Faculty of Engineering and Department of Physics

Engineering Physics 131

Midterm Examination

Monday February 27, 2012; 7:00 pm – 8:30 pm

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1. No notes or textbooks allowed.
2. Formula sheets are included (may be removed).
3. The exam has **7** problems and is out of **50 points**. Attempt all parts of all problems.
4. Show all work in a neat and logical manner. Questions 1 to 3 do not require detailed calculations and only the final answers to these questions will be marked. For Questions 4 to 7, details and procedures to solve these problems will be marked.
5. Write your solution directly on the pages with the questions. Indicate clearly if you use the backs of pages for material to be marked.
6. Non-programmable calculator allowed. Turn off all cell-phones, laptops, etc.

DO NOT separate the pages of the exam containing the problems.

LAST NAME: ______________________________________

FIRST NAME: ______________________________________

ID#: ____________________

Please circle the name of your instructor:

B01: Chow

B02: Fenrich

B03: Schiavone

B04: Lavoie

B05: Wheelock

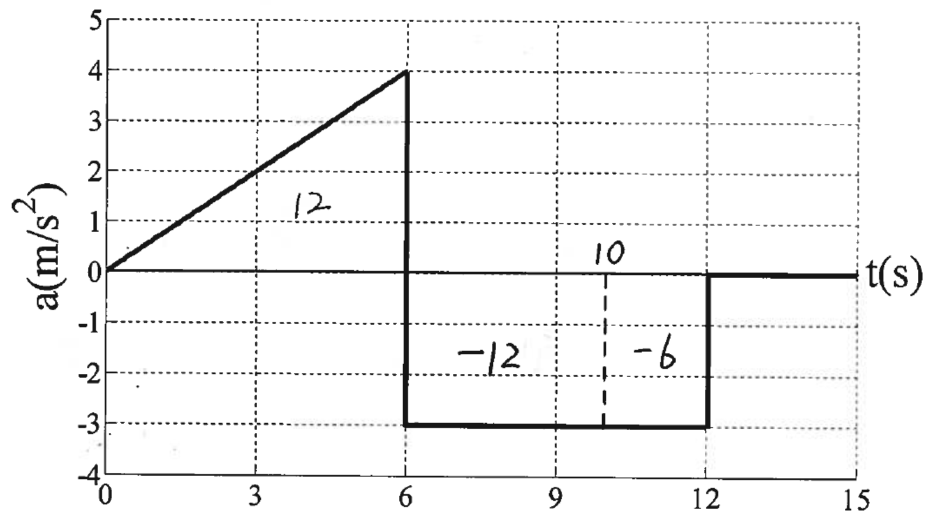
B06: Tang

Please do not write in the table below.

Question	**Value (Points)**	**Mark**
1	5	
2	4	
3	6	
4	10	
5	9	
6	9	
7	7	
Total	50	

1. [5 Points]

A particle starts from rest at $t = 0$ and undergoes an acceleration as shown in the figure below. Answer the questions given below.



(a) At what time does the velocity reach its maximum and what is the velocity at this time?

Answer: $t = 6s$ $v = 12$ m/s

(b) What is the velocity of the particle at $t = 12$ s?

Answer: -6 m/s

(c) What is the average acceleration of the particle during the time interval $t = 0$ s to $t = 12$ s?

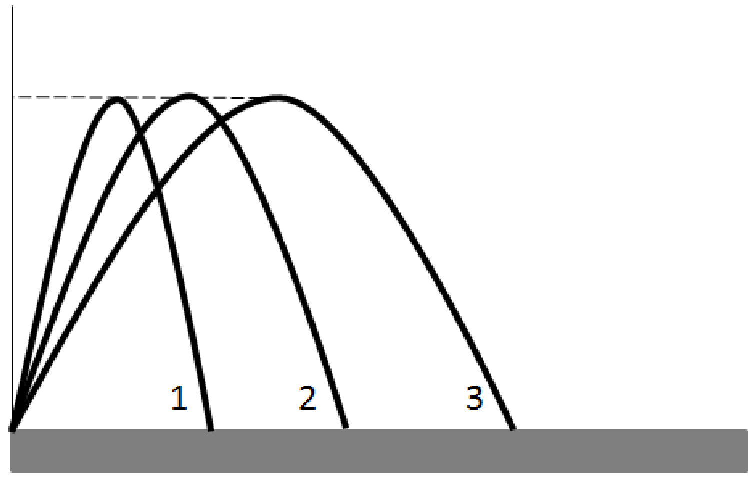
Answer: -0.5 m/s^2

(d) At what time during the $t = 0$ s to $t = 12$ s interval does the particle reach its maximum displacement?

Answer: 10 s

2. [4 Points]

Three projectiles are launched from the same point and follow the paths indicated below, reaching the same maximum height.



(a) At the top of each path, rank the paths according to the following. Rank the paths from greatest to least, and indicate any ties.

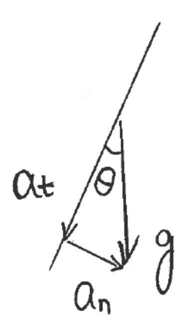
(i) The magnitude of the tangential acceleration. Answer: $1 = 2 = 3 \ (= 0)$

(ii) The magnitude of the normal acceleration. Answer: $1 = 2 = 3 \ (= g)$

(b) When each particle is at half its maximum height, rank the paths according to the following. Rank the paths from greatest to least, and indicate any ties.

(i) The magnitude of the tangential acceleration. Answer: $1 > 2 > 3$

(ii) The magnitude of the normal acceleration. Answer: $3 > 2 > 1$

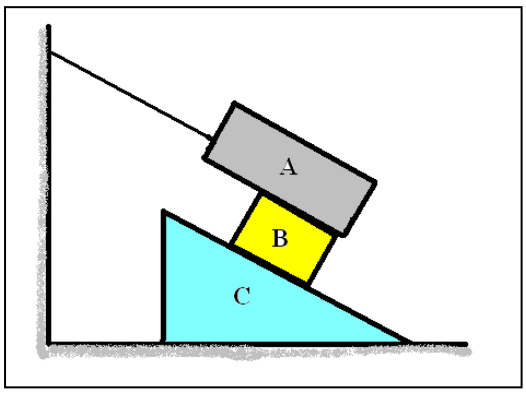


$a_t = g\cos\theta$

$a_n = g\sin\theta$

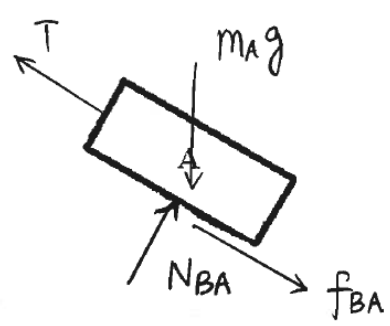
3. [6 Points]

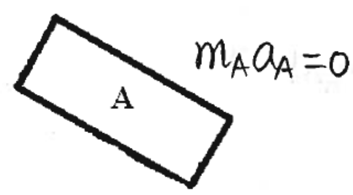
Block A is held by a rope to a wall and it is static. It rests on top of Block B which is sliding down the inclined wedge C. The wedge C is sliding on the floor because of the motion of B. All the surfaces (i.e. A, B, C and the floor) are rough. Draw the free body diagram and the kinetic diagram for Block A, Block B, and wedge C.
[Be sure to clearly indicate the **correct directions** of **any vector** on each diagram. All symbols shown in your diagrams are assumed to take on positive values.]

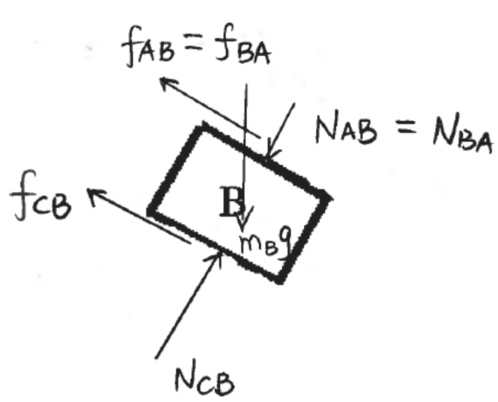


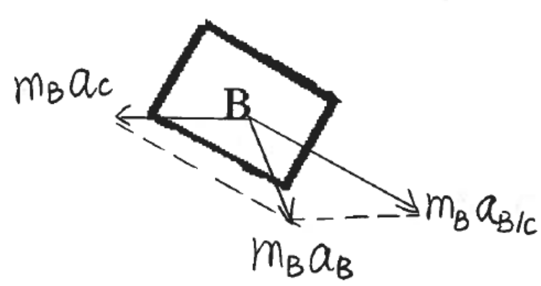
Free Body Diagrams:

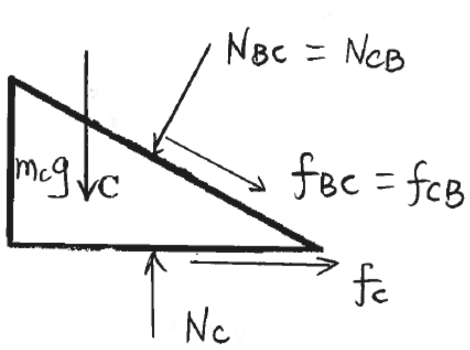
Kinetic Diagrams:

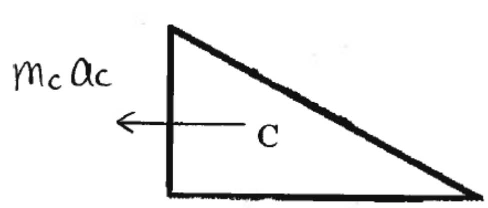












4. [10 Points]

A dragster starts from rest and travels along a straight track with an acceleration given by the following:

$0 \le s \le 75$: $\quad a = 0.01s^2$

$75 \le s \le s'$: $\quad a = 75 - s$

where s is in m and a is in m/s^2.

Construct the v-s graph for $0 \le s \le s'$, and determine the total distance s' traveled before the dragster again comes to rest.

$0 \le s \le 75$:

$$a\,ds = v\,dv$$

$$\Rightarrow \quad 0.01 s^2 ds = v\,dv$$

$$\Rightarrow \quad \int_0^s 0.01 s^2 ds = \int_0^v v\,dv$$

$$\Rightarrow \quad \frac{0.01}{3} s^3 = \frac{1}{2} v^2$$

$$\Rightarrow \quad v = \sqrt{\frac{0.02}{3}}\, s^{3/2} = 0.0816\, s^{3/2}$$

$$v(s=75) = 53.0 \quad m/s$$

$75 \le s \le s'$

$$a\,ds = v\,dv$$

$$\Rightarrow \quad (75 - s)\, ds = v\,dv$$

$$\Rightarrow \quad \int_{75}^{s} (75 - s)\, ds = \int_{53.0}^{v} dv$$

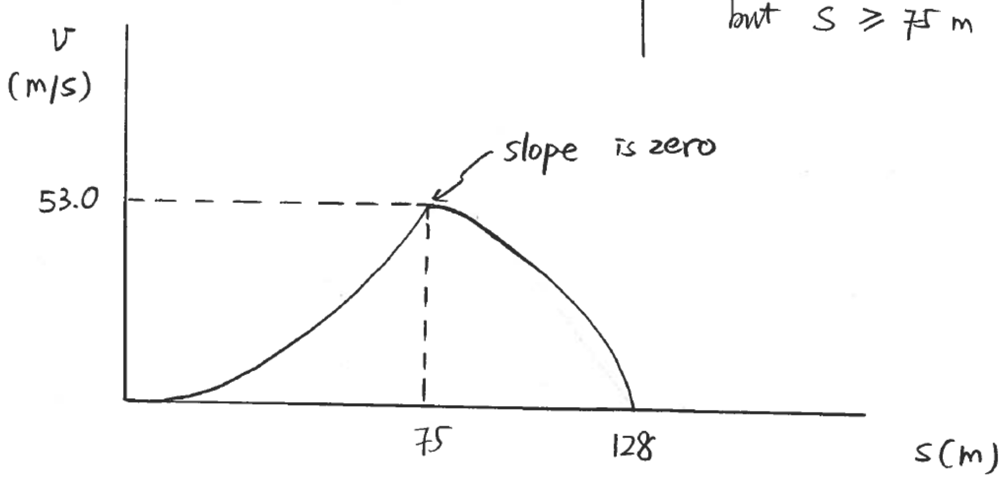
$$\Rightarrow \quad -\frac{1}{2}(75-s)^2 = \frac{1}{2}(v^2 - 53.0^2)$$

$$\Rightarrow \quad v = \sqrt{2809 - (75-s)^2}$$

$$v = 0 \quad \Rightarrow \quad 75 - s = \pm 53.0$$

$$\Rightarrow \quad s = 22 \text{ m} \quad \text{or} \quad 128 \text{ m}$$

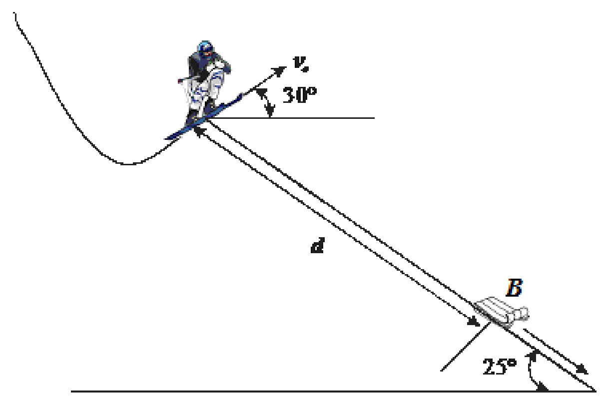
but $s \ge 75$ m $\Rightarrow$ $s = 128$ m



5. [9 Points]

A skier leaves a ski jump with an initial velocity of v_o at an angle of 30° above the horizontal. After 2.75 seconds he lands on the toboggan at point B, a distance *d* down the inclined plane.

(a) Find the initial speed v_o of the skier and the distance *d* down the inclined plane.
(b) If the toboggan has a velocity of 18 m/s directed down the incline just before the skier lands on it, determine the velocity of the skier relative to the toboggan at this instant.



(a) motion of the skier:

$$d\cos 25^o = v_o \cos 30^o (2.75) \qquad (1)$$

$$-d\sin 25^o = v_o \sin 30^o (2.75) - \frac{1}{2} g (2.75)^2 \qquad (2)$$

Ratio of (2) to (1) gives $-v_o \cos 30^o \tan 25^o = v_o \sin 30^o - \frac{1}{2} g (2.75)$ $\Rightarrow v_o = 14.9 m/s$

Substitute the result into (1) to get $d = 39.2m$

(b) Velocity of the toboggan: $\vec{v}_t = 18\left(\cos 25^o \vec{i} - \sin 25^o \vec{j}\right) = 16.314\vec{i} - 7.607\vec{j} \quad m/s$

Velocity of the skier when he lands:

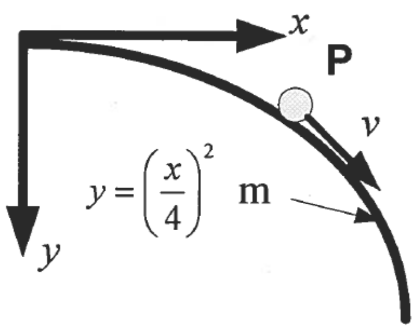
$$v_x = v_o \cos 30^o = 12.904\ m/s$$
$$v_y = v_o \sin 30^o - g(2.75) = -19.528 m/s$$
$$\Rightarrow \vec{v}_s = 12.904\vec{i} - 19.528\vec{j} \quad m/s$$

$$\Rightarrow \vec{v}_{s/t} = \vec{v}_s - \vec{v}_t = -3.41\vec{i} - 11.9\vec{j} \quad m/s$$

6. [9 Points]

Particle P moves down the hill with speed given by $v = \sqrt{2gy}$, where g = 9.81 m/s^2 and y is in m. When x = 4 m determine

(a) the normal component of the acceleration, and
(b) the tangential component of the acceleration.



(a) $v = \sqrt{2gy} = \frac{\sqrt{2g}}{4} x$

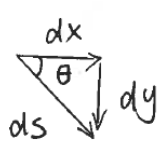
$\Rightarrow$ at $x = 4$ m, $v = 4.43$ m/s

$a_n = \frac{v^2}{\rho}$ $\qquad \rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|}$

$\frac{dy}{dx}\Big|_{x=4} = \frac{x}{8}\Big|_{x=4} = 0.5$ $\qquad \frac{d^2y}{dx^2}\Big|_{x=4} = \frac{1}{8} = 0.125$

$\Rightarrow \rho = 11.18$ m $\qquad \Rightarrow a_n = \frac{4.43^2}{11.18} = 1.76$ m/s^2

(b) $a_t = v\frac{dv}{ds} = \frac{\sqrt{2g}}{4} x \cdot \frac{\sqrt{2g}}{4} \frac{dx}{ds} = \frac{g}{8} x \frac{dx}{ds}$



$\tan\theta = \frac{dy}{dx} = 0.5 \Rightarrow \theta = 26.57°$

$\Rightarrow \frac{dx}{ds} = \cos\theta = 0.894$

$\Rightarrow a_t = \frac{9.81}{8}(4)(0.894) = 4.39$ m/s^2

Alternate solution for part (b):

$$a_t = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{\sqrt{2g}}{4}x\right) = \frac{\sqrt{2g}}{4}\frac{dx}{dt} = \frac{\sqrt{2g}}{4}v_x = \frac{\sqrt{2g}}{4}v\cos\theta$$

$$= \frac{\sqrt{2g}}{4}\frac{\sqrt{2g}}{4}x\cos\theta = \frac{9.81}{8}(4)\cos 26.57^o = 4.39\, m/s^2$$

7. [7 Points]

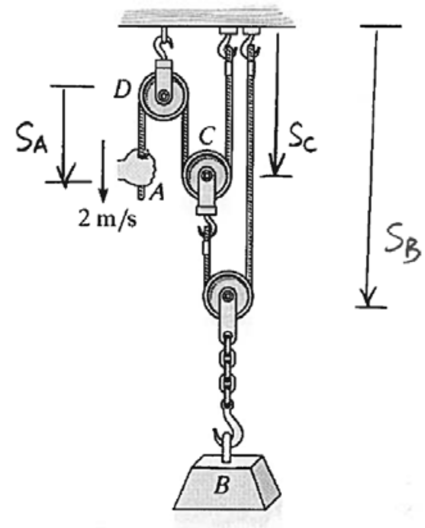
In the pulley system shown below,

(a) If the rope is pulled downward at point A with a constant speed of 2 m/s, what is the velocity of pulley C? Clearly specify its magnitude and direction.

$S_A + 2S_C = l_1$

$\Rightarrow \quad v_C = -\frac{1}{2} v_A = -1 \text{ m/s},$

i.e., 1 m/s upward



(b) Now consider that the system is initially at rest, and then the rope is pulled downward with a speed of 0.5*t* m/s (where *t* is in seconds). How long does it take to raise block B through a vertical distance of 2 m?

$S_B + S_B - S_C = l_2 \quad \Rightarrow \quad v_B = \frac{1}{2} v_C = -\frac{1}{4} v_A = -\frac{1}{8} t$

$\Rightarrow \quad \frac{dS_B}{dt} = -\frac{1}{8} t$

$\Rightarrow \quad -\frac{1}{8} t \, dt = dS_B$

$\Rightarrow \quad -\frac{1}{8} \int_0^t t \, dt = \int_0^{-2} dS_B$

$\Rightarrow \quad -\frac{1}{16} t^2 = -2 \quad \Rightarrow \quad t = 5.66 s$

Fundamental Equations of Dynamics

KINEMATICS

Particle Rectilinear Motion

Variable a	*Constant $a = a_c$*
$a = \dfrac{dv}{dt}$	$v = v_0 + a_c t$
$v = \dfrac{ds}{dt}$	$s = s_0 + v_0 t + \frac{1}{2}a_c t^2$
$a\,ds = v\,dv$	$v^2 = v_0^2 + 2a_c(s - s_0)$

Particle Curvilinear Motion

x, y, z Coordinates		*r, θ, z Coordinates*	
$v_x = \dot{x}$	$a_x = \ddot{x}$	$v_r = \dot{r}$	$a_r = \ddot{r} - r\dot{\theta}^2$
$v_y = \dot{y}$	$a_y = \ddot{y}$	$v_\theta = r\dot{\theta}$	$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$
$v_z = \dot{z}$	$a_z = \ddot{z}$	$v_z = \dot{z}$	$a_z = \ddot{z}$

n, t, b Coordinates

$v = \dot{s}$	$a_t = \dot{v} = v\dfrac{dv}{ds}$
	$a_n = \dfrac{v^2}{\rho} \quad \rho = \dfrac{[1 + (dy/dx)^2]^{3/2}}{\lvert d^2y/dx^2 \rvert}$

Relative Motion

$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A} \quad \mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$

Rigid Body Motion About a Fixed Axis

Variable $\mathbf{a}$	*Constant* $\mathbf{a} = \mathbf{a}_c$
$\alpha = \dfrac{d\omega}{dt}$	$\omega = \omega_0 + \alpha_c t$
$\omega = \dfrac{d\theta}{dt}$	$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha_c t^2$
$\omega\,d\omega = \alpha\,d\theta$	$\omega^2 = \omega_0^2 + 2\alpha_c(\theta - \theta_0)$

For Point P

$s = \theta r \quad v = \omega r \quad a_t = \alpha r \quad a_n = \omega^2 r$

Relative General Plane Motion—Translating Axes

$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A(\text{pin})} \quad \mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A(\text{pin})}$

Relative General Plane Motion—Trans. and Rot. Axis

$\mathbf{v}_B = \mathbf{v}_A + \mathbf{\Omega} \times \mathbf{r}_{B/A} + (\mathbf{v}_{B/A})_{xyz}$

$\mathbf{a}_B = \mathbf{a}_A + \dot{\mathbf{\Omega}} \times \mathbf{r}_{B/A} + \mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{r}_{B/A}) + 2\mathbf{\Omega} \times (\mathbf{v}_{B/A})_{xyz} + (\mathbf{a}_{B/A})_{xyz}$

KINETICS

Mass Moment of Inertia $I = \int r^2\,dm$

Parallel-Axis Theorem $I = I_G + md^2$

Radius of Gyration $k = \sqrt{\dfrac{I}{m}}$

Equations of Motion

Particle	$\Sigma\mathbf{F} = m\mathbf{a}$
Rigid Body (Plane Motion)	$\Sigma F_x = m(a_G)_x$ $\Sigma F_y = m(a_G)_y$ $\Sigma M_G = I_G\alpha$ or $\Sigma M_P = \Sigma(\mathcal{M}_k)_P$

Principle of Work and Energy

$T_1 + U_{1-2} = T_2$

Kinetic Energy

Particle	$T = \frac{1}{2}mv^2$
Rigid Body (Plane Motion)	$T = \frac{1}{2}mv_G^2 + \frac{1}{2}I_G\omega^2$

Work

Variable force	$U_F = \int F\cos\theta\,ds$
Constant force	$U_F = (F_c\cos\theta)\,\Delta s$
Weight	$U_W = -W\,\Delta y$
Spring	$U_s = -(\frac{1}{2}ks_2^2 - \frac{1}{2}ks_1^2)$
Couple moment	$U_M = M\,\Delta\theta$

Power and Efficiency

$P = \dfrac{dU}{dt} = \mathbf{F}\cdot\mathbf{v} \quad \epsilon = \dfrac{P_{\text{out}}}{P_{\text{in}}} = \dfrac{U_{\text{out}}}{U_{\text{in}}}$

Conservation of Energy Theorem

$T_1 + V_1 = T_2 + V_2$

Potential Energy

$V = V_g + V_e$, where $V_g = \pm Wy$, $V_e = +\frac{1}{2}ks^2$

Principle of Linear Impulse and Momentum

Particle	$m\mathbf{v}_1 + \Sigma\int \mathbf{F}\,dt = m\mathbf{v}_2$
Rigid Body	$m(\mathbf{v}_G)_1 + \Sigma\int \mathbf{F}\,dt = m(\mathbf{v}_G)_2$

Conservation of Linear Momentum

$\Sigma(\text{syst. } m\mathbf{v})_1 = \Sigma(\text{syst. } m\mathbf{v})_2$

Coefficient of Restitution $e = \dfrac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$

Principle of Angular Impulse and Momentum

Particle	$(\mathbf{H}_O)_1 + \Sigma\int \mathbf{M}_O\,dt = (\mathbf{H}_O)_2$ where $H_O = (d)(mv)$
Rigid Body (Plane motion)	$(\mathbf{H}_G)_1 + \Sigma\int \mathbf{M}_G\,dt = (\mathbf{H}_G)_2$ where $H_G = I_G\omega$ $(\mathbf{H}_O)_1 + \Sigma\int \mathbf{M}_O\,dt = (\mathbf{H}_O)_2$ where $H_O = I_O\omega$

Conservation of Angular Momentum

$\Sigma(\text{syst. } \mathbf{H})_1 = \Sigma(\text{syst. } \mathbf{H})_2$

APPENDIX A

Mathematical Expressions

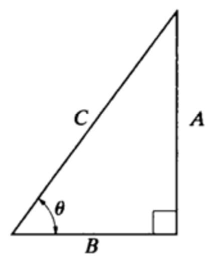
Quadratic Formula

If $ax^2 + bx + c = 0$, then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Hyperbolic Functions

$\sinh x = \dfrac{e^x - e^{-x}}{2}, \cosh x = \dfrac{e^x + e^{-x}}{2}, \tanh x = \dfrac{\sinh x}{\cosh x}$

Trigonometric Identities



$\sin\theta = \dfrac{A}{C}, \csc\theta = \dfrac{C}{A}$

$\cos\theta = \dfrac{B}{C}, \sec\theta = \dfrac{C}{B}$

$\tan\theta = \dfrac{A}{B}, \cot\theta = \dfrac{B}{A}$

$\sin^2\theta + \cos^2\theta = 1$

$\sin(\theta \pm \phi) = \sin\theta\cos\phi \pm \cos\theta\sin\phi$

$\sin 2\theta = 2\sin\theta\cos\theta$

$\cos(\theta \pm \phi) = \cos\theta\cos\phi \mp \sin\theta\sin\phi$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$\cos\theta = \pm\sqrt{\dfrac{1 + \cos 2\theta}{2}}, \sin\theta = \pm\sqrt{\dfrac{1 - \cos 2\theta}{2}}$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

$1 + \tan^2\theta = \sec^2\theta \qquad 1 + \cot^2\theta = \csc^2\theta$

Power-Series Expansions

$\sin x = x - \dfrac{x^3}{3!} + \cdots \qquad \sinh x = x + \dfrac{x^3}{3!} + \cdots$

$\cos x = 1 - \dfrac{x^2}{2!} + \cdots \qquad \cosh x = 1 + \dfrac{x^2}{2!} + \cdots$

Derivatives

$$\frac{d}{dx}(u^n) = nu^{n-1}\frac{du}{dx}$$

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$$

$$\frac{d}{dx}(\cot u) = -\csc^2 u\frac{du}{dx}$$

$$\frac{d}{dx}(\sec u) = \tan u \sec u\frac{du}{dx}$$

$$\frac{d}{dx}(\csc u) = -\csc u \cot u\frac{du}{dx}$$

$$\frac{d}{dx}(\sin u) = \cos u\frac{du}{dx}$$

$$\frac{d}{dx}(\cos u) = -\sin u\frac{du}{dx}$$

$$\frac{d}{dx}(\tan u) = \sec^2 u\frac{du}{dx}$$

$$\frac{d}{dx}(\sinh u) = \cosh u\frac{du}{dx}$$

$$\frac{d}{dx}(\cosh u) = \sinh u\frac{du}{dx}$$

Integrals

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C,\ n \neq -1$$

$$\int \frac{dx}{a+bx} = \frac{1}{b}\ln(a+bx) + C$$

$$\int \frac{dx}{a+bx^2} = \frac{1}{2\sqrt{-ba}}\ln\left[\frac{a+x\sqrt{-ab}}{a-x\sqrt{-ab}}\right] + C,\ ab < 0$$

$$\int \frac{x\,dx}{a+bx^2} = \frac{1}{2b}\ln(bx^2+a) + C,$$

$$\int \frac{x^2\,dx}{a+bx^2} = \frac{x}{b} - \frac{a}{b\sqrt{ab}}\tan^{-1}\frac{x\sqrt{ab}}{a} + C,\ ab > 0$$

$$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left[\frac{a+x}{a-x}\right] + C,\ a^2 > x^2$$

$$\int \sqrt{a+bx}\,dx = \frac{2}{3b}\sqrt{(a+bx)^3} + C$$

$$\int x\sqrt{a+bx}\,dx = \frac{-2(2a-3bx)\sqrt{(a+bx)^3}}{15b^2} + C$$

$$\int x^2\sqrt{a+bx}\,dx = \frac{2(8a^2-12abx+15b^2x^2)\sqrt{(a+bx)^3}}{105b^3} + C$$

$$\int \sqrt{a^2-x^2}\,dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\sin^{-1}\frac{x}{a}\right] + C,\ a > 0$$

$$\int x\sqrt{x^2 \pm a^2}\,dx = \frac{1}{3}\sqrt{(x^2 \pm a^2)^3} + C$$

$$\int x^2\sqrt{a^2-x^2}\,dx = -\frac{x}{4}\sqrt{(a^2-x^2)^3} + \frac{a^2}{8}\left(x\sqrt{a^2-x^2} + a^2\sin^{-1}\frac{x}{a}\right) + C,\ a > 0$$

$$\int \sqrt{x^2 \pm a^2}\,dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2\ln\left(x+\sqrt{x^2 \pm a^2}\right)\right] + C$$

$$\int x\sqrt{a^2-x^2}\,dx = -\frac{1}{3}\sqrt{(a^2-x^2)^3} + C$$

$$\int x^2\sqrt{x^2 \pm a^2}\,dx = \frac{x}{4}\sqrt{(x^2 \pm a^2)^3} \mp \frac{a^2}{8}x\sqrt{x^2 \pm a^2} - \frac{a^4}{8}\ln\left(x+\sqrt{x^2 \pm a^2}\right) + C$$

$$\int \frac{dx}{\sqrt{a+bx}} = \frac{2\sqrt{a+bx}}{b} + C$$

$$\int \frac{x\,dx}{\sqrt{x^2 \pm a^2}} = \sqrt{x^2 \pm a^2} + C$$

$$\int \frac{dx}{\sqrt{a+bx+cx^2}} = \frac{1}{\sqrt{c}}\ln\left[\sqrt{a+bx+cx^2} + x\sqrt{c} + \frac{b}{2\sqrt{c}}\right] + C,\ c > 0$$

$$= \frac{1}{\sqrt{-c}}\sin^{-1}\left(\frac{-2cx-b}{\sqrt{b^2-4ac}}\right) + C,\ c > 0$$

$$\int \sin x\,dx = -\cos x + C$$

$$\int \cos x\,dx = \sin x + C$$

$$\int x\cos(ax)\,dx = \frac{1}{a^2}\cos(ax) + \frac{x}{a}\sin(ax) + C$$

$$\int x^2\cos(ax)\,dx = \frac{2x}{a^2}\cos(ax) + \frac{a^2x^2-2}{a^3}\sin(ax) + C$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax} + C$$

$$\int xe^{ax}\,dx = \frac{e^{ax}}{a^2}(ax-1) + C$$

$$\int \sinh x\,dx = \cosh x + C$$

$$\int \cosh x\,dx = \sinh x + C$$