The gravitational pull of a black hole is so strong that not even light can escape, but this does not mean that we cannot ask what is happening in its interior

Black holes: the inside story

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In 1930 the late Subrahmanyan Chandrasekhar showed that the pressure due to the quantum degeneracy of electrons could not support a cold star with a mass of more than about 1.4 times that of the Sun. Is was the first indication that not all stars end their lives peacefully as white dwarfs - objects that slowly contract under their own gravity to the size of the Earth and cool to invisibility. Astrophysicists are now almost certain that burnt-out stars more than a few times the mass of the sun collapse to form black holes - objects whose gravitational pull is so strong that not even light can escape from them.

What happens as the star collapses? As the external gravitational field settles in the wake of the collapse like a quivering soap bubble, every irregularity and distinguish- ing feature is swallowed up or radiated away. Observers outside the hole see the gravitational field asymptotically approaching a standard configuration known as the Kerr-Newman field and characterized by just three numbers: mass, angular momentum and charge. In the words of John Wheeler: "a black hole has no hair". Chandrasekhar has called them "the most perfect macroscopic objects in the universe".

1 Black hole basics

What lurks behind the smooth, hairless facade of the "event horizon" (the hole's surface)? This is a question that it might be prudent not even to raise. With black holes only just recently admitted into the fold, questions of this sort beg to be labeled as airy-fairy fantasy, and anyone professing to take them seriously as a simpleton, charlatan or crackpot. Whatever may happen inside the hole, we will never observe it, nor will it affect anything we can observe.

What is more, in 1965 Roger Penrose, then at Birkbeck College in London, proved that there must be a singularity within the hole, signalling a breakdown of all the laws of physics. Moreover, various irregularities swept inward with the collapse must be expected to grow and accumulate, creating inner chaos. On the face of it, this hardly seems a fit topic for serious scientists of sound views.

In fact, the prospects are less bleak than we have painted them because of a key property of the hole's interior that we have not yet mentioned. Descent into a black hole is fundamentally a progression in time. Inside the hole, every form of matter and every causal influence is condemned to fall inwards (i.e. towards smaller radii). The future is inseparably linked with diminishing radius, so the radial coordinate, r, becomes time. This means that we are confronting an evolutionary, not a structural, problem. Moreover, we don't even need a quantum theory of gravity until space-time is bent on scales of the Planck length ($\approx 10^{33}$). This is because simple causality prevents our ignorance of the inner, high-curvature regions from affecting the description of the outer and preceding layers afforded by today's standard, well established theories.

Finally, thanks to the no-hair property and work by Richard Price, initial conditions for the evolution are known with precision. This should be contrasted with the situation in cosmology, where the initial state of the universe is a matter of pure speculation. In 1972 Price, then at the California Institute of Technology, showed that initially - that is, near the event horizon - the gravitational field has a Kerr-Newman form, perturbed by a tail of gravitational waves decaying as v^p, where v is "advanced time" and $p > 12$. (Advanced time is a time coordinate that stays fixed for photons propagating radially inwards: in flat space, $u = t + r/c$, where c is the speed of light). Because of this rapid decay, the tail has no significant external effects; but internally, as we shall see, its effect is critical.

Exploration of the outer layers of a black hole thus emerges as a standard type of initial-value problem, and so it is possible to make progress. The task is to integrate the partial differential equations of Einstein's theory forward in time (and inwards into the hole), beginning from Price's initial conditions

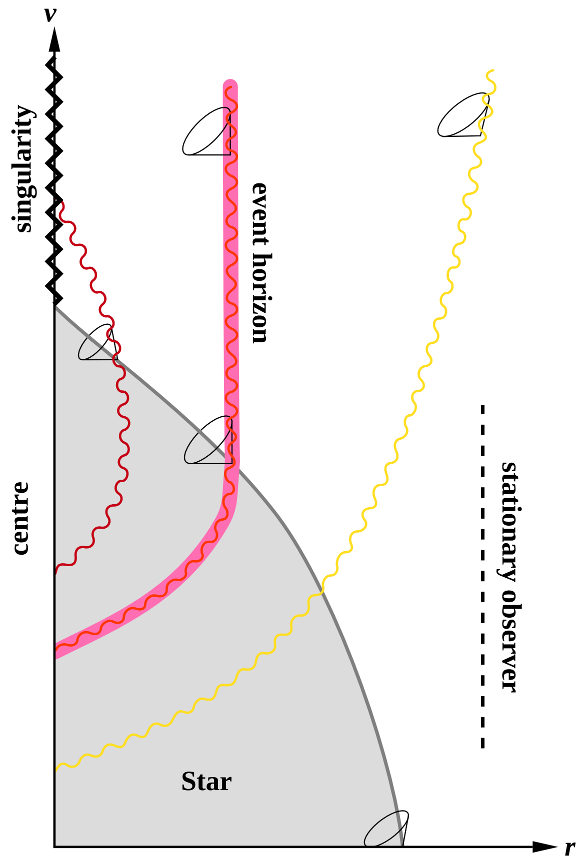

Figure 1: The space-time history of the collapse of an uncharged spherical star. The horizontal axis is radius and the vertical axis is advanced time, v. In this space-time a photon moving radially inwards would be represented by a horizontal line (not shown). The wavy lines are photons traveling radially outwards. As the star collapses, its gravitational pull increases, and eventually the photons are unable to escape. Photons emitted at the critical moment, E, marginally fail to escape. They are hold captive by gravity and mark the "event horizon" or boundary of the hole. Matter and radiation inside the hole must propagate within the future light-cones shown (since they cannot move faster than light), and are condemned to fall towards the central singularity at $r = O$.

at the event horizon. We have stressed this because it is important to scotch the notion that what is being attempted is metaphysical or hopelessly speculative. It is, in fact, no more problematical than following the motion of a fluid, using Euler's hydrodynamical equations, up to the onset of turbulence or a shock.

2 No spin or charge

The simplest black holes are those formed when a star with zero charge and zero angular momentum collapses. This does not exclude the possibility that initially the star was ellipsoidal (or cubic). However, to begin with, let us Iook at the collapse of an exactly spherical star. Figure 1 depicts the space-time history: the star contracts to radius $r = O$, where a singularity is created that extends indefinitely into the future (in the sense of increasing advanced time v).

The event horizon formed in the collapse can be pictured as a sphere of imaginary photons, initially expanding but then caught and held stationary (by the growing surface gravity of the star) at the Schwarzschild radius, $^{2Gm}/_{c^2}$, where m is the mass of the hole. For a black hole of six solar masses the Schwarzschild radius is less than 20 km. On a grander scale, the two billion solar- mass black hole in the nucleus of the giant galaxy M87 - whose presence was recently con firmed by spectrograms taken from the Hubble Space Telescope - would swallow up the orbit of Pluto if relocated in the solar system.

In the space-time of figure 1 the outgoing photons follow bent paths, while the track of a stationary

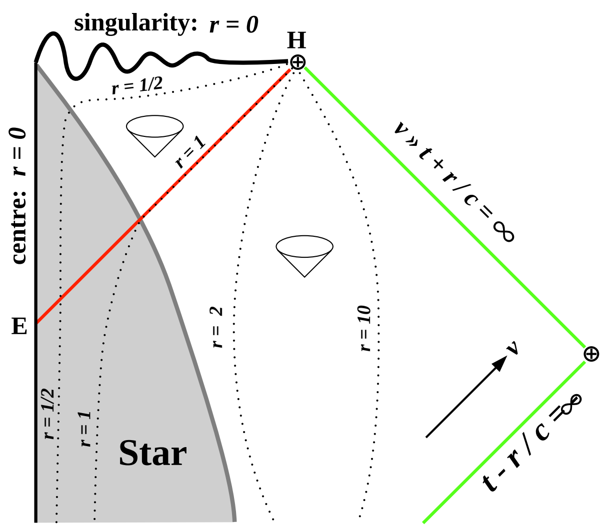

Figure 2: The same collapse as 1 in a Penrose map. Tracks of radially moving photons are inclined at $\pm 45°$ (not shown), and infinity has been artificially "compactified" by a coordinate transformation as explained in the text. The Penrose map grossly distorts distances at the "points" marked by circled crosses (where infinitely many points are crammed together), and along the "lines" joining these points. Curves of constant radial coordinate r (in units of the Schwarzschild radius) are shown by dotted lines; outside the event horizon these lines represent tracks of stationary observers. The light cones show that, inside the event horizon, all matter and radiation is condemned to move towards smaller r; outside the event horizon the light cones show that it is still possible to escape to infinity.

observer (i.e. constant r) appears as a vertical straight line. In the curved space of general relativity there are, of course, no genuinely straight lines; "straightness" is merely a convention of the mapping. In figure 2 we show the same spherical collapse in a "Penrose map" in which outgoing and ingoing photon tracks have been "straightened" into lines running at $\pm 45°$ to the horizontal (and the stationary-observer tracks are bent). For instance, the event horizon is now represented by a straight line. At the same time a coordinate transformation of the form $u = \tan v$' has been applied. This preserves the directions of photon tracks ($u =$constant), but compresses the $u = \infty$ part of the space-time onto a finite line $u' = \pi/2$ on the map.

The Penrose map brings out the causal structure of space-time. Tracks of material particles and causal signals moving radially at speeds less than that of light - are always confined between the north-west and north-east directions on the map.

If the star is not spherical, then an unproven but plausible conjecture known as "osmic censorship" posits that an event horizon should still form. After some initial agitation, the gravitational field and geometry should settle to a static, spherical form as v goes to infinity. Indeed, in 1980 Igor Novikov and Andrei Doroshkevich, then at the Applied Mathematics Institute in Moscow, argued that this happens both outside and inside the event horizon. Thus the end-state of any non-spinning uncharged collapse should be a spherical, Schwarzschild black hole, perturbed by a dying tail of gravitational waves.

Before admitting spin and charge to this picture, we should point out in passing that stars and holes bearing electric charge are not of direct astrophysical interest - any excess charge would be rapidly neutralized by accretion from the interstellar plasma. However, charged black holes have a role as models for the realistic black holes formed in a spinning collapse. They share many of the same features, yet are simpler to deal with because they can be assumed to be spherical.

3 Collapse with charge and spin

The event horizon is essentially the last outpost from which a doomed astronaut falling into the hole can still flash news to the outside. In the collapses discussed so far, the astronaut can still receive signals - for example, she will still be able to see the stars in the night sky - all the way to the singularity at $r = O$. However, once the smallest amount of charge or spin is introduced, a new feature - a second, inner horizon - appears, which seems to alter this picture radically. This horizon marks the last moment the astronaut can still receive news (figure 3).

The final state of a spherical, charged collapse is known as a "Reissner-Nordström" black hole. The contracting star passes in turn through the outer (event) and inner horizons. At this point, gravity at its surface actually becomes repulsive because of the large amount of electric field energy that is now located outside the shrunken star and no longer contributing to its effective internal mass. At event B the star rebounces at a non-zero radius and reexpands into a new infinite universe. It will continue to pulsate, each time entering a new universe.

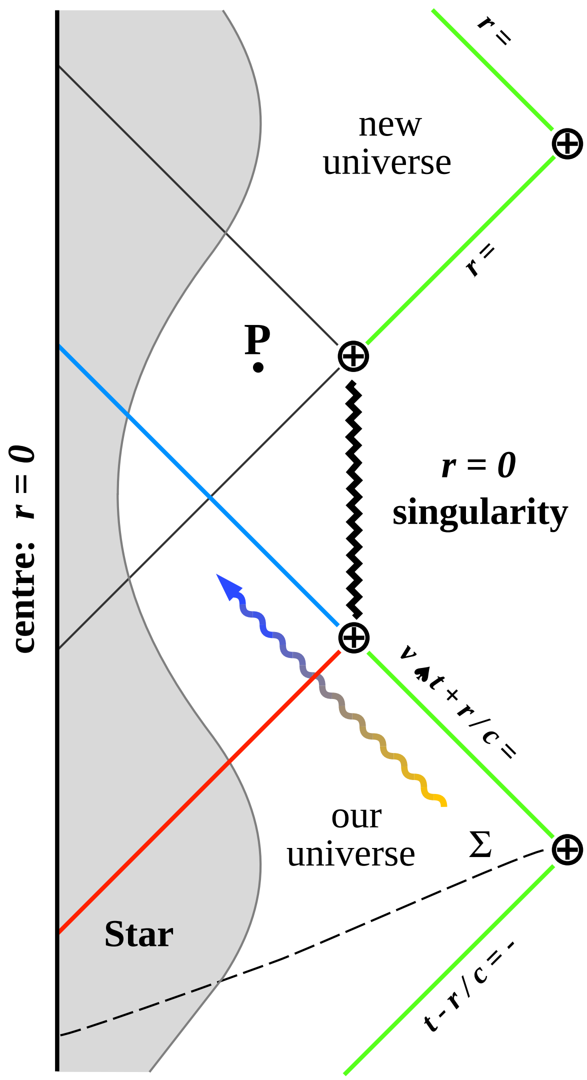

Figure 3: Penrose map showing the collapse of a spherical star with charge. The star contracts within the even horizon, then rebounds at B and expands into a new universe. An astronaut falling into the hole descends from the event horizon to the Inner horizon in a time which is finite by her wristwatch - from a fraction of a second for a solar-mass hole up to eight hours for the black hole in M87. During this descent she is still able to receive light signals from the outside, no matter how late these are sent. Thus she is able to view the entire future of the outer universe before her own demise when she hits the enormous densities and forces along the inner horizon.

The prospect of travel to other universes via a black hole sounds like science fiction, and before we get carried away we should check this picture for stability. If cosmic censorship holds, inclusion of spin and other non-sphericities should not prevent an event horizon from forming and the external field from converging asymptotically to a stationary condition. These stationary states are uniquely the three-parameter Kerr-Newman geometries, and they have a lattice-like structure entirely similar to figure 3.

However, there is a snag, first noted by Penrose in 1968. Unlike the simplest forms of collapse (shown in figures 1 and 2), conver- gence is now non-uniform and breaks down completely along the inner horizon. What is the reason for this instability? Remember that the inner horizon marks the last moment at which our astronaut can still receive news; but she then gets all of the news. In the few moments remaining before she plunges through this surface and into the core of the hole, the entire history of the outer universe will be flashed in fast motion before her eyes, This compression, or blue shift, has drastic consequences. The gentle waving of a lady's fan far outside the hole, continued for long enough, would generate gravitational waves that would hit the poor astronaut like a pneumatic drill.

In practice, such time-dependent disturbances are unavoidable. The gravitational-wave tail left by a non- spherical collapse is partially absorbed by the hole. Moreover, the natural power-law decay of the gravitational radiation predicted by Price is completely swamped near the inner horizon by the blue shift of the radiation, which grows as $exp(2\kappa_0 v)$, where κ_0 is the "surface gravity" of the inner horizon.

If our unfortunate astronaut was lucky enough to survive the passage through this "curtain" of blue-shifted radiation along the inner horizon, figure 3 suggests that she might then pass into to an open, spacious region marked by a breakdown of predictability. The value of a physical field at an event P in this region would no longer be uniquely determined by the initial conditions at the onset of collapse (the surface Σ in figure 3), because of unpredictable influences originating from the singularity at $r = O$ where the field equations break down. This horizon is known as the "Cauchy horizon" after the Cauchy initial value problem in mathematical physics. However, there is no reason to assume that the core of a real black hole will be anything like the Kerr-Newman or Reissner-Nordström lattices pictured in figure 3.

This picture of an unstable Cauchy horizon, which becomes singular under exposure to radiative tails, was established through numerical integrations by Penrose and Michael Simpson at Birkbeck in 1973, and by several analytical studies in the following decade. However, this earlier work assumed a fixed (Reissner-Nordström) background, and did not attempt to estimate the effect of the blue-shifted radiation on the gravitational field.

The first attempts to take this back-reaction into account were made by William Hiscock of the University of Montana in 1981. More representative models were examined by Eric Poisson and one of the authors (WI) at the University of Alberta in 1999, and by Amos Ori, then at CalTech, in 1991. To simplify the mathematics for a first reconnaissance, these early models made several approximations. It was assumed that rotating (non-spherical) black holes could be adequately modeled by spherical charged holes, since their horizon structures are similar. Gravitational (quadrupole) wave tails were modeled by spherical scalar waves. More general studies by Ori, by Patrick Brady, Chris Chambers and John Smith at the University of Newcastle-upon-Tyne in the UK, and by our group at the University of Alberta, confirm that these simplified models capture most of the essential features of the generic case.

The most dramatic of these is an effect that has been dubbed "mass inflation". The blue-shifted influx due to the gravitational wave tail causes the local mass function to blow up exponentially near the Cauchy horizon (figure 4). The mass function $m(u, v)$, is the mass inside a given radius. More explicitly, $m(u, v) \approx Cv^p e^{\kappa_0(u+v)}$, Where C is a constant and u is an internal retarded time that increases from $-\infty$ as we move inwards from the event horizon. (Localized mass is not well defined in general relativity and it would perhaps be better to speak of the blow-up of local conformal curvature $\Psi \approx {}^m\!/_{r^3}$.)

We hasten to reassure the reader that no trace of this inflation can be detected externally. News of the drastic change of internal field travels at the speed of light (as a gravitational wave) and cannot escape from the hole. So outside observers register a mass no larger than that of the original star.

The singularity along the Cauchy horizon produced by mass inflation appears quite gross but is actually spread over the surface of the Cauchy horizon. Although a free-falling object experiences infinite tidal forces at the Cauchy horizon, these forces do not grow fast enough to deform the object significantly before it reaches the horizon.

4 Mechanics of mass inflation

To demystify these phenomena, a simple mechanical example may help. Consider a concentric pair of thin spherical shells, of masses $m_{\rm con}$ and $m_{\rm exp}$, contracting and expanding at the speed of light, and remember that in Einstein's nonlinear theory, gravitational energy also exerts a gravitational attraction on matter. The mutual potential energy of the shells acts as a debit (a binding energy) of order $-G^{m_{\rm exp}m_{\rm con}}/r$ on the gravitational mass-energy of whichever is the outer shell. When the shells cross, say at radius r_0, this debit is transferred from the contracting shell to the expanding one, and their masses change. The exact calculation shows that the new masses, $m'_{\rm con}$ and $m'_{\rm exp}$ are

$$
\begin{aligned}
m'_{\rm con} &= m_{\rm con}\left(1 - 2\frac{2Gm_{\rm exp}}{c^2 r_0}\right)^{-1} \\
m'_{\rm exp} &= m_{\rm exp}\left(1 - 2\frac{2Gm'_{\rm con}}{c^2 r_0}\right)
\end{aligned}
$$

The total mass and energy are, of course, conserved. If the encounter takes place just outside a horizon, the infalling mass can become arbitrarily large. The total energy (kinetic plus potential) $m'_{\rm exp}c^2$ is then negative, which means that the expanding shell is now trapped inside a black hole. This simple model already provides a fair schematic picture of what happens near the Cauchy horizon, with the two shells representing streams of infalling and back-scattered radiation.

Mass inflation is not just a catch-phrase for something of merely formal significance. If the infalling light were absorbed by a lump of charcoal at the centre, it would contribute the full value of its inflated mass-energy to the lump.

5 Inside knowledge

Our message is that the interior of a generic black hole is not completely inscrutable: the investigations we have described lead to a working picture of at least the outer, classical layers of the hole.

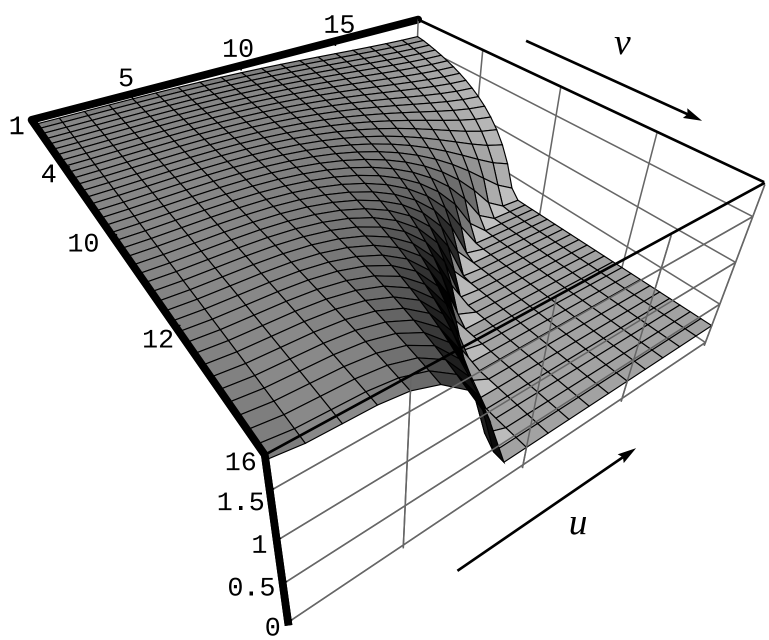

Figure 4: Mass inflation near the Cauchy horizon. This shows how $exp(^{-2Gm}/_{c^2 r})$, plotted vertically, goes sharply to zero (and hence $m \to \infty$ at the Cauchy horizon. From numerical integrations by Brady and Smith at Newcastle University.

The generic picture is not so very different from the simplest pictures we began with. The one striking new feature is the appearance of a locally mild, precursory singularity to the strong central singularity. It sits on the Cauchy horizon, which is a three-dimensional cylinder inside the hole. It extends indefinitely into the past and contracts towards the future, finally tapering to the crushing singularity at its future endpoints, represented by C in figure 5. In effect the Cauchy horizon serves as a bridge, linking the quiescent, asymptotically hairless, final phases of the hole (at late advanced time) to the "hairy" crunch near C that is associated with its formation.

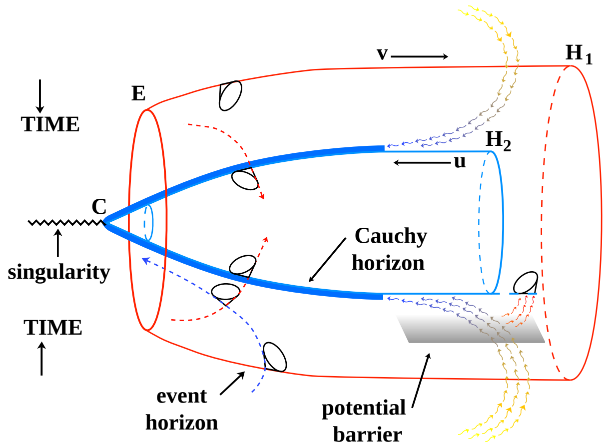

Figure 5: The inside of a black hole with one angular variable suppressed. Inside the hole, increasing time coincides with decreasing radius, r. Radiation flowing into the hole is partially scattered by a ridge of curvature midway between the two horizons. The unscattered portion gets strongly blue-shifted and accumulates along the Cauchy horizon. The scattered radiation initiates contraction of the Cauchy horizon, thereby catalysing the inflationary conversion of potential energy into material energy.

Objects tossed into the hole at progressively later advanced times (v) fall towards the Cauchy horizon at progressively earlier internal retarded times (u). What happens to these objects when they reach the Cauchy horizon?

We can only speculate on the answer to this question. Our classical models break down just before the Cauchy horizon because the curvature of space-time begins to approach the Planck scale and we need a quantum theory of gravity. So far we have only charted a coastline: we do not yet know whether it heralds a narrow strip of land or a continent. The most conservative answer would be that the question is not well posed because, for a real black hole, the Cauchy horizon never "happens". It is always forestalled - either by evaporation of the hole or by its merging with other black holes in a cosmological big crunch.

Therefore, the fate of the objects that are suddenly catapulted from near the Cauchy horizon into the remote future is bound up with the fate of the universe as a whole.

6 Further reading

- A Bonnano, S Droz, W Israel and S M Morsink 1995 Structure of the spherical black hole interior *Proc, Roy. Soc. A* **450** 553

- P R Brady and J D Smith 1995 Black hole singularities: a numrical approach *Phys. Rev. Lett.* **75** 1256

- R d'Inverno 1992 *Introducing Einstein's Relativity* (Oxford University Press)

- E Poisson and W Israel 1990 Internal structure of black holes *Phys. Rev.* **D41** 1796.