Chapitre 24 – Capacité et diélectriques [24 au 26 mai]

DEVOIR : 24.50, 24.60, 24.64, 24.68, 24.70

24.1. Condensateurs et capacité

- P. 816: définitions de condensateur et capacité (Eq. (24.1))
- Unité de capacité : farad (F)
- Eq. (24.2) : plaques parallèles
- Les exemples aux pp. 818-820 contiennent les principaux cas.

24.6. IDENTIFY: \[ C = \frac{Q}{V_{ab}}. \]

SET UP: When the capacitor is connected to the battery, enough charge flows onto the plates to make \( V_{ab} = 12.0 \text{ V} \).

EXECUTE: (a) \( 12.0 \text{ V} \)

(b) (i) When \( d \) is doubled, \( C \) is halved. \( V_{ab} = \frac{Q}{C} \) and \( Q \) is constant, so \( V \) doubles. \( V = 24.0 \text{ V} \).

(ii) When \( r \) is doubled, \( A \) increases by a factor of 4. \( V \) decreases by a factor of 4 and \( V = 3.0 \text{ V} \).

EVALUATE: The electric field between the plates is \( E = \frac{Q}{P_a A} \). \( V_{ab} = Ed \). When \( d \) is doubled \( E \) is unchanged and \( V \) doubles. When \( A \) is increased by a factor of 4, \( E \) decreases by a factor of 4 so \( V \) decreases by a factor of 4.

24.8. INCREASE: \[ C = \frac{Q}{V_{ab}}. \]

SET UP: We want \( E = 1.00 \times 10^4 \text{ N/C} \) when \( V = 100 \text{ V} \).

EXECUTE: (a) \[ d = \frac{V_{ab}}{E} = \frac{1.00 \times 10^2 \text{ V}}{1.00 \times 10^4 \text{ N/C}} = 1.00 \times 10^{-2} \text{ m} = 1.00 \text{ cm} \].

\[ A = \frac{Cd}{P_a} = \frac{(5.00 \times 10^{-12} \text{ F})(1.00 \times 10^{-2} \text{ m})}{8.854 \times 10^{-12} \frac{\text{C}^2}{\text{N} \cdot \text{m}^2}} = 5.65 \times 10^{-3} \text{ m}^2. \]

\[ A = \pi r^2 \text{ so } r = \sqrt{\frac{A}{\pi}} = 4.24 \times 10^{-2} \text{ m} = 4.24 \text{ cm} \].

(b) \( Q = CV_{ab} = (5.00 \times 10^{-12} \text{ F})(1.00 \times 10^2 \text{ V}) = 5.00 \times 10^{-10} \text{ C} = 500 \text{ pC} \)

EVALUATE: \( C = \frac{P_a A}{d} \). We could have a larger \( d \), along with a larger \( A \), and still achieve the required \( C \) without exceeding the maximum allowed \( E \).

24.12. IDENTIFY: Apply the results of Example 24.3. \( C = \frac{Q}{V} \).

SET UP: \( r_a = 15.0 \text{ cm} \). Solve for \( r_a \).

EXECUTE: (a) For two concentric spherical shells, the capacitance is \( C = \frac{1}{k} \left( \frac{r_b}{r_b - r_a} \right) \).

\[ kCr_a - kCr_b = r_a r_b \text{ and } r_b = \frac{kCr_a}{kC - r_a} = \frac{k(116 \times 10^{-12} \text{ F})(0.150 \text{ m})}{k(116 \times 10^{-12} \text{ F} - 0.150 \text{ m})} = 0.175 \text{ m}. \]

(b) \( V = 220 \text{ V} \) and \( Q = CV = (116 \times 10^{-12} \text{ F})(220 \text{ V}) = 2.55 \times 10^{-8} \text{ C} \).
**Evaluate:** A parallel-plate capacitor with \( A = 4\pi r_a r_b = 0.33 \text{ m}^2 \) and \( d = r_b - r_a = 2.5 \times 10^{-2} \text{ m} \) has
\[
C = \frac{P_0 A}{d} = 117 \text{ pF},
\]
in excellent agreement with the value of \( C \) for the spherical capacitor.

**24.2. Condensateurs en série et en parallèle**
- Série : mêmes \( Q \), on additionne les \( V \). \( C_{eq} \) en Eq. (24.5)
- Parallèle: mêmes \( B \), on additionne les \( Q \), \( C_{eq} \) en Eq. (24.7)
- Lire p. 823, exemple 24.6

**24.15. Identify:** Replace series and parallel combinations of capacitors by their equivalents. In each equivalent network apply the rules for \( Q \) and \( V \) for capacitators in series and parallel; start with the simplest network and work back to the original circuit.

**Set Up:** Do parts (a) and (b) together. The capacitor network is drawn in Figure 24.15a.

**Execute:** Simplify the circuit by replacing the capacitor combinations by their equivalents: \( C_1 \) and \( C_2 \) are in series and are equivalent to \( C_{12} \) (Figure 24.15b).

\[
\frac{1}{C_{12}} = \frac{1}{C_1} + \frac{1}{C_2}
\]

\[
C_{12} = \frac{C_1 C_2}{C_1 + C_2} = \frac{(4.00 \times 10^{-6} \text{ F})(4.00 \times 10^{-6} \text{ F})}{4.00 \times 10^{-6} \text{ F} + 4.00 \times 10^{-6} \text{ F}} = 2.00 \times 10^{-6} \text{ F}
\]

\( C_{12} \) and \( C_3 \) are in parallel and are equivalent to \( C_{123} \) (Figure 24.15c).

\[
C_{123} = C_{12} + C_3
\]

\[
C_{123} = 2.00 \times 10^{-6} \text{ F} + 4.00 \times 10^{-6} \text{ F}
\]

\[
C_{123} = 6.00 \times 10^{-6} \text{ F}
\]

\( C_{123} \) and \( C_4 \) are in series and are equivalent to \( C_{1234} \) (Figure 24.15d).

\[
\frac{1}{C_{1234}} = \frac{1}{C_{123}} + \frac{1}{C_4}
\]

\[
C_{1234} = \frac{C_{123} C_4}{C_{123} + C_4} = \frac{(6.00 \times 10^{-6} \text{ F})(4.00 \times 10^{-6} \text{ F})}{6.00 \times 10^{-6} \text{ F} + 4.00 \times 10^{-6} \text{ F}} = 2.40 \times 10^{-6} \text{ F}
\]

The circuit is equivalent to the circuit shown in Figure 24.15e.
\[ V_{1234} = V = 28.0 \text{ V} \]
\[ Q_{1234} = C_{1234}V = \left(2.40 \times 10^{-6} \text{ F}\right)\left(28.0 \text{ V}\right) = 67.2 \text{ }\mu\text{C} \]

**Figure 24.15e**

Now build back up the original circuit, step by step. \( C_{1234} \) represents \( C_{123} \) and \( C_4 \) in series (Figure 24.15f).

\[ Q_{123} = Q_4 - Q_{1234} = 67.2 \text{ }\mu\text{C} \]
(charge same for capacitors in series)

**Figure 24.15f**

Then \( V_{123} = \frac{Q_{123}}{C_{123}} = \frac{67.2 \text{ }\mu\text{C}}{6.00 \text{ }\mu\text{F}} = 11.2 \text{ V} \)

\[ V_4 = \frac{Q_4}{C_4} = \frac{67.2 \text{ }\mu\text{C}}{4.00 \text{ }\mu\text{F}} = 16.8 \text{ V} \]

Note that \( V_4 + V_{123} = 16.8 \text{ V} + 11.2 \text{ V} = 28.0 \text{ V} \), as it should.

Next consider the circuit as written in Figure 24.15g.

\[ V_3 = V_{12} = 28.0 \text{ V} - V_4 \]
\[ V_3 = 11.2 \text{ V} \]
\[ Q_3 = C_3V_3 = (4.00 \text{ }\mu\text{F})(11.2 \text{ V}) \]
\[ Q_3 = 44.8 \text{ }\mu\text{C} \]
\[ Q_{12} - C_{12}V_{12} = (2.00 \text{ }\mu\text{F})(11.2 \text{ V}) \]
\[ Q_{12} = 22.4 \text{ }\mu\text{C} \]

**Figure 24.15g**

Finally, consider the original circuit, as shown in Figure 24.15h.

\[ Q_1 = Q_2 = Q_{12} = 22.4 \text{ }\mu\text{C} \]
(charge same for capacitors in series)

\[ V_1 = \frac{Q_1}{C_1} = \frac{22.4 \text{ }\mu\text{C}}{4.00 \text{ }\mu\text{F}} = 5.6 \text{ V} \]
\[ V_2 = \frac{Q_2}{C_2} = \frac{22.4 \text{ }\mu\text{C}}{4.00 \text{ }\mu\text{F}} = 5.6 \text{ V} \]

**Figure 24.15h**

Note that \( V_1 + V_2 = 11.2 \text{ V} \), which equals \( V_3 \) as it should.

Summary: \( Q_3 = 22.4 \text{ }\mu\text{C}, V_3 = 5.6 \text{ V} \)
\( Q_2 = 22.4 \text{ }\mu\text{C}, V_2 = 5.6 \text{ V} \)
\( Q_3 = 44.8 \text{ }\mu\text{C}, V_3 = 11.2 \text{ V} \)
\( Q_4 = 67.2 \text{ }\mu\text{C}, V_4 = 16.8 \text{ V} \)

(c) \( V_{ad} = V_3 = 11.2 \text{ V} \)

**EVALUATE:** \( V_1 + V_2 + V_4 = V \), or \( V_2 + V_4 = V \). \( Q_1 = Q_2, Q_1 + Q_3 = Q_4 \) and \( Q_4 = Q_{1234} \).
24.18. **IDENTIFY:** For capacitors in parallel the voltages are the same and the charges add. For capacitors in series, the charges are the same and the voltages add. \( C = \frac{Q}{V} \).

**SET UP:** \( C_1 \) and \( C_2 \) are in parallel and \( C_3 \) is in series with the parallel combination of \( C_1 \) and \( C_2 \).

**EXECUTE:** (a) \( C_1 \) and \( C_2 \) are in parallel and so have the same potential across them:

\[
V_1 = V_2 = \frac{Q}{C_2} = \frac{40.0 \times 10^{-6} \text{ C}}{3.00 \times 10^{-6} \text{ F}} = 13.33 \text{ V}.
\]

Therefore, \( Q_1 = V_1 C_1 = (13.33 \text{ V})(3.00 \times 10^{-6} \text{ F}) = 80.0 \times 10^{-6} \text{ C} \). Since \( C_3 \) is in series with the parallel combination of \( C_1 \) and \( C_2 \), its charge must be equal to their combined charge:

\[
C_3 = 40.0 \times 10^{-6} \text{ C} + 80.0 \times 10^{-6} \text{ C} = 120.0 \times 10^{-6} \text{ C}.
\]

(b) The total capacitance is found from:

\[
\frac{1}{C_{\text{tot}}} = \frac{1}{C_{12}} + \frac{1}{C_3} = \frac{1}{9.00 \times 10^{-6} \text{ F}} + \frac{1}{5.00 \times 10^{-6} \text{ F}}
\]

\[
C_{\text{tot}} = 3.21 \mu \text{F}.
\]

\[
V_{\text{ab}} = \frac{Q_{\text{tot}}}{C_{\text{tot}}} = \frac{120.0 \times 10^{-6} \text{ C}}{3.21 \times 10^{-6} \text{ F}} = 37.4 \text{ V}.
\]

**EVALUATE:**

\[
V_3 = \frac{Q}{C_3} = \frac{120.0 \times 10^{-6} \text{ C}}{5.00 \times 10^{-6} \text{ F}} = 24.0 \text{ V}.
\]

\[
V_{\text{ab}} = V_1 + V_3.
\]

24.22. **IDENTIFY:** Simplify the network by replacing series and parallel combinations of capacitors by their equivalents.

**SET UP:** For capacitors in series the voltages add and the charges are the same; \( \frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots \)

For capacitors in parallel the voltages are the same and the charges add; \( C_{\text{eq}} = C_1 + C_2 + \cdots \ C = \frac{Q}{V} \).

**EXECUTE:** (a) The equivalent capacitance of the 5.0 \( \mu \text{F} \) and 8.0 \( \mu \text{F} \) capacitors in parallel is 13.0 \( \mu \text{F} \). When these two capacitors are replaced by their equivalent we get the network sketched in Figure 24.22. The equivalent capacitance of these three capacitors in series is 3.47 \( \mu \text{F} \).

(b) \( Q_{\text{tot}} = C_{\text{tot}} V = (3.47 \ \mu \text{F})(50.0 \ \text{V}) = 174 \mu \text{C} \)

(c) \( Q_{\text{tot}} \) is the same as \( Q \) for each of the capacitors in the series combination shown in Figure 24.22, so \( Q \) for each of the capacitors is 174 \( \mu \text{C} \).

**EVALUATE:** The voltages across each capacitor in Figure 24.22 are \( V_{10} = \frac{Q_{\text{tot}}}{C_{10}} = 17.4 \text{ V} \),

\[
V_{13} = \frac{Q}{C_{13}} = 13.4 \text{ V} \quad \text{and} \quad V_9 = \frac{Q}{C_9} = 19.3 \text{ V}.
\]

\[
V_{10} + V_{13} + V_9 = 17.4 \text{ V} + 13.4 \text{ V} + 19.3 \text{ V} = 50.1 \text{ V}.
\]

The sum of the voltages equals the applied voltage, apart from a small difference due to rounding.

![Figure 24.22](image)

24.3. **Stockage de l’énergie dans les condensateurs et énergie du champ**

- Eq. (24.9) donne l’énergie emmagasinée dans un condensateur. La preuve est contenue dans Eq. (24.8).
- Eq. (24.10) définit la densité d’énergie, donnée par Eq. (24.11).
- Eq. (24.11) est générale, pour tout champ \( \mathbf{E} \), même si elle n’a été obtenue que
pour un condensateur plan.

24.24. **IDENTIFY:** Apply \( C = Q / V \), \( C = \frac{P_q A}{d} \). The work done to double the separation equals the change in the stored energy.

**SET UP:** \( U = \frac{1}{2} CV^2 = \frac{Q^2}{2C} \).

**EXECUTE:**
(a) \( V = Q / C = (2.55 \mu C)/(920 \times 10^{-12} \text{ F}) = 2770 \text{ V} \)

(b) \( C = \frac{P_q A}{d} \) says that since the charge is kept constant while the separation doubles, that means that the capacitance halves and the voltage doubles to 5540 V.

(c) \( U = \frac{Q^2}{2C} = \frac{(2.55 \times 10^{-6} C)^2}{2(920 \times 10^{-12} \text{ F})} = 3.53 \times 10^{-3} \text{ J} \). When if the separation is doubled while \( Q \) stays the same, the capacitance halves, and the energy stored doubles. So the amount of work done to move the plates equals the difference in energy stored in the capacitor, which is \( 3.53 \times 10^{-3} \text{ J} \).

**EVALUATE:** The oppositely charged plates attract each other and positive work must be done by an external force to pull them farther apart.

24.28. **IDENTIFY:** After the two capacitors are connected they must have equal potential difference, and their combined charge must add up to the original charge.

**SET UP:** \( C = Q / V \). The stored energy is \( U = \frac{Q^2}{2C} = \frac{1}{2} CV^2 \).

**EXECUTE:**
(a) \( Q = CV_o \).

(b) \( V = \frac{Q}{C} = \frac{Q_1}{C_2} \) and also \( Q_1 + Q_2 = Q = CV_o \). \( C_1 = C \) and \( C_2 = \frac{C}{2} \) so \( \frac{Q_1}{C} = \frac{Q_2}{(C/2)} \) and \( Q_1 = \frac{Q_2}{2} \).

\[
Q = \frac{3}{2} Q_1, \quad Q_1 = \frac{2}{3} Q \quad \text{and} \quad V = \frac{Q_1}{C} = \frac{2}{3} \frac{Q}{C} = \frac{2}{3} V_o.
\]

(c) \( U = \frac{1}{2} \left( \frac{Q_1^2}{C_1} + \frac{Q_2^2}{C_2} \right) = \frac{1}{2} \left[ \frac{\left( \frac{3}{2} Q \right)^2}{C} + \frac{\left( \frac{2}{3} Q \right)^2}{C} \right] = \frac{1}{3} Q^2 \left( \frac{1}{C} + \frac{1}{C} \right) = \frac{1}{3} CV_o^2 \).

(d) The original \( U \) was \( U = \frac{1}{2} CV_o^2 \), so \( \Delta U = -\frac{1}{6} CV_o^2 \).

(e) Thermal energy of capacitor, wires, etc., and electromagnetic radiation.

**EVALUATE:** The original charge of the charged capacitor must distribute between the two capacitors to make the potential the same across each capacitor. The voltage \( V \) for each after they are connected is less than the original voltage \( V_o \) of the charged capacitor.

24.32. **IDENTIFY:** The two capacitors are in series. \( \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots \). \( C = \frac{Q}{V} \). \( U = \frac{1}{2} CV^2 \).

**SET UP:** For capacitors in series the voltages add and the charges are the same.

**EXECUTE:**
(a) \( \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} \) so \( C_{eq} = \frac{C_1 C_2}{C_1 + C_2} = \frac{(150 \text{ nF})(120 \text{ nF})}{150 \text{ nF} + 120 \text{ nF}} = 66.7 \text{ nF} \).

\( Q = CV = (66.7 \text{ nF})(36 \text{ V}) = 2.4 \times 10^{-6} \text{ C} = 2.4 \mu C \)

(b) \( Q = 2.4 \mu C \) for each capacitor.

(c) \( U = \frac{1}{2} C_{eq} V^2 = \frac{1}{2}(66.7 \times 10^{-9} \text{ F})(36 \text{ V})^2 = 43.2 \mu J \)
(d) We know $C$ and $Q$ for each capacitor so rewrite $U$ in terms of these quantities.

$$U = \frac{1}{2}CV^2 = \frac{1}{2}C(Q/C)^2 = \frac{Q^2}{2C}$$

150 nF: $U = \frac{(2.4 \times 10^{-6} \text{ C})^2}{2(150 \times 10^{-9} \text{ F})} = 19.2 \text{ } \mu\text{J}$ ; 120 nF: $U = \frac{(2.4 \times 10^{-6} \text{ C})^2}{2(120 \times 10^{-9} \text{ F})} = 24.0 \text{ } \mu\text{J}$

Note that 19.2 $\mu\text{J}$ + 24.0 $\mu\text{J}$ = 43.2 $\mu\text{J}$ , the total stored energy calculated in part (c).

(e) 150 nF: $V = \frac{Q}{C} = \frac{2.4 \times 10^{-6} \text{ C}}{150 \times 10^{-9} \text{ F}} = 16 \text{ } \text{V}$ ; 120 nF: $V = \frac{Q}{C} = \frac{2.4 \times 10^{-6} \text{ C}}{120 \times 10^{-9} \text{ F}} = 20 \text{ } \text{V}$

Note that these two voltages sum to 36 V, the voltage applied across the network.

**EVALUATE:** Since $Q$ is the same the capacitor with smaller $C$ stores more energy ($U = Q^2 / 2C$) and has a larger voltage ($V = Q / C$).

24.36. **IDENTIFY:** Apply Eq.(24.11).

**SET UP:** Example 24.3 shows that $E = \frac{Q}{4\pi\varepsilon_0 r^2}$ between the conducting shells and that

$$E = \left( \frac{r_r}{r_b} \right) V_{ab} \cdot$$

**EXECUTE:** $E = \left( \frac{r_r}{r_b} \right) \frac{V_{ab}}{r^2} = \frac{\left[ 0.125 \text{ m} \right] \left[ 0.148 \text{ m} \right]}{\left[ 0.148 \text{ m} - 0.125 \text{ m} \right]} \frac{120 \text{ V}}{r^2} = \frac{96.5 \text{ V} \cdot \text{m}}{r^2}$

(a) For $r = 0.126 \text{ m}$ , $E = 6.08 \times 10^3 \text{ V/m} \cdot u = \frac{1}{2} \varepsilon_0 E^2 = 1.64 \times 10^{-7} \text{ J/m}^3$.

(b) For $r = 0.147 \text{ m}$ , $E = 4.47 \times 10^3 \text{ V/m} \cdot u = \frac{1}{2} \varepsilon_0 E^2 = 8.85 \times 10^{-7} \text{ J/m}^3$.

**EVALUATE:** (c) No, the results of parts (a) and (b) show that the energy density is not uniform in the region between the plates. $E$ decreases as $r$ increases, so $u$ decreases also.

24.4. **Diaélectriques**

- Commencer par la Fig. 24.15, qui montre que la présence d’un matériau diaélectrique entre les plaques d’un condensateur réduit le champ électrique, tel que donné par Eq. (24.14).
- Le tableau 24.1 donne les constantes diaélectriques $K$ de divers matériaux.
- Eq. (24.17) décrit la permittivité.
- Eq. (24.20) décrit la densité d’énergie.

24.38. **IDENTIFY:** $V = Ed$ and $C = Q / V$ . With the dielectric present, $C = KC_0$.

**SET UP:** $V = Ed$ holds both with and without the dielectric.

**EXECUTE:** (a) $V = Ed = (3.00 \times 10^4 \text{ V/m})(1.50 \times 10^{-3} \text{ m}) = 45.0 \text{ V}$ .

$Q = C_0V = (5.00 \times 10^{-12} \text{ F})(45.0 \text{ V}) = 2.25 \times 10^{-10} \text{ C}$ .

(b) With the dielectric, $C = KC_0 = (2.70)(5.00 \text{ pF}) = 13.5 \text{ pF}$ . $V$ is still 45.0 V, so $Q = CV = (13.5 \times 10^{-12} \text{ F})(45.0 \text{ V}) = 6.08 \times 10^{-10} \text{ C}$ .

**EVALUATE:** The presence of the dielectric increases the amount of charge that can be stored for a given potential difference and electric field between the plates. $Q$ increases by a factor of $K$. 
24.40. **IDENTIFY:** Capacitance depends on geometry, and the introduction of a dielectric increases the capacitance.

**SET UP:** For a parallel-plate capacitor, \( C = K \varepsilon_o A/d \).

**EXECUTE:** (a) Solving for \( d \) gives

\[
d = \frac{K \varepsilon_o A}{C} = \frac{(3.0)(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(0.22 \text{ m})(0.28 \text{ m})}{1.0 \times 10^{-9} \text{ F}} = 1.64 \times 10^{-3} \text{ m} = 1.64 \text{ mm}.
\]

Dividing this result by the thickness of a sheet of paper gives \( \frac{1.64 \text{ mm}}{0.20 \text{ mm/sheet}} = 8 \) sheets.

(b) Solving for the area of the plates gives

\[
A = \frac{Cd}{K \varepsilon_o} = \frac{(1.0 \times 10^{-9} \text{ F})(0.012 \text{ m})}{(3.0)(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)} = 0.45 \text{ m}^2.
\]

(c) Teflon has a smaller dielectric constant (2.1) than the posterboard, so she will need more area to achieve the same capacitance.

**EVALUATE:** The use of dielectric makes it possible to construct reasonable-sized capacitors since the dielectric increases the capacitance by a factor of \( K \).

24.44. **IDENTIFY:** \( C = Q/V, C = K \varepsilon_o, V = Ed \).

**SET UP:** Table 24.1 gives \( K = 3.1 \) for mylar.

**EXECUTE:** (a) \( \Delta Q = Q - Q_o = (K - 1)Q_o = (K - 1)C \varepsilon_o V_o = (2.1)(2.5 \times 10^{-7} \text{ F})(12 \text{ V}) = 6.3 \times 10^{-6} \text{ C} \).

(b) \( \sigma = \alpha(1-1/K) \) so \( Q = (9.3 \times 10^{-6} \text{ C})(1-1/3.1) = 6.3 \times 10^{-6} \text{ C} \).

(c) The addition of the mylar doesn’t affect the electric field since the induced charge cancels the additional charge drawn to the plates.

**EVALUATE:** \( E = V/d \) and \( V \) is constant so \( E \) doesn't change when the dielectric is inserted.

24.46. **IDENTIFY:** \( C = K \varepsilon_o, C = Q/V, V = Ed \).

**SET UP:** Since the capacitor remains connected to the battery the potential between the plates of the capacitor doesn’t change.

**EXECUTE:** (a) The capacitance changes by a factor of \( K \) when the dielectric is inserted. Since \( V \) is unchanged (the battery is still connected), \( \frac{C_{\text{after}}}{C_{\text{before}}} = \frac{Q_{\text{after}}}{Q_{\text{before}}} = \frac{45.0 \text{ pC}}{25.0 \text{ pC}} = K = 1.80 \).

(b) The area of the plates is \( \pi r^2 = \pi(0.0300 \text{ m})^2 = 2.827 \times 10^{-3} \text{ m}^2 \) and the separation between them is thus \( d = \frac{P \varepsilon_o A}{C} = \frac{(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(2.827 \times 10^{-3} \text{ m}^2)}{12.5 \times 10^{-12} \text{ F}} = 2.00 \times 10^{-3} \text{ m} \). Before the dielectric is inserted, \( C = \frac{P \varepsilon_o A}{d} = \frac{Q}{V} \) and \( V = \frac{Qd}{P \varepsilon_o A} = \frac{(25.0 \times 10^{-12} \text{ C})(2.00 \times 10^{-3} \text{ m})}{(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(2.827 \times 10^{-3} \text{ m}^2)} = 2.00 \text{ V} \). The battery remains connected, so the potential difference is unchanged after the dielectric is inserted.

(c) Before the dielectric is inserted, \( E = \frac{Q}{P \varepsilon_o A} = \frac{25.0 \times 10^{-12} \text{ C}}{(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(2.827 \times 10^{-3} \text{ m}^2)} = 1000 \text{ N/C} \)

Again, since the voltage is unchanged after the dielectric is inserted, the electric field is also unchanged.

**EVALUATE:** \( E = \frac{V}{d} = \frac{2.00 \text{ V}}{2.00 \times 10^{-3} \text{ m}} = 1000 \text{ N/C} \), whether or not the dielectric is present. This agrees with the result in part (c). The electric field has this value at any point between the plates. We need \( d \) to calculate \( E \) because \( V \) is the potential difference between points separated by distance \( d \).