sparse inversion reconstruction proposed by Sacchi and Ulrych (1996), Sacchi et al. (1998) and Zwartjes and Gisolf (2007). In our algorithm, however, the discrete Fourier basis is replaced by a discrete Gabor basis.

Theory

We start our analysis by considering data in the f-x (frequency-space) domain. The discrete spatial signal for one monochromatic frequency component \( f \) is denoted by \( d(n, f) \), where \( n \) indicates the spatial index for a trace at position \( x = (n - 1)\Delta x \). To avoid notational clutter we will represent the data via \( d(n) \), \( n = 0 \cdots N - 1 \) and understand that the analysis is carried out in space for one monochromatic frequency component \( f \). The discrete Gabor expansion of \( d(n) \) is given by the following expression (Bastiaans and Geilen, 1996)

\[
d(n) = \sum_m \sum_k a_{mk} g(n - mL)e^{i2\pi kn/K}.
\]

(1)

The Gabor coefficient \( a_{mk} \) corresponds to the amplitude of the elementary signal at spatial location \( mL \) and wavenumber \( k_{rad} = 2\pi k/K, k = 0 \cdots K - 1 \). The kernel \( g(n) \) denotes the Gabor synthesis window. We express equation (1) in matrix form and, in addition, we include a noise term to tolerate realistic data scenarios

\[
\tilde{d} = G\tilde{a} + \tilde{n},
\]

(2)

where \( \tilde{d} \) is the \( N \times 1 \) vector of spatial data and \( \tilde{a} \) is the \( N_a \times 1 \) vector containing the Gabor coefficients. We chose \( K \) and \( L \) such that \( N_a >> N \). We now have an underdetermined problem that admits an infinite number of solutions. Among all possible solutions, we will
chose a solution that is sparse (Sacchi and Ulrych, 1996; Olshausen and Field, 1996; Sacchi et. al, 1998; Hennenfent and Herrmann, 2006). This is achieved by minimizing the following cost function

\[ J = \| T(\tilde{d} - G \tilde{a}) \|_2^2 + \mu^2 \sum_{mk} \ln(1 + \frac{a^*_{mk} a_{mk}}{\beta^2}). \]  

(3)

The first term in the right hand side of (3), the misfit function, measures how well the Gabor expansion describes the signal. The second term, the Cauchy norm, measures the sparseness of the solution. The regularization parameter \( \mu \) determines the importance of the regularization term relative to the data misfit. The parameter \( \beta \) is the Cauchy scaling constant. We have also included a sampling operator \( T \) to deemphasize the influence of missing observations (Liu and Sacchi, 2004). A preconditioned conjugate gradients algorithm minimizes the cost function given by equation (3). Rough estimates of the trade-off parameters (\( \mu, \beta \)) can be obtained by training the algorithm with synthetic data sets that have undergone decimation. The idea is to find parameters that favour solutions with the fewest number of non-zero Gabor coefficients. The estimated Gabor coefficients are used to model data at all spatial locations via equation (1).

Finally, we stress that the matrix \( G \) does not have an explicit matrix representation. The preconditioned conjugate gradients method builds the solution via a sequence of steps where the operator \( G \) and its adjoint \( G^* \) are implicitly applied to vectors.

**Examples**

Figure 1 portrays the application of the Gabor reconstruction algorithm to a 1D problem. In this case, the data represent a temporal hyperbolic chirp with missing observations. This is a clear example of a signal that cannot be modelled with a sparse distribution of complex exponentials. Figure 1a shows the regularly sampled signal. Figure 1b portrays the available data used to estimate the Gabor coefficients. Figure 1c is the reconstruction of the chirp using the Gabor transform. The error signal is provided in Figure 1d. In addition, Figure 1e and 1f show the amplitude of the Gabor coefficients in the frequency-time space for the classical Gabor transform and for the coefficients inverted with the procedure presented in this paper, respectively. We stress that both time-frequency panels were computed from the data with missing observations (Figure 1b).

We also applied the Gabor reconstruction algorithm in the \( f-x \) domain. The test was carried out with a marine data set from the Gulf of Mexico. The data prior to decimation is portrayed in Figure 2a. The decimated data after removing 60\% of the traces is portrayed in Figure 2b. Figure 2c shows the reconstructed data. Finally, Figure 2d shows the reconstruction error. The \( f-x \) Gabor reconstruction was capable of recovering the missing data. Strong variations of structural dip have not impinged on the performance of the method.

**Conclusions**

We have presented an \( f-x \) Gabor reconstruction algorithm. The method uses the Gabor expansion of the spatial data to reconstruct missing observations. The Gabor coefficients are retrieved via the solution of an inverse problem. A unique and stable solution is found by including a sparsity constraint in the form of a Cauchy prior. Our results demonstrate that the sparse Gabor expansion in the \( f-x \) domain provides an efficient signal representation for seismic data reconstruction.
Figure 1: a) Hyperbolic chirp. b) Hyperbolic chirp with missing samples. c) Gabor reconstruction using the sparse inversion algorithm described in the text. d) Residual signal (original signal minus reconstruction). e) Classical Gabor spectrum of the chirp in b). f) Gabor spectrum obtained via sparse inversion.

Figure 2: a) Common offset section from a marine survey. b) Decimated section. c) f-x Gabor reconstruction. d) Error section (original data minus reconstruction).
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References


