Musical transformations of time

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I.  Background: Philosophical investigations

Temporality and referentiality are two qualities that strongly shape the subjective perception and experience of objects possessing them. Unlike spatiality, temporality implies a directed flow. When it characterizes the object of perception, this flow sharply conditions experience in the temporal domain. Willy-nilly, the human being is carried along by the flow of time as presented by the object of perception. The quality of referentiality means that an object points away from itself, and therefore directs experience away from perception, from the percept (as signifier) to the referent (as signified). On the other hand abstraction, or non-referentiality, means that an object points nowhere (or to itself only). Taken together, the combination of the qualities of temporality and abstraction in an object is powerful because upon fixedly perceiving such an object experience is then wholly absorbed by the temporal flows suggested by the object.

As a creative art form combining temporality and abstraction, music (via its objects, or performances) is therefore uniquely suited to manipulate the subjective experience of time, to creative alternative temporal realities. In this paper I speculatively elaborate upon this idea, starting with a philosophical consideration of music in relation to the other arts, and then moving to a more practical exploration of how music transforms ordinary time, and what some of the effects or functions of those temporal transformations might be. The paper is not empirical, based neither upon controlled laboratory data (as generated by music cognition), nor ethnographic data (as is typically generated by ethnomusicologists). Rather discussion remains at a theoretical and speculative level. However many concrete research directions are indicated, and I will point these out whenever possible.

* * *

By manipulating perception, the arts create alternative realities, juxtaposed with the more ordinary reality of everyday social space-time and communication. Music maps time to abstract sound structures. Essentially, music is non-referential, and temporal. These facts place music opposite to the pictorial arts, but also distinguish it from temporal sonic arts (e.g. drama, verbalized poetry) by the abstractness of its structures, and from atemporal abstract visual arts (e.g. abstract painting) by its rootedness in time.

<table>
<thead>
<tr>
<th>Temporal</th>
<th>Referential</th>
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<tr>
<td>Y</td>
<td>N</td>
</tr>
<tr>
<td>Y</td>
<td>Drama, cinema, literature (ritual)</td>
</tr>
<tr>
<td>N</td>
<td>Pictorial visual art</td>
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Figure 1. The status of various art forms along the twin dimensions of temporality and referentiality.

This diagram shows the unique position music occupies. As temporal and non-referential, music not only induces a temporal transformation, but envelops bodily experience more than other arts. Generally, the temporal arts envelope experience far more than atemporal arts. Whereas the atemporal arts are always embedded within a broader ordinary temporal reality of communication and social interaction (e.g. I am at a museum, viewing paintings; I move from one to another taking care not to bump into others, perhaps while conversing with my friend), the temporal arts create their own reality. And whereas the referential arts tend to invoke an extra-aesthetic context (by reference), non-referential arts do not. That is, atemporal
arts are linked to extra-aesthetic temporal reality by context, while referential arts are linked to ordinary temporal reality by reference. Music need not link to either, and thus envelopes all the more completely.

Because music is essentially embedded in time, yet not essentially referential, its primary action upon human experience (though that action need not always be successful) is to plunge consciousness into an alternative temporal reality, one that inevitably represents a transformation of ordinary time perception. Indeed, I view such a temporal transformation as music’s primary function. Referentiality, which would distract from this function by turning attention from sign to signified, is not required for musical power and meaning to emerge.

Music is a function of and in time, which does not require external referents, and which addresses them in a very limited and inconsistent manner (as compared to language). Whereas many laypeople think of music as the art of tones (pitches), the tonal realm is special to the musical universe, whereas time straddles both musical and non-musical experience. Music’s most significant effect, then, is not its manipulation of tone, but its ability to transformation the perception of time during musical experience, since such a transformation has meaning outside of musical experience as well.

Listeners, to the extent that they fixedly concentrate upon a musical performance, find their sense of time (bound up, as it is, with the temporal flow of that music) transformed. The effect is naturally limited by the capabilities of composer and performers, by the attentiveness of the listener (backgrounded music produces a much lesser effect), and by the extent to which the music happens to be referential for a particular listener (thus distracting from fixed concentration upon itself). Still, music carries the potential for temporal transformations, and it is this potential which is to be explored here.

Since music is a function mapping a domain (time) into a range (sound), the significance of music’s temporal transformations means that, unlike most communicative systems (including most arts), the significance of the ‘musical function’ is not primarily in its range (sound), but rather in the way that range organizes its domain.

Music’s abstraction is consequential beyond the mere negation of reference, enabling more flexible, complex, and variable perceptual processing to take place, generating a myriad of Gestalten unlimited by restrictions of references. In this music contrasts sharply with visual perception. Thus take for instance a visual scene. The left vertical edge of my computer monitor connects to the horizontal line of a bookshelf across my office; taken together the two lines should form the numeral 7, and yet it is extremely difficult to make myself perceive this Gestalt, so strongly do I associate visual form with meaning, that is reference (the example is adapted from Wertheimer 1938). In music there is no reference to interfere with Gestalt formation; music enables the pure play of perception.

This fact which derives essentially from music’s non-referentiality is at the same time responsible for the power of music to represent time complexly. If music is a function whose significance is the interpretation of its organization of its domain (time), then that interpretive process, unconstrained by representation, is freed to assume much more complex, varied, and ambiguous forms, straying far from the objective sonic signal which putatively induces it. Interpretations of that signal may include, for instance, simultaneous, overlapping, hierarchical, and stochastic forms of temporal organization. Most of the interpretations constitute reductions (informationally), which are simultaneously augmentations (structurally), of the raw sonic signal. An acoustic reality thus induces a psychoacoustic one which is totally different in substance and structure, and indeed quasi independent of it. New psychoacoustic streams induced by fresh acoustic data complexly interact with pre-existing ones, and a relatively wide variety of individual interpretations are always possible, as compared with the visual field. Nevertheless certain tendencies, likelihoods, or perhaps only possibilities, can be identified. It is the task of this paper to examine some of the possible interpretations of the sonic signal, considering always the ways in which that signal (potentially) organizes time.

This importance of music’s temporal transformation is often manifested in evaluative musical discourse, much of which overtly or tacitly emphasizes the temporal domain for defining aesthetic criteria. Thus salient aesthetic values such as “groove” in most popular musics, “swing” in jazz, fixed “beats per minute” in genres of electronica, “phrasing” and “tempo” in classical music, or “lift-up-over-sounding” (for the Kaluli of Papua New Guinea (Feld 1988)) are closely linked in each case to a particular music’s temporal properties. The question remains: what is the relation between such discourses and the actual acoustic
signals being evaluated? In this paper it is not my attention to probe specific ethnographic cases, or to provide an objective empirical study relating subjective sensations of temporal transformations to aesthetic concepts. Rather I aim to provide a speculative map and inventory of some of the ways in which music may potentially induce a transformation in the listener’s sense of time.

Because music tends to envelope its perceiver, establishing a complete and emotionally (often socially) charged context, it is ideally suited to the more general task of altering consciousness (and social reality). Many mystical traditions, from the sacred bata drumming of Cuban Santeria, to Sufi (mystical Islamic) dhikr, deploy music, not only to provide an emotional basis for trance, but to cut participants off from ordinary physical reality, while linking them with each other and with some posited metaphysical reality. Music thus not only establishes an alternative abstract reality, detached from ordinary reality (via properties of temporality and abstractness), it is also capable of establishing that reality as a social space. This renders all of its effects all the more powerful.

II. Temporal transformations

I am claiming that the art of manipulating sound (music) can be most profoundly described as the art of manipulating time. That is to say, music is more than any other art (and more than anything else) empowered to transform time perception, that this power is in turn contingent upon music’s deep abstractness and temporality, and that music’s larger significance (in the broadest terms of human evolution) is bound up with its capacity to deploy such power creatively, as a deliberate and willful act of manipulation, by listener (through choice), composer (through technique), and performer (to the extent that the performer is allowed, through improvisatory or expressive license, to determine the music’s final sounded form). In this we should seek the general psychological and social function of the musical art, whose tonal characteristics (as magnificent as they can be) do not relate to any transmusical qualities of experience which might lend music functional value.

In the remainder of this paper I would like to move away from these philosophical considerations, towards a survey some of the temporal transformations of which music is capable. Again, music as a sonic signal can at most be said to induce, or perhaps only suggest, these transformations, which are really cognitive interpretations of the sonic signal, whose abstraction enables the most diverse interpretations, as I have indicated. A priori, there is no reason to believe that all human beings should make the same interpretations, and in fact there is plenty of evidence that they do not – no musical language is universal (though the capacity to make music is). Some of the differences will be cultural variables, while others may be individual. This is because the cognitive interpretation of music depends not only on universal psychological capabilities of the human mind, but also a particular listening history. However in what follows I outline only a set of possible transformations, backed by specific examples. Each is naturally associated with further questions for research.

The consequences of the temporal transformations induced by music’s acoustic signal can be profitably, though often only speculatively, analyzed for emotional and social effects, since it is an understanding of these effects which I believe holds the key to an appreciation of music’s functional significance. Examples will also be provided whenever possible.

The ordinary time of waking life is far from being homogeneous, much less isomorphic to the mathematical concept of a one-dimensional continuum. Time speeds up, when one is enjoying oneself, or slows down (in boredom). Nevertheless, there is nothing in quotidian life experience (which is dominated by uncontrolled, uncoordinated, often chaotic signals produced by the social and natural worlds) to establish any regular transformation of time, much less to do so as a conscious, purposeful, willful act. Though it is no doubt an oversimplification, I will use the expression ‘T’ to denote the reality and experience of this quotidian time, which we may naively take as roughly isomorphic to continuous segments of the real continuum R. The time variable will be represented as ‘t’, and the sonic signal as ‘s(t)’.

Informally, we will say that s(t) induces (or suggests) a temporal transformation from T to S if (1) s(t) supports the interpretation M(t); (2) M can be naturally defined as a function on some subspace of S (instead of on T), i.e. M(t) = g(h(t)); (3) S is as simple as possible.

III. Varieties of temporal transformation
In this section, I sketch a variety of temporal transformations, cast in the formal framework described above. The list is far from being sufficiently exhaustive, detailed, or scientifically verified. Its purpose is only to raise awareness of the possibilities of temporal transformations (supported by a few examples), and to stimulate further research (empirical or ethnographic) in each area.

A. Framed (bounded) time

To begin, it is worth pointing to the obvious: a musical piece transforms time by establishing a beginning and an ending, a temporal frame, a finite interval \([a,b]\) within \(T\). Thus music transforms time from an eternal flow, or at least an indeterminately bounded flow, to a definitely bounded one. Of course many other events besides music present definite beginning and ending points, but music often establishes temporal boundaries in an especially powerful way, due to its abstractness temporality, affectivity, and ‘enveloping’ properties. When the music starts at \(t=a\), one can be removed from ordinary temporal-referential reality surprisingly quickly, and this sudden shift in temporal reality (whose transformative qualities have not even yet been fully realized) serves to emphasize the boundary itself. When the music ends at \(t=b\), just the opposite occurs. The audio signal does not merely distinguish boundary points \(a\) and \(b\) within a temporal continuum; but rather marks these moments as transitions in and out of a different kind of time, within which various other temporal transformations may occur. A good example is provided by groove-based music, since the groove typically starts and ends abruptly, emphasizing the boundaries.

B. Discrete event time

While both a naïve representation of music as sonic signal and a full insider’s understanding of musical processes (including individual and social aspects of performance) indicate that music is continuous on several levels, musical notation (the score), and to a large extent the musical conceptualizations of composers and performers, suggests that music perception comprises a sequence of discrete events, occurring on a variety of aggregative levels. Score comprises symbols in a temporal sequence, realized as a sequence of musical events.

At the lowest organizational level, each instrument produces a stream of musical events, delimited by sonic start and stop points. The start (or attack) point is followed by an attack curve which is one of the characteristics of a musical instrument; the stop point is preceded by a decay curve. Since psychologically time is directional, the start point is more effective in marking an event time than the stop point, though the salience of the former may be reduced by a very gradual attack curve, and the salience of the latter may be enhanced by a very sudden decay curve. The significance of start and stop points may be mitigated by some sudden transformation of sound intensity in between the two. Nevertheless, the start points, what I term “attack points”, typically define a perceptually salient sequence (if not uniformly so) which qualifies as an \(M'\) function, an interpretation of \(M\). In this case the music transforms time to become a discrete sequence \(S\) of real values, such that \(h(t) = \{t \text{ if } t \text{ is a attack point, undefined otherwise}\}\), and \(g(t) = M(t)\). Then \(M'(t) = g(h(t))\).

The reduction to discrete time \(S\) is precisely the reduction employed by MIDI encoding favored for synthesizers and sequencers, which includes only start and stop points, together with timbre, pitch, and volume information. Most music lovers cringe at the thought of listening to MIDI encoded music for enjoyment, even though data entry might have transpired via expert performers, precisely because so much information has been omitted. On the other hand, for certain kinds of music the approximation is not bad, and in any case it is clear that the encoding, whatever its aesthetic failings, is a reasonable representation of an interpretation of \(M\).

Attack point-level event representations work especially well for music employing instruments whose attack and decay curves are uniform and hence predictable. A good example is the harpsichord, whose attack and decay cannot be controlled by the performer; the only free variable is timing. The principal limitation of MIDI representations is then practical (the quality of the sound generator) rather than theoretical (the reduction of continuous time to discrete time). As an example, one may compare two examples of Bach’s two-part Invention number 1 [BWV 772], one recorded live, and the other via MIDI. Far less satisfying is the temporal reduction induced by a MIDI version of Beethoven’s Violin Concerto in D (Op.61) 3rd Movement, Rondo – Allegro.

Besides the lowest level event stream, determined primarily by the sequence of attack points, are higher
level events determined by cognitively salient groups of such points: a chord, a change in timbre, the start of a harmonic progression, phrase, accent grouping, theme, or structural section. However the same principle – that time is transformed from continuous to discrete – applies in all cases.

C. Temporal transformations relating to the tactus

1. Regular discrete time: the steady tactus

Musical sound signals typically (though not always) support an interpretation consisting a sequence of equally spaced, accented beats, implied (if not always stated) by the stream of musical events, such that between any two adjacent beats there is no third. Such an interpretation is called a pulse line, or tactus. Supposing the separation between adjacent pulses to be the quantum t, the tactus is a reduction representing a temporal transformation from continuous time T to the regular discrete time series \( S = \{ a + k*t \} \), for \( k=0,1,2,\ldots,n \), where \( a \) is the start point, \( n*t < b \), and \( (n+1)*t > b \). The existence of such tactus, which is a psychological construct, enables one to define the music’s tempo, normally calculated as \( 60/t \), the number of pulses per minute.

A regular tactus is predictable and thus effective in socially organizing the performance context, as well as producing an emotional response: the feeling of motoric drive, an inexorable progression of discrete time. In writing for ensembles, composers usually rely upon a tactus (which is nearly always implied by musical notations) at the very least as a functional scaffolding for performers, typically to be turned (like the externalized pipes of the modernist Pompidou Centre in Paris) to aesthetic interest for listeners as well (though in some complex 20th century music the listener will not hear the tactus, which acts rather like a watch mechanism, functioning to keep musicians together, but hidden from external view).

Typically \( t \) will not exceed about one second; slower rates will not sufficiently ensure social uniformity, and will lead to the emergence of subdivisions, producing higher-level structures to be considered below. At the other extreme, a tactus will not be perceived for \( t \) less than about one-tenth of a second. The specific conditions governing these limits, and the relation between the stream of music events and perception of tactus (how often must the pulse line be reinforced by a real musical event?), are problems requiring empirical experimentation.

2. Regular discrete time: the accelerating (decelerating) tactus

The quantum t may vary continuously as a function of time, without sacrificing the theoretical concept of a tactus. Let \( f(t) \) be continuous and positive, with a bounded derivative, so that over a small interval of time \( f \) doesn’t change too much. Then the following series \( S \) is defined inductively: \( s_0 = a; s_n = s_{n-1} + f(s_{n-1}) \).

Such a series can represent a variable pulse line, where \( t \) at \( s_n \) is simply \( s_n - s_{n-1} \). The specific conditions under which \( S \) can be perceived as a potential pulse line (for any musical signal) need to be determined by experiment.

Under certain conditions (also requiring empirical investigation) a particular stream of musical events may support such an interpretation \( S \). In this case, the music transforms \( T \) to \( S \). Here the tempo (defined ‘instantaneously’ or as an average) is variable, since the number of pulses per second is not constant, but may increase or decrease according to \( f \). \( S \) is accelerating (or decelerating) time.

What are the effects of temporal acceleration? Musical acceleration induces acceleration of biological processes of performers, and (depending on their level of engagement) listeners as well. Such acceleration produces an impression of emotion, perhaps since other contexts producing such acceleration (e.g. fear, ecstasy) are typically emotional, or perhaps because such an bodily acceleration is the very stuff of emotion. Sociologically, a uniformly accelerating (slow to fast) pulse line tends to produce a unified corporate ecstasy. Slower tempos enable individual variation (since social unity is generally not required between pulses), and the rate is close to that of ordinary life. Such a tempo can thus engage a diverse group of musical participants (musicians, singers, dancers, listeners). Once the participating social group has been thus engaged, an accelerating tempo tends to gather them together, driving towards a state that is at once affectively heightened, and socially unified. Such a musical process is extremely valuable for corporate rituals that aim to produce individual affective energy (for catharsis, personal transformation, or validation of metaphysical belief), which is also channeled towards social unity (ensuring group solidarity, as well as feeding back to enhance individual affect). A good example of such acceleration can be seen in
the “buildup” characterizing the Sufi dhikr (“remembrance of God”) in Egypt. This ceremony overtly aims at creating a feeling of ecstatic connection to the Divine, by combining regular chant, body movements, pulsed music, and mystical poetry in performance. The accelerating pulse line not only swells emotion, but also provides a sense of temporal directionality and unified target, as realizing in musical form the theosophical goal of reaching towards the One God.

Less common but also effective is uniform deceleration (fast to slow), providing emotional relaxation. Whereas some music accelerates or decelerates over a long stretch of time (accelerando, ritard), regular undulating changes in the absolute value of the tactus unit may also take place over shorter intervals. Complex undulating patterns can be contrived, such as Radiohead’s remarkable “Pyramid Song”. At the extremes, time can be made to stop entirely, ironically either by slowing or speeding the pulse line to the point that it is no longer perceptible as a pulse.

3. Breath time

Certain tempo fluctuations may resemble natural cycles of the body, particularly the heartbeat and breath, gaining power not only from a general iconicity with bodily process, but also by entraining the biological cycles of participants (performers and listeners alike), thereby enhancing the psychic and social power of the music. While positing a straightforward causal relation between music which simply mimics biological rhythms and the emotional power of that music is no doubt simplistic, many psychologists admit that some such relationships are important (e.g. Davies 1978:192ff). Certainly, musical performance/perception and bodily processes are closely linked.

Indeed in most cultural contexts, a rigid tactus makes for bad music. Music teachers often warn their students not to play mechanically, to make the music ‘breathe’ through subtle tempo fluctuations. Such an instruction aims at inducing the student to establish a homology between musical time and the continuously varying frequency of bodily periodicities, such as breath or heart rate, thereby establishing a relationship between musical time and the body itself. The most rigid tactus occurs in synthetic sequencer-driven music, particularly popular electronica, part of whose appeal lies precisely in the deliberate glorification of synthetic and mechanical art (e.g. “industrial” music).

Biological time, on the other hand, is implied by irregular undulations of a variable (but steady state) tactus. Such variation naturally occurs in nearly all non-sequenced music, but is occasionally exaggerated, amplified for effect. Such is the case for instance in certain traditional Japanese genres, such as Noh, which display a surprising elastic pulse line. Subtler instances of the same phenomenon appear as variable measure lengths in jazz (Owens in Progler 1995:27), and in the well-known rubato (‘stolen’ time) interpretations preferred for performing late Romantic piano music of Chopin and Liszt.

4. Irrational time

The existence of a tactus implies a kind of rational temporal transformation, in which two musical time intervals are always comparable as the ratio of two integers, since every such interval is composed of an integral number of pulses. Fuzzy time, which captures the notion that music events may exhibit probabilistic distributions around a theoretical pulse or metric position (these ideas are developed below), may be considered a source of irrationality. But in this section I want to consider a deterministic version of irrationality as injected into a fundamentally rational system. I am not simply claiming that music produces irrational time; the continuum T already contains the irrational. Rather, I claim that music may highlight the idea of the irrational by juxtaposing certain irrational elements against a fundamentally rational framework.

Pragmatically, ensemble music often relies upon such rationality as a means of ensuring social synchronization in performance (and the equivalent ‘click track’ is used when overdubbing in the studio). Conventional musical notations, whose interpretation is predicated on the existence of a tactus, at least for the performer, also require rationality.

But solo music need not employ a tactus for pragmatic reasons, and is thus freer to explore the temporal continuum; because notation is difficult, such music is often improvisatory. Indeed, by removing one constraint (rationality), the absence of a tactus facilitates improvisation. Thus Arab solo song is notoriously difficult to transcribe into any usual notation, because there is no common interval (much less a perceptually utilized one) out of which all durations can be constructed.
The same considerations apply in ensemble music, whenever the requirement for social synchronization is partially relaxed. Thus the Egyptian mawwal, a genre of unpulsed improvisatory song, can be superimposed upon a tactus, because the soloist is not required to synchronize with other musicians (or he is only required to do so at particular cadential points in his improvisatory phrases). Here the notion of temporal transformation, since the music clearly juxtaposes rational and irrational time, in such a way as to highlight the latter.

The mawwal is rather unusual for exhibiting large scale irrationality, at time spans exceeding that of the tactus, a supra-tactal irrationality. More common is the sub-tactal kind. The pulse unit tends to be short, as a means of ensuring synchronization among performers at time scales equal to or exceeding the pulse unit itself. But irrationality is then possible at time scales smaller than the tactus itself, since at this sub-tactal time scale synchronization is not necessary, and irrational deviations have no deleterious effects.

A good example is found in bebop (jazz). In this context, the tactus consists of a steady stream of pulses often conceptualized as “quarter notes”, although the soloist often plays faster. One aspect of jazz’s mythical “swing” is captured by the manner in which a quarter note is divided into two parts.

It is well known that jazz players do not always perform “swing eighths” in the same way, the primary source of variability lying in the position of the second eighth note. Although jazz pedagogues often notate solos artificially using triplets (a rational division of the tactus into three parts) such that the second swing eighth comes precisely on the last third of the beat, such a division is not practically necessary, since synchronization is already guaranteed by the larger-scale tactus of quarter notes. The performer is thus freed to place the second swing eighth along a continuum, since this irrational operation will not contradict the tactus. In actual performance, this is what happens. The results can hardly be described. More research is required to better understand the musical processes going on here (See Progler 1995).

<table>
<thead>
<tr>
<th>Principal tactus</th>
<th>X</th>
<th>X</th>
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<tr>
<td>Artificial swing</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Real swing</td>
<td>X</td>
<td>X?</td>
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**Figure 2. The simplified structure of jazz swing. The line labeled ‘real swing’ exhibits irrationality at the points marked X? (the question mark indicates that placement is individualistic, and rules for placement need to be induced from empirical data).**

The irrational placement of an attack point within the tactus framework often carries a heightened significance. Psychoacoustically, the irrational attack point is highly affective, perhaps deriving from the theoretical fact that it carries infinite information (since its representation in terms of rational tactal units is infinite). In a social sense, irrationality is likewise functional, both for generating affectivity (to be distributed through and amplified by the group), and for enabling performers to develop idiosyncratic styles, identifiable definitions of swing.

A similar procedure typifies West African music, which Gunther Schuller viewed as one of jazz’s primary sources (Schuller 1968). The Ewe of southern Ghana employ polyphonic drum ensembles in which most parts sit squarely on two principal pulse lines, both of which are constantly emphasized by periodic bell and rattle parts, and which holds percussion, singing, and dancing together. Only one part (the kagan) tends to avoid accentuating the principal tactus, playing instead with irrational divisions inside each principal pulse. While teachers of African music in the West may simply the situation by explaining that the kagan part divides the beat into three equal parts, in fact its strokes often anticipate those parts, sliding forward along a temporal continuum that cannot be captured by any tactus. The powerful effect is to add “salt to the stew” as Ewes sometimes say, and to keep the ensemble from dragging; here personal style can also come to the fore.

<table>
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<th>Principal tactus</th>
<th>X</th>
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Figure 3. The simplified structure of typical Ewe music, showing the relation of normative (rational) and actual (irrational) kagan parts. Variable irrational placements are marked as ‘X?’.

Whereas the tactus defines rationality in music, and ensures social cohesion and synchronization, it also limits temporal expressivity by reducing it to a binary function of the tactus. Irrational timings, incommensurate with the tactus, are infinitely expressive.

5. Multiple simultaneous temporal transformations

We have noted in passing that a musical signal may admit multiple tactus interpretations; an extreme example being given by stratified central Javanese gamelan music. Multiple tacti may be nested (one tactus subdividing another), but may also be offset. One clear example is the interlock of Balinese gamelan gong kebyar ensembles. The rapid-fire gangsas (bronze metallophones) are divided into two parts, polos and sangsih, which present complex interlocking figures called kotekan; each part suggests a different tactus. In the simplest case each part plays on a tactus at the same tempo, but offset by 180 degrees in phase (within the larger music/dance context, it is the polos which is generally interpreted as “on-beat”).

Polos  x  x  x  x  x  
Sangsih x  x  x  x  x  

Figure 4. Simple kotekan style (called “Nuutin”, or “following”) in the Balinese gamelan gong kebyar.

Multiple tacti correspond to multiple simultaneous temporal transformations; these interact with one another and produce powerful psychic and social effects. For instance, the Balinese kotekan can be performed very fast (since the resultant speed is double the capability of a single player); at the same time their interlocking patterns have been taken as expressive metaphors of (and reinforcement for) the tightly interlocked social structure of Balinese villages generally. As it may be argued that metric cycles are really composed of a nested set of tacti, it is logical to consider meter next

D. Temporal transformations relating to cycles

1. Periodic (cyclic) time

Meter is a psychoacoustic interpretation of s(t) most conveniently defined as a periodic function (of pulse number modulo meter length) whose values are various accents levels. Like the tactus, the metric function is not necessarily performed as such, but is rather implied by musical material (especially melodic phrases and rhythmic patterns), in conjunction with listener expectations (as culturally conditioned). Musical meter effects a temporal transformation from the continuum T to the space of integers modulo k, where k is the number of pulses in the metric cycle.

Accent level  4  1  2  1  3  1  2  1  
Pulse equivalence class  0  1  2  3  4  5  6  7  

Figure 5. Common meter (4/4) represented as a periodic function of pulse number mod 8. Accents range from 1 (weakest) to 4 (strongest).
Meter may also be regarded as an extension of tactus. We mentioned earlier that more than one tactus may be operative. Generalizing, one can define meter as a set of tacti operating simultaneously, a weighted polyphonic tactus. Typically each level will contain the next one, nested from fastest to slowest (i.e. the tacti are in-phase in a harmonic series), although some meters can only be defined in terms of a set phase offset (extending the notion of interlock presented earlier).

| Tactus 1 | 1 |
| Tactus 2 | 1 |
| Tactus 3 | 1 | 1 | 1 |
| Tactus 4 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| Total accent | 4 | 1 | 2 | 1 | 3 | 1 | 2 | 1 |
| Pulse equivalence class | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |

**Figure 6.** Common meter (4/4) represented as a polyphony of four in-phase tacti, in the harmonic series f, 2f, 4f, 8f. Note that the total accent is identical to that shown in the preceding diagram.

| Accent | 4 | 1 | 1 | 2 | 1 | 3 | 3 | 2 | 1 | 1 |
| Pulse equivalence class | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

**Figure 7.** The Arabic meter “sama‘i thaqil” represented as a periodic function of pulse number mod 10.

| weight |
| Tactus 1 | 1 | 1 |
| Tactus 2 | 2 | 1 |
| Tactus 3 | 2 | 1 |
| Tactus 4 | 1 | 1 |
| Tactus 5 | 1 | 1 |
| Tactus 6 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| Total accent | 4 | 1 | 2 | 1 | 3 | 3 | 2 | 1 | 1 |
| Pulse equivalence class | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

**Figure 8.** The Arabic meter “sama‘i thaqil” represented as a weighted polyphony of out-of-phase tacti, each a periodic function of pulse number mod 10.

As in the case of the tactus, the relation between musical material and meter is delicate, since implication of the latter by the former is often covert. Suppose the musical signal s(t) strongly implies a metric interpretation, M. Once established, that meter may persist even when s(t) shifts to support M more
ambivalently. But if the music changes too drastically, M will disappear in favor of a new meter, or perhaps no meter at all. The listener’s interpretation of meter at time t is not based purely on the sound signal s(t), but is conditioned also by a hysteresis effect (what the meter was a moment ago). Such is the case for tactus also. But meter, more than tactus, is also conditioned by a lifetime’s listening experience, which is in turn related to cultural patterns of conditioning. Here considerable empirical experimentation is required to determine limits and boundaries of metric perception, as a function of culture.

Thus an Ewe from southern Ghana will immediately infer certain meters from complex bell patterns which would baffle the typical North American. In southern Ghana dancers typically move to the 12-pulse pattern \( \times \times \times \times \times \times \times \times \times \times \times \times \times \) suggesting that they interpret it as a periodic accent function analogous to (accent is again measured on a scale from 1 to 4): 411211311211. An unenculturated listener is unlike to derive such an interpretation, at least not immediately, or might even claim not to feel any meter at all, while the Ghanaian listener is able to maintain the same metric pattern even in the face of considerable acoustic data to the contrary.

In some musics something very close to the meter is overtly stated, usually by percussion instruments charged with maintaining the ‘groove’. Thus in Arabic music rhythmic cycles are more often realized in sound, by cyclic patterns more or less tantamount to meter. Musicians need not repeat a single pattern mechanically, but at least one percussionist will typically remain close to it.

<table>
<thead>
<tr>
<th>Accent</th>
<th>4</th>
<th>1</th>
<th>1</th>
<th>2</th>
<th>1</th>
<th>3</th>
<th>3</th>
<th>2</th>
<th>1</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drum stroke</td>
<td>Dum</td>
<td>Tek</td>
<td>Dum</td>
<td>Dum</td>
<td>Tek</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pulse equivalence class</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
</tr>
</tbody>
</table>

Figure 9. The Arabic “Sama‘i thaqil”, as an abstract accent pattern, and as realized on a drum. The “dum” stroke is heavy and low; “tek” is lighter and higher.

In other musics, there is no core periodic rhythm and the meter is maintained by a psychoacoustic inference from the sound signal s(t). Thus in Hindustani music, where the metric concept is called tala, the tabla player establishes the meter with a basic pattern, but then swiftly moves away from it to improvise, while performers and listeners maintain the same metric interpretation, even in the presence of considerable contrary sonic evidence, out of a sense of cultural expectation, including the knowledge that the tala exists and does not change. Indeed in such musics it is precisely the clash between the ongoing meter and more temporary temporal transformations (as suggested by ephemeral patterns in the musical signal’s event stream) which is often highly valued, both as a measure of musician’s skill, and for a pleasant sensation by which two different temporal transformations (an event transformation, and a metric one) collide with one another.

2. A general formalism for metric transformations

Thus far meter has been explained as a cycle, though the concept of tactal polyphony correctly suggests that meter may be explained more generally as a set of simultaneous, possibly phase-offset, cycles.

Formally, a meter can be defined as a periodic accent function, and the set of all meters is isomorphic to the set of expressions E generated inductively:

(1) E contains all small integers, n.

(2) If a and b are in E, then so are: (1) the product m(a) for any small integer m; (2) the sequential sum a + b; (3) the simultaneous sum a \( \times b \) [n]

The expression so derived has the following rhythmic interpretation: Each integer n represents an accented pulse followed by n-1 unaccented pulses; m(a) means that rhythm a is repeated m times, the first pulse of the first repeat being accented; a + b means that rhythms a and b are concatenated sequentially (one following the other); a \( \times b \) [n] means that a and b are overlaid simultaneously (polyphony) and repeated as
needed until the lengths match, then b is phase shifted \( n \) pulses (forwards for \( n > 0 \), backwards for \( n < 0 \)); \( [n] \) can be omitted when \( n = 0 \).

Thus the metric pattern represents a particular way of representing an integer \( n \) in terms of an ordered sequence of sums and products.

3. Toral (bicyclic) time

Any meter consisting of a cycle of cycles, i.e. which contains at least one product in its E expression, is technically a compound meter, although the term is usually reserved for meters whose products contain both \( m = 2 \) or \( 4 \) and \( m = 3 \), the latter in an interior portion of the E expression (such as \( 4(3) \) or \( 2(3(2)) \)).

While most meters contain products, many (such as \( 4/4 \)) are short and self-similar (comprising powers of two). Longer meters involving larger and contrastive (often relatively prime) factors induce a stronger sense of cycles operating within cycles. Thus \( 4(3) \) strongly suggests two levels of cycles. Such multicyclic meters are often termed ‘compound’. Non-compound meters are termed ‘simple’.

If simple meter induces a temporal transformation mapping the continuous line into a circle, then a compound meter consisting of a cycle of cycles should be represented as a curve spiraling around the cross product of two circles, or a torus (doughnut surface). Bicyclic time implies toral time.

A good example is again provided by the Ewe’s 12-pulse music (common also throughout Africa and the Caribbean), typically organized as 4 groups of 3. Each cycle (4 and 3) can be represented as a number of points on the periphery of a circle; combining the two is equivalent to the cross product of the four-point circle with the three-point circle, resulting in 12 points on the surface of a torus. We may imagine the smaller dimension of circumference 3, and the larger of circumference 4. Then the meter loops four times around the smaller circumference, as it loops also once around the larger circumference, before returning to its starting point.

We have not yet had occasion to mention the fact that simple sequential additions should be represented as the union of two circles; the implication is that time alternates between cycling one and cycling the other. Then a product such as \( 3(3 + 2) \) implies two nested tori (one inside the other).

4. Hypertoral (multicyclic) time

As an extension, one may consider longer factorizations implying more than two cycles. The basic Ewe meter is better considered as three cycles \( (2(2(3))) \), while Shona mbira music is frequently performed in the 48 pulse framework \( 2(2(2(3))) \). To each multicyclic meter corresponds a hypertorus, the cross product of more than two constituent cycles. Visually, these cannot be easily represented in three dimensions, though the music implies a temporal transformation corresponding to such a higher dimension torus.

In Ewe and Shona music the cyclic layers are often implied rather than stated by the texture overtly. Javanese gamelan provides a good example of multicyclic time in which different cycles (differentiated by density, phase, or both) are sonically distinguished, by being performed explicitly on instruments (various kinds of metallophones) of contrasting timbres and/or pitch ranges. For this reason Javanese gamelan is often called “stratified” music.

| Bonang panerus: | xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx |
| Bonang barung:  | .x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x |
| Saron:          | .x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x |
| Kethuk:        | .x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x |
| Kenong:        | x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x |
| Kempul:        | x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x |
| Gong:          | x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x.x |
| Accent:        | w.m.w.S.w.m.w.S.w.m.w.S.w.m.w.S.w.m.w.S.w.m.w.S.w.m.w.S.w.m.w.v |

Figure 10. The principal seven strata of a central Javanese gamelan ensemble for the composition
“Bubaran Kembang Pacar”, showing one complete metric cycle (four such cycles completes a melodic unit). Beats are marked by ‘x’. Accent levels are weak (w), medium (m), strong (s), very strong (v) (after Sutton 1996:332-336).

The multiple cycles of Javanese gamelan, occasionally coinciding, are frequently described by scholars as homologous to Javanese Hindu-Buddhist conceptions of time (Becker 1979); as gamelan music accompanies shadow puppet representations of Hindu epics, its multicyclic musical structure reinforces the Hindu world view by effecting a real temporal transformation. More prosaically, such a structure enables a wide range of playing abilities and personal temperaments; gamelan can thus become an open, participatory community music.

5. Spectral time, harmonic time, timbral time

As a periodic function, meter can be spectrally analyzed in the frequency domain. The simplest cases to analyze are expressions in E’, where:

(1) E’ contains all small integers, n.
(2) If a is in E’, then so is m(a) for any small integer m

In this case all frequencies are harmonics of a fundamental, combined in equal proportions.

We have already observed that 4/4 ‘common’ time (as 2(2(2))) is represented by the harmonic sum f + 2f + 4f + 8f (hence three octaves, corresponding to intervals A1 + A2 + A3 + A4); likewise 2/4 (as 2(2)) is f + 2f + 4f, and 3/4 ‘waltz’ time (as 3(2)) is f + 2f + 6f (an octave plus a twelfth, corresponding to intervals A1 + A2 + E2). More generally, a recursive calculation is possible: if the meter is represented by an expression m(a) (where a is an expression in E’), then it contains the frequency sum f + m* (frequency sum for a); and the frequency sum for a single integer (m) is f + m*f (here multiplying by m means raising the frequency by the factor m).

Meter implies a temporal transformation; these derivations suggest that this transformation can be examined in the frequency domain as a metric “harmony” or “timbre”; if the meter were sounded fast enough, it could be heard as such.

6. Polymeter: refactorization and permutation

By definition, the product expressions representing compound meters can be rewritten without altering the length of the meter, by choosing different factors for the same product, or permuting the same factors. Musically, and temporally, what does it mean to transform the meter in this way?

We return to our Ewe example. The most commonly perceived factorization (as judged by music performed, dance patterns, and the discourse of participants) appears to be 2(2(3)) which corresponds to an accent pattern (again deploying the four point scale) of 411211311211. The same pattern is commonly permuted in two ways:

2(3(2)) = 412121312121
3(2(2)) = 412131213121

Each permutation gives rise to a completely different metric cycle, though always amounting to 12 pulses. Considering that each meter represents a different hypertorus, their temporal transformations are quite different. Another neat way of highlighting the differences is to use the same hypertorus for all three, but then traverse its three angular dimensions in a different order for each. This procedure is more intuitive in three dimensions, sufficient for representing the torus corresponding to 3(4) and 4(3). If in the first case one may wind around the narrow dimension faster, circling slowly around the wider dimension, then in the latter case one does the opposite.

Polymeter arises from the potential of compound meters containing heterogeneous factors to be refactored and permuted. When the two factorizations or permutations of a cycle are presented sequentially, the listener experiences a polymetric modulation (metric modulations are further discussed below). When they happen simultaneously, the listener experiences polymeter in the true sense of the word. Subjective evidence suggests that listeners typically maintain one factorization as primary, while the other is heard as a
contrasting rhythm. (The extent to which it is possible to maintain simultaneously and equally two
contrastive meters from a musical signal is not known, so far as I know.) Unlike the product notation
(which specifies a particular factorization), the notation implies polymeter without prejudice. Since
polyrhythm corresponds to traversing a torus in two directions at once, it suggests toral time much more
strongly than a single compound meter. Simultaneous and sequential polymeter is quite common in Ewe
music, which therefore strongly projects a toral geometry.

Several emotional and social effects are enabled by polymetric temporal transformations. The intense affect
generated by polymeter is difficult to describe, but may be likened to the miraculous sensation of seeing a
flat object suddenly become three dimensional. At the same time, polymeter enables social flexibility,
since different individuals may groove to the music differently, according to their state (just as allowing
spontaneous harmony parts may enable a singer to drop down to a more comfortable pitch level).

7. Phase shifts: backbeat, Agbadza

The possibility of multiple simultaneous metric interpretations of the sonic signal \( s(t) \) leads to multiple
simultaneous temporal transformations, which may interact in interesting ways. We have seen this
phenomenon twice already: as a phase-shifted interlock (of two tacti), and in non-phase shifted polymeter.

The backbeat, so familiar to African-influenced musics from jazz to reggae and blues, provides an
interesting case of phase-shifted metric simultaneity. The backbeat refers to the rhythmic accentuation of
beats two and four, within a 4/4 context which would ordinarily emphasize beats one and three. Formally
we have \( 2(2) \ 2(2) [1] \), or (in terms of accent patterns) \( 4232 \ 2324 \) (note that the sum could also be
defined as a metric pattern superimposed on its own retrograde, although our notation does not allow this).
Typically melodic patterns emphasize 4232 (suggesting that 4/4 is in fact the meter), while percussion and
bass loudly proclaim 2324. In jazz the two flows tend to cancel each other out, producing a nearly
unaccented flow of beats, which is merely the resultant pattern obtained by adding the two cycles.

Phase shifts may also be combined with polymeter. Thus in Ewe music it is not uncommon to find \( 4(3)
3(4) [3] \), as in one pattern used in a piece called Agbadza.

<table>
<thead>
<tr>
<th>Kidi drum</th>
<th>x</th>
<th>x</th>
<th>x</th>
</tr>
</thead>
<tbody>
<tr>
<td>Claps</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
</tbody>
</table>

Figure 11. Two periodic lines which may occur in Agbadza.

8. Metric modulation

Any metric interpretation only persists so long as it is supported by a stream of musical events. If that
stream changes significantly, the meter may change as well (though often without changing the tactus, to
ensure continuity). Thus we have the concept of metric modulation, analogous to the more familiar
modulations in tonality. Changes in meter imply corresponding changes in the music’s temporal
transformation; such changes highlight both meters (by juxtaposition) and call attention to the boundary
between them. This process may thereby generate emotion, or enable some form of social reorganization.
Besides polymetric modulation, which has already been discussed, two special cases are examined below.

a) Spectral modulation

Earlier we demonstrated how meter can be represented as a harmonic spectrum. Such a representation is
particularly useful in understanding a special kind of metric modulation. The modulation from 4/4 (accent
pattern \( 41213121 \)) to 2/4 (accent pattern \( 3121 \)), while maintaining a continuous pulse might appear to be a
qualitative change, by which half of 4/4 is cut away. But upon examining the two spectra one observes
that 4/4 (\( f + 2f + 4f + 8f \)) and 2/4 (\( f' + 2f' + 4f' \)) are closely related. If the pulse remains continuous, then
\( f' = 2f \), so that 2/4 is (\( 2f + 4f' + 8f \)). It then becomes clear that this modulation merely represents an
emphasis of the upper harmonics of 4/4, which could even be effected continuously (by gradually
reducing the fundamental \( f \)). Furthermore, if 2/4 is slowed by half (by doubling the pulse), it becomes (\( f +
2f + 4f \)), which is remarkably close to 4/4. That is, 2/4 is at once the upper partials of 4/4, and a near copy
of 4/4, played twice as fast.
An excellent example is provided by popular Arabic music. The following three patterns are often used in sequence. Qualitatively, it is clear that each emphasizes the upper harmonics of the previous; moreover, the third is nearly a copy of the second, but at double speed.

<table>
<thead>
<tr>
<th>Wahda</th>
<th>4</th>
<th>3</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maqsum</td>
<td>4</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>Bamb</td>
<td>4</td>
<td>3</td>
<td>2</td>
</tr>
</tbody>
</table>

**Figure 12. Spectral acceleration. The relation between three Arabic metric cycles commonly used in sequence. Numbers indicate relative accent levels.**

The beauty of a change in temporal transformation from a meter to its “octave” superior is in creating the impression of a sudden acceleration (by emphasizing upper harmonics) without changing the tactus, or clashing with the original meter, which can even be maintained by listeners. (The effect is analogous to an octave jump in pitch, which paradoxically raises the pitch without changing it.) Emotionally, this technique produces a quantum leap in energy, while maintaining continuity.

Both tactal and spectral acceleration are deployed by the great Egyptian religious singer, Shaykh Yasin al-Tuhami, in performing his version of the dhikr (with percussion accompaniment). The following diagram shows how a gradual tempo increase explodes in a sudden spectral acceleration. At this point there is a qualitative increase in group energy and emotion; dhikr movements may double in speed, but there is no discontinuity in tactus.

**Figure 13. A typical tempo curve for a performance by Shaykh Yasin al-Tuhami of Egypt. The vertical line is scaled in pulses (beats) per second. A single line indicates wahda, a double line indicates maqsum, and a triple line indicates bamb. The entire performance (nearly three hours) contains six ‘buildups’ combining tempo and spectral acceleration.**

**b) Hypermetric and hypometric modulation**

Because it tends not to be periodic over the long term, hypermeter has not been represented as a product cycle; however changes in hypermetric organization could be represented as a change in the outer factors of a product (since the inner factors represent a constant meter). Thus if four cycles are grouped twice, and then two cycles are grouped thrice, one might write the modulation as: \(2(4(m)) \rightarrow 3(2(m))\), where \(m\) represents the constant meter.

In hypometric modulation factors are introduced at the innermost parenthetical level. Since the tactus is connected to the innermost level, such a modulation amounts to a shift in the ratio between tactus and other cyclic components of the meter. The principal example of this modulatory technique occurs in Javanese gamelan, where it is known as a change in irama. The Javanese concept of irama is a measure of the ratio between tactus (performed by the highest frequency stratum) and the core melody line (played by saron). Thus in Figure 10 above that ratio is four. After one or more cycles, the tempo slows down, and the fastest stratum doubles the tactus pulse rate, pushing the ratio to eight. This process can repeat, leading ultimately to a ratio of 32. Throughout, the pulse rate maintains approximately the same value, varying within a narrow range. Thus a series is defined: \(M(4) \rightarrow M(8) \rightarrow \ldots \rightarrow M(32)\), where \(M\) represents a fixed “outer” structure. The emotional sensation of these shifts in temporal transformation is powerful, as if a huge machine were changing gears, or as if an aural microscope were revealing new microstructure by slowing down time (Sutton 1996:336-337). As the piece concludes, the progression moves in reverse.

**E. Hierarchical time**

Music theorists have long noted that whereas meter as represented by “bars” in music notation fulfills a pragmatic function (helping the performer to read a notation by visually accentuating part of the psychoacoustic organization of the resultant sound), the concept of meter as a principle of temporal organization extends beyond bars towards higher levels of organization. While these levels may not be
important enough to mark in the musical score (although performers should understand them in order to play musically), they are real in a psychoacoustic sense. The hierarchical organization of “bars”, i.e. meter in its ordinary sense of relatively short pulse-groupings, is called hypermeter.

But whereas the level of meter is cyclic, hence periodic, over relatively long stretches of time (the accent function repeats many times before changing or ending), hypermetric organization typically is not. Hypermeter is nevertheless clearly felt, implied by melodic, harmonic, and rhythmic features, as a higher-level organization of metrical units.

Thus hypermeter maps continuous time into a hierarchical tree structure, in which elementary bits of temporal organization (pulses organized into a metric structure) are themselves grouped together at a higher level, with the first portion of the grouping (like a downbeat) receiving a stronger ‘accent’. However these higher-level patterns do not repeat periodically for long enough to be established as higher-order factors in a temporal transformation (which could then be represented on a hypertorus). Sometimes two measures may be grouped together; other times three. The temporal depth (number of active levels) and grouping patterns can quickly change, and composers exploit this fact to good effect.

Formally, the set of such hierarchies are isomorphic to expressions from a set $E$, where:

1. The string $m$ is in $E$
2. If $x_1, x_2, \ldots, x_n$ are in $E$, then so is $(x_1 + x_2 + \ldots + x_n)$

The hierarchical interpretation of an expression $y$ in $E$ is evident. If $y = (x_1 + x_2 + \ldots + x_n)$ then the top hierarchical level contains $n$ branches, represented respectively by $x_1, x_2, \ldots, x_n$, and so on recursively.

Listeners generally expect hypermeter to organize cycles ‘subharmonically’ in groups of 2, 4, or sometimes 3; since these factors represent ratios already encountered at the metric level, their presence in the hypermeter fulfills an expectation of self-similarity or automorphism. Indeed factors of two and four are especially common in many musical systems, as when a melody (occupying two or four metric cycles) is repeated twice, following which the same procedure is applied to another melody, and so on. The self-similarity of such structures makes them easy to remember, and be retained in oral traditions.

A striking example is provided by the Brahms’ First Piano Concerto, which opens with the hypermetrical structure: $(m + (5(m + m)))$ (here the numeral 5 is a metalanguage shorthand indicating that the following unit should be repeated five times). Listening to this passage, one understands that composers use irregular groupings to contradict the mechanical regularity of periodic meter, as a means of generating emotional interest.

**F. Deleted time**

A number of authors have attempted to determine, through ethnographic, phenomenological, or empirical methods, the essence of “swing”. Swing is a key parameter in the discourse of jazz musicians and aficionados, but also appears to be supported by a real acoustic (and psychoacoustic) basis. We have already considered swing as a locus for irrational time. In this section I take a completely different tack, treating swing as the deletion of time along the temporal continuum $T$. Let us first of all return to the pedagogue’s approximation that swing entails the rational triplet division of the principal tactus; the argument that follows can easily be extended to irrational divisions.

First, let us measure how often each pulse (now considered a series of triplets, each comprising one larger beat) occurs, by tabulating a histogram of use. We would presumably find that the central pulse of each triplet is hardly ever employed.

Let us then postulate a difference between silence and existence within a metric structure. A silence indicates that a particular segment of the meter is not currently being sounded in a particular iteration of the cycle, though it is often sounded in other iterations. However the statistical structure of swing triplets implies that the central third of each triplet is virtually never used. In this case I posit that the metric structure (as based upon the stream of musical events) effectively implies the deletion of this middle third.

What I would like to propose, then is that the ineffable quality of swing corresponds to a perception of a metric structure that has implied the deletion of time from the continuum. Or, to state the process the other
way round, one may consider swing to begin with a duple division of each beat, after which a small temporal “vacuum” is inserted between each duple pair.

Besides swing, the concept of deleted time may be usefully applied to other musical contexts; one remarkable example is Radiohead’s “Pyramid Song”.

G. Probabilistic time

1. Fuzzy tactus, fuzzy meter

Earlier I argued that much music defines a sequence of discrete events, and implies a regular (if accelerating) tactus, whose function is part aesthetic and part social. However exact synchronization with the tactus is evidently impossible, and if one were to gather empirical data about the relation of attack points to the tactus (by superimposing the neighborhood of every pulse, scaled for $t$), one might discover a distribution (perhaps normal, perhaps skewed), which might not even be centered at the pulse point. Data might also be collected to generate such a distribution at each point in the metric cycle. Such randomness represents several combined effects of unintentional human error, and (intentional) use of unpredictability along the continuum for the sake of expressivity.

Some jazz singers famously lag the pulse, while the Ewe kagan player often anticipates it. But the point is not that such deviations present a new deterministic system, but rather that they can be approximated by independent random distributions of a form which must be empirically determined. Swing may exhibit patterned randomness as well. The upshot is that rather than defining a sequence of discrete events, music presents a sequence of probabilistic distributions governing deviation from the tactus. In doing so, music transforms time into a random, or at least unpredictable, structure. The detailed investigation of such distributions constitutes an important empirical task towards understanding how live music works to create its emotional effects.

2. Probabilistic meter and information

Thus far we have considered meter as a psychoacoustic periodic accent function established (and sometimes challenged) by the musical event stream. The existence of meter in this sense is understood primarily from introspection.

Suppose we adopt a more objective, empirical approach to meter, accepting only the attack point event stream within a regular tactus as our data, and ignoring higher level events (phrase endings, harmonic changes, etc.) as well as issues of (culturally shaped) listener expectation for the moment. How can some concept of meter (perhaps not the only one) be statistically inferred from the attack point event stream?

This event stream presents us with a discrete time series; at each point in time (along the tactus) some number of events are occurring, which number may be taken as an estimate of accent. For simplicity we may start by reducing each pulse to two states: accented, or unaccented. As a first step, we can compute an autocorrelation function to determine whether any periodicities are salient in this data. If we discover a strong harmonic series, let us then assume the fundamental to represent the metric frequency, and its reciprocal the metric period, $P$.

The histogram measurement which (presumably) revealed that the central third of each swing triplet is hardly every used suggests our next step here. For $k=0,...,P-1$ we calculate the fraction of pulses, whose pulse number mod $P$ falls into the equivalence class $k$, contain an accent. This calculation provides us with a kind of histogram, a function $M(k)$ of cyclic pulse number representing the usage of each pulse within the metric cycle; that usage may be interpreted as a probability. Rather than using this histogram to infer meter, we now take this $M$ to be the meter. With this interpretation, music can be understood as transforming time from the continuum to a cycle of probabilities. It is furthermore now possible to characterize the temporal transformation $M$ in terms of its total information content, $I = -[M(k) \log (M(k)) + (1-M(k)) \log (1-M(k))]$. Note that in this calculation pulses which are always accented, or never accented, will contribute zero information (as they are completely predictable).

IV. Future directions

The preceding theoretical discussion has far from exhausted the possibilities by which music may induce temporal transformations. Further theorizing should assess the potential significance of other musical
phenomena, including as higher-level event streams (e.g. melodic, harmonic, timbral, dynamic events at multiple levels of organization), and unique culture-specific musical techniques. Such theorizing should be followed up in three ways:

(1) Controlled psychoacoustic experimentation in the laboratory (including subjects of diverse cultural backgrounds), to explore the validity of testable theories, to determine all testable parameters suggested by theory, and to understand the relation between perceptual and individual variables.

(2) Ethnographic investigation of music and musical discourse in various cultural contexts, in order to determine (for each) which temporal transformations appear to be operative in practice, what are their affective or social effects, how they are recognized in discourse, and how they relate to more general aesthetic, temporal, cosmological, or metaphysical practices and concepts.

(3) Functionalist analysis (psychological, and cultural) in order to probe the cognitive and social value of temporal transformations within psychic and social systems, both as a response to particular cultural and social contexts, and for the human species in general.

Reference List


