Solving Systems of Simultaneous Equations Using MATLAB

MATLAB is capable of solving sets of equations in a number of ways. This lab will explore a few of them for a small linear system.

Exercises

The electrical circuit shown in Figure 1 has two voltage sources connected to a resistor network.

The governing equations for this system can be shown to be:

\[(R_1 + R_2)i_1 - R_2i_2 = V_1\]

\[-R_2i_1 + (R_2 + R_3 + R_4)i_2 - R_4i_3 = 0\]

\[-R_4i_2 + (R_4 + R_5)i_3 = -V_2\]

1) Write this set of equations in the form \[A\]{x} = {c}.

2) Usually when we create a model we check it with conditions for which we already know the answer to make sure that the model is correct. Using the values \(R_1 = R_2 = R_3 = R_4 = R_5 = 1\) ohm for the parameters in \[A\] and \(V_1 = V_2 = 5\) volts for the parameters in \{c\}, calculate the currents \(i_1, i_2,\) and \(i_3\) in five ways:

a) By hand calculating the inverse using determinants (Cramer’s Rule) and then solving \{x\} = \([A]\)^{-1}\{c\};

b) Using Gauss-Siedel iteration (by hand, by spreadsheet, or preferably with a Matlab m-file), starting with this first guess at a trial solution:
\[ x = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \]

and of course not forgetting to check the row-sum criterion before you start;

c) Using the `solve` command for a set of symbolic expressions for the equations;

d) Using the `inv` command to find the inverse of \([A]\) and then solve for \(\{x\} = [A]^{-1}\{c\}\);

e) Using right division to find the solution to \(\{x\} = [A]^{-1}\{c\}\) using the syntax

\[ x = A \backslash c \]

which calculates the elements of the vector \(x\) using Gaussian elimination with partial pivoting.

3) Write an m-file program in MATLAB that asks the user to enter the circuit parameters, solves the system of equations, and prints the resulting currents at the command window. You can use whichever solution method you prefer. (You might want to use a variable name such as \(x\) rather than \(i\), because the variable name \(i\) is usually reserved in MATLAB for complex numbers.)

4) Test your program using your known solution.

5) Try some different values for the system parameters. Check the solution by multiplying the matrix \(A\) by the solution vector, subtract the vector of forcing constants, and see whether the residual is acceptable.

References: