EN PH 131 Final Review

From ESSC

Created from Stefan Damkjar’s Winter 2012 Notes

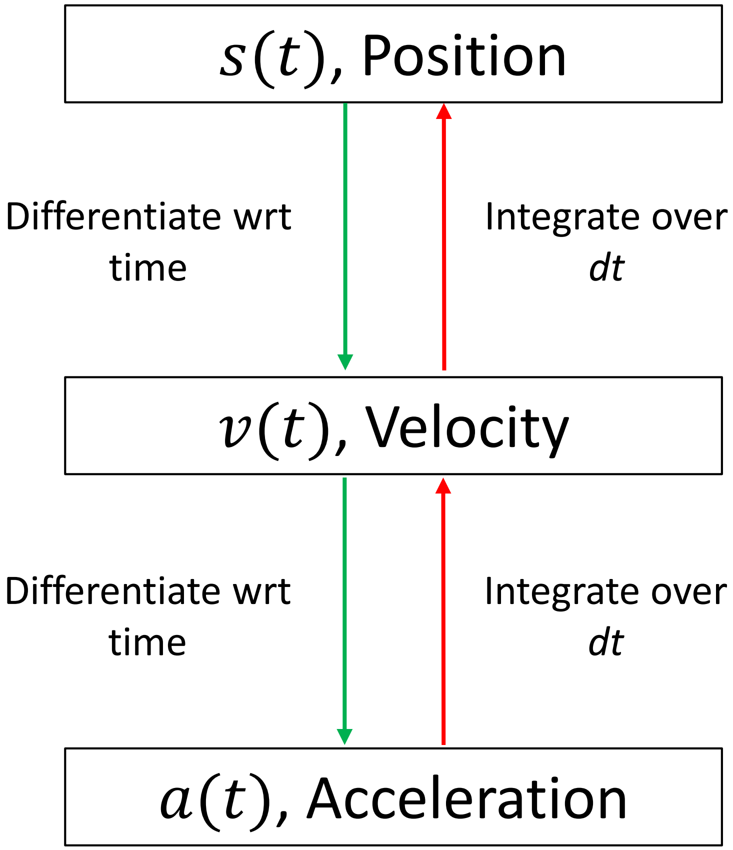
Chapter 2

Kinematics of a particle

Rectilinear Motion

- Particle travels along a straight path
- Examples
  - Car traveling in one direction
  - Tossing a ball upwards
  - Falling object

Rectilinear Motion - Fundamentals



Other relationship

$$ads = vdv$$

Rectilinear Motion – Constant Acceleration

1. $v = v_0 + a_c t$
2. $s = s_0 + v_0 t + \frac{1}{2} a_c t^2$
3. $v^2 = v_0^2 + 2a_c(s - s_0)$
4. $s - s_0 = \frac{v + v_0}{2} t$

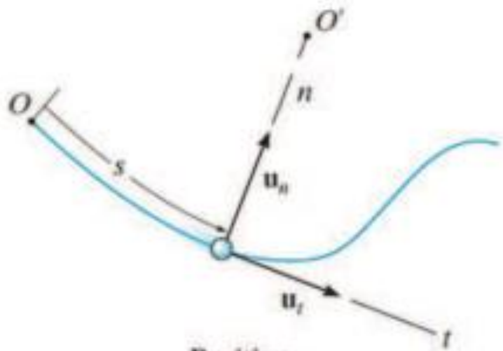
Curvilinear Motion

- Particle moves along a curve
- Examples
  - Car driving around a bend
  - Projectile
- Common coordinate systems
  - Horizontal and vertical direction ($\boldsymbol{i}$ and $\boldsymbol{j}$)
  - Tangential and normal direction ($\boldsymbol{n}$ and $\boldsymbol{t}$)

Curvilinear Motion

- Velocity vector **only** has tangential component
- Acceleration vector can have **both** tangential and normal component
  - Tangential: correspond to change in speed
    - $a_t = \dot{v}$
  - Normal: correspond to change in direction
    - $a_n = v^2/\rho$

$$\rho = \frac{\left(1 + \left(\frac{d}{dx}(y)\right)^2\right)^{\frac{3}{2}}}{\left|\frac{d^2}{dx^2}(y)\right|}$$



Position

Dependent Motion

- Motion of multiple objects are related
- Examples:
  - System of pulleys
  - Connected blocks

Dependent Motion - Procedure

1. Define positions relative to a fixed datum line
    - Different datum lines can be used for each particle
2. Relate positions to cord length
    - Unchanged segments can be ignored
    - Separate equations for each cord
3. Differentiate to relate velocities and acceleration

Relative Motion

- $\vec{\boldsymbol{r}}_B$ and $\vec{\boldsymbol{r}}_A$ are position vectors in the perspective of a fixed observer
- $\vec{\boldsymbol{r}}_{B|A}$ is the position vector of *B* as seen from *A*

$$\vec{\boldsymbol{r}}_{B|A} = \vec{\boldsymbol{r}}_B - \vec{\boldsymbol{r}}_A$$

$$\vec{\boldsymbol{v}}_{B|A} = \vec{\boldsymbol{v}}_B - \vec{\boldsymbol{v}}_A$$

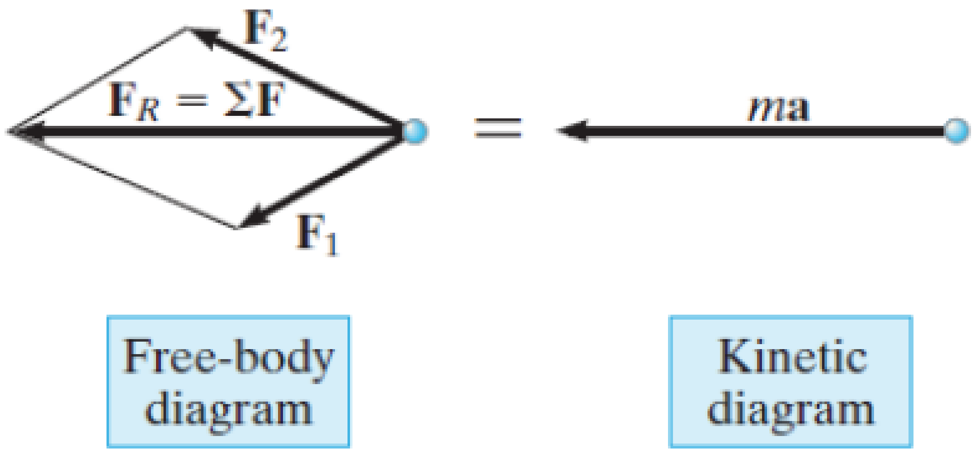
$$\vec{\boldsymbol{a}}_{B|A} = \vec{\boldsymbol{a}}_B - \vec{\boldsymbol{a}}_A$$

Chapter 3

Kinetics of a particle

Kinetics of a particle

- Cause of motion to external forces
- Newton's second law: $\sum \vec{F} = \vec{F}_R = m\vec{a}$
- Valid for an inertial frame of reference
  - observer is fixed or moving at a constant speed in a straight line



Kinetics of a particle - Procedure

1. Select appropriate inertial coordinate system (x-y, normal-tangential, cylindrical)
2. Draw free body diagram (FBD) of all external forces
   - Weight, normal forces, friction forces, and other applied forces
   - Friction forces act opposite to direction of motion
3. Draw kinetic diagram (KD) with its inertial force $m\vec{a}$
4. Solve the equation of motion $\sum \vec{F} = m\vec{a}$
5. Use kinematics (Chapter 2) to find velocity and position from acceleration if necessary

Chapter 4

Work and Energy

Work

- A force does work on a particle if the particle moves along the line of action
- Work is positive if the force is in the same direction as displacement
  - Moving a particle with weight W results in negative work

$$U\Big|_a^b = \int_{r_a}^{r_b} \vec{F} \cdot d\vec{r}$$

Principle of Work and Energy

- Only translational kinetic energy is considered
- Equation applies for a single particle or a system of particles

$$\sum U \Big|_a^b = T_b - T_a = \frac{1}{2} m\vec{v}_b^2 - \frac{1}{2} m\vec{v}_a^2$$

Power and Efficiency

- Power is amount of work performed per unit of time
- SI Unit is Watt (W) which equates to 1 J/s
- Efficiency is defined as the ratio of output power to input power

$$P = \frac{d}{dt}U = \vec{F} \cdot \vec{v}$$

$$\epsilon = \frac{P_o}{P_i}$$

Conservation of Energy

- Applies when a particle is only acted by conservative forces
- A force is conservative if the work done is only dependent on start and end point (i.e. independent of the path)
  - Gravitational force
  - Elastic spring
- Work of non-conservative forces are path dependent
  - Friction

$$T_a + V_a = T_b + V_b = \text{Constant}$$

Chapter 5

Impulse and Momentum

Principle of Linear Impulse and Momentum

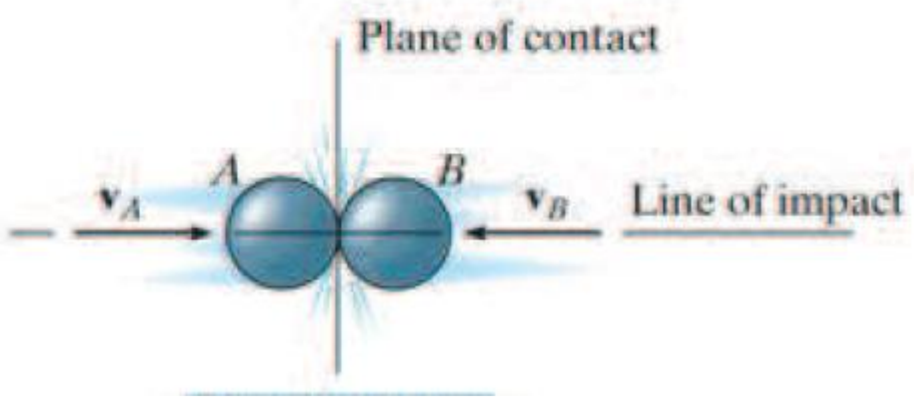
- Relates the initial and final velocity of a particle as result of an impulse
- Can be applied on a single particle or system of particles

$$m\vec{v}_1 + \sum \int_{t_1}^{t_2} \vec{F} dt = m\vec{v}_2$$

Impact

- Line of impact is the line that crosses the center of mass of both particles
- Impact force is applied along the line of impact (LOI)
- The coefficient of restitution (e) relates the velocities before and after impact along the LOI

$$e = \frac{(v_{B,LOI})_2 - (v_{A,LOI})_2}{(v_{A,LOI})_1 - (v_{B,LOI})_1}$$



Impact Questions - Procedure

- Define coordinate system so that the x-axis is along the LOI
- Apply conservation of linear momentum for the system of particle along the LOI
  - *e* can be used to relate velocities in the x-axis (LOI)
- Apply conservation of linear momentum for each particle along y-axis

<u>x-axis (LOI)</u>

$$\sum m(v_x)_a = \sum m(v_x)_b$$

$$e = \frac{((v_B)_x)_b - ((v_A)_x)_b}{((v_A)_x)_a - ((v_B)_x)_a}$$

<u>y-axis</u>

$$m_A((v_A)_y)_a = m_A((v_A)_y)_b$$

$$m_B((v_B)_y)_a = m_B((v_B)_y)_b$$

Chapter 6 – 8

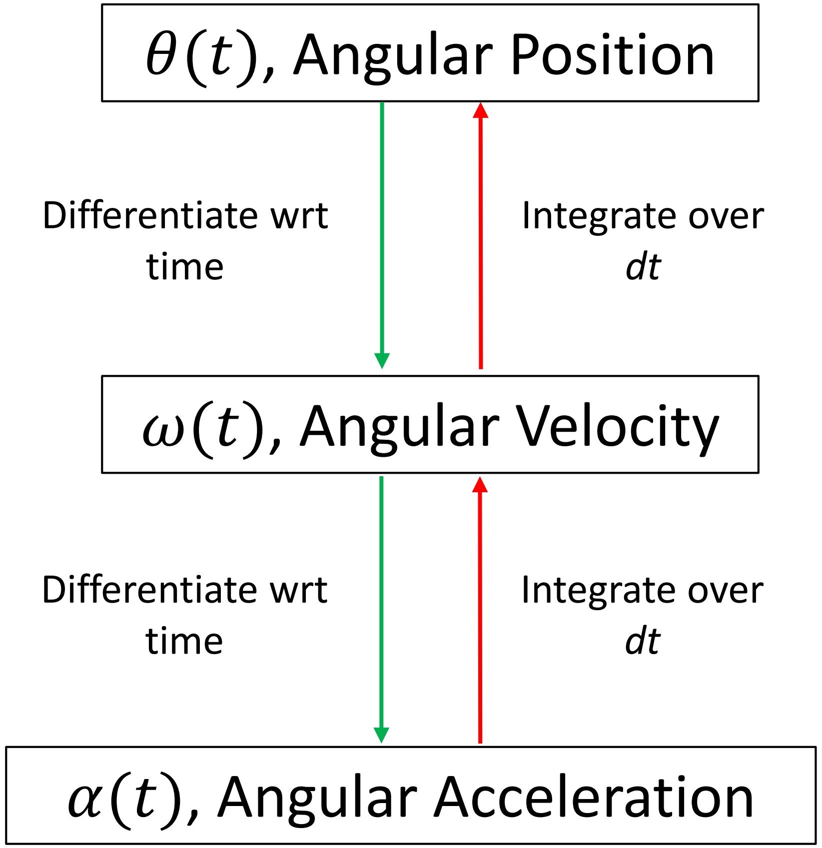
Planar Kinematics of a Rigid Body

Planar Kinetics with Work and Energy

Planar Kinematics of Rigid Body

- Size and shape of body is considered
- Both translation and rotation about center of mass (COM) are considered
- Deformation not considered, hence the name rigid body

Angular Motion - Fundamentals

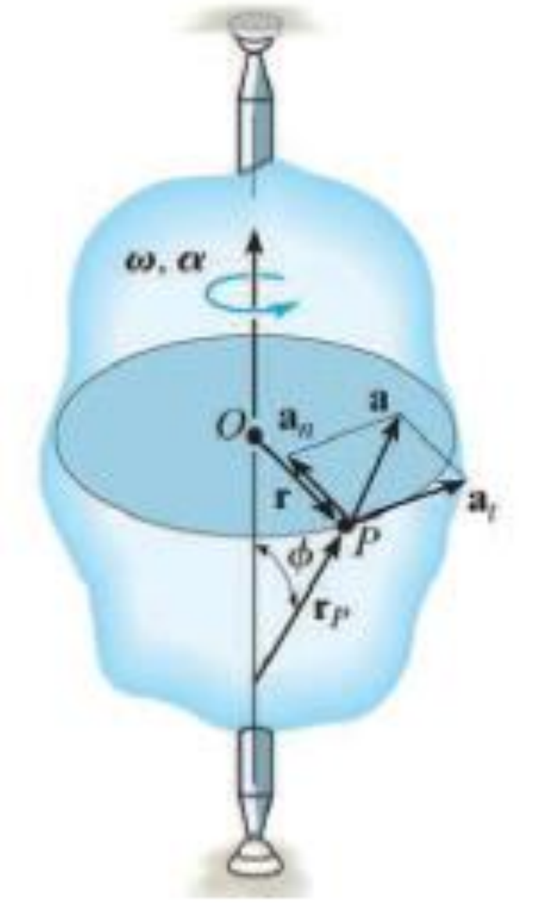


Constant acceleration α_c

- $\omega = \omega_0 + \alpha_c t$
- $\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha_c t^2$
- $\omega^2 = \omega_0^2 + 2\alpha_c(\theta - \theta_0)$
- $\theta - \theta_0 = \frac{\omega+\omega_0}{2} t$

Pure Rotation

- Rotation but no translation
  - Gear turning
  - Fan spinning
- Velocity
  - $v = \omega r$
- Acceleration
  - $a_t = \alpha r$
  - $a_n = \omega^2 r$
- Vector notation
  - $\vec{v} = \vec{\omega} \times \vec{r}$
  - $\vec{a} = \vec{\alpha} \times \vec{r} - \omega^2 \vec{r}$

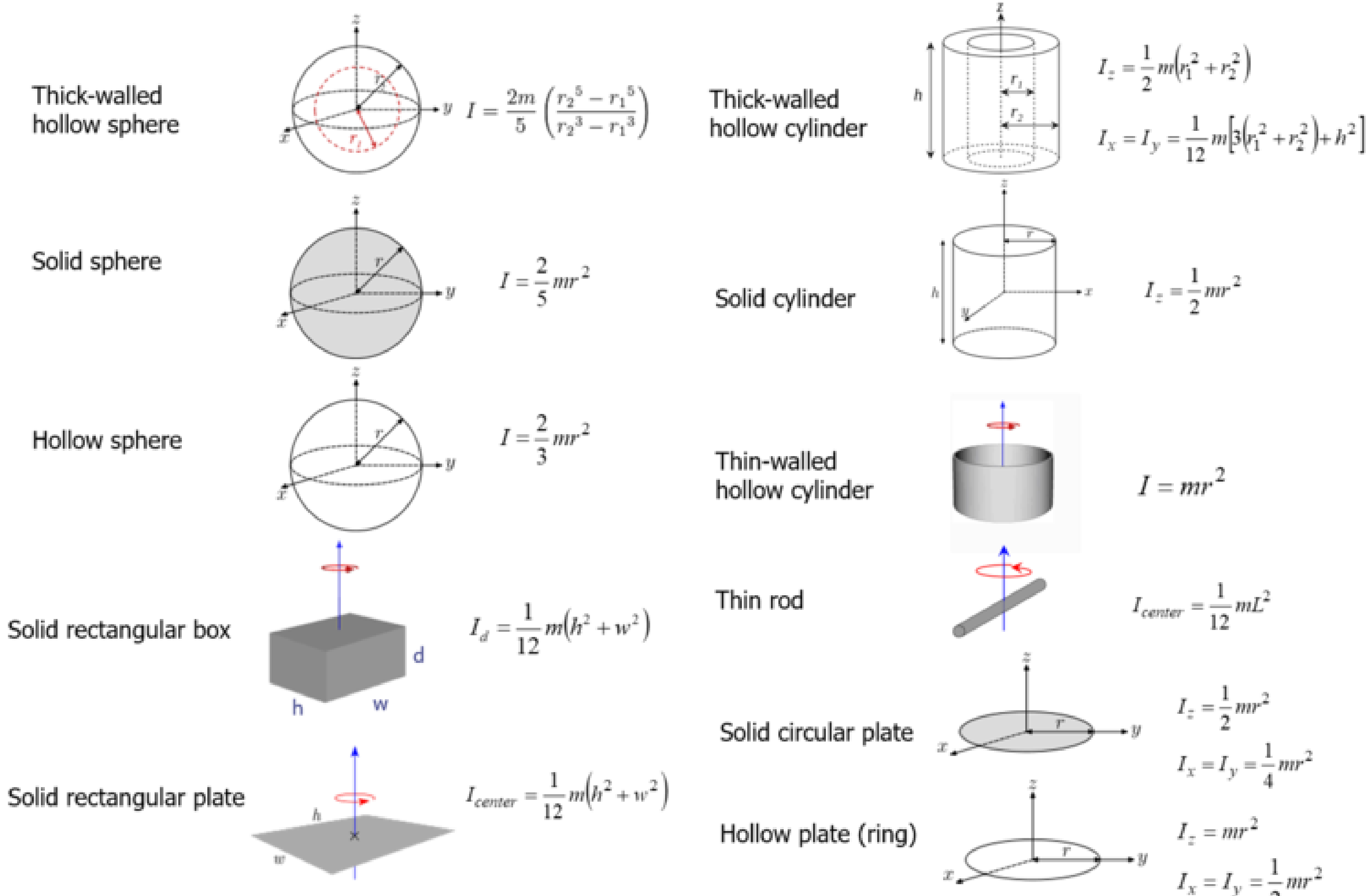


Moment of Inertia

- Mass moment of inertia $I = \int_m r^2 dm$
  - Table can be used for simple shapes (see next slide)
- Mass moment of inertia is usually given for an axis that goes through the COM
  - Parallel axis theorem to shift I_G another parallel axis
- Moment of inertia can also be given through radius of gyration, *k*

$$I = I_G + md^2 \qquad\qquad I = mk^2$$

Moment of Inertia



Taken from https://efcms.engr.utk.edu/ef151-2019-01/pilot/classmgr.php?c=43&p=mmi

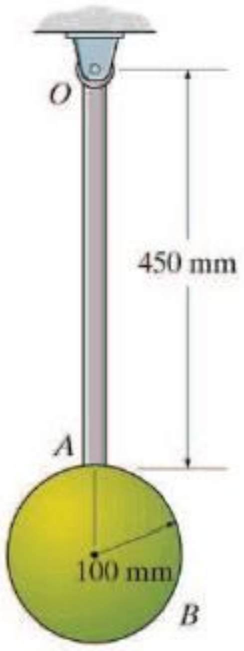
Moment of Inertia - Composite Bodies

- For a body made up of multiple simple shapes, I_P can be determined by summing the moment of inertia of each individual body
  - I_P of each individual body must be along the same axis before summing
  - Use parallel axis theorem to translate to the same axis

I_G found using table

$$I_O = I_G + (m)(d)^2$$

$$I_O = (I_O)_{rod} + (I_O)_{sphere}$$



Rigid Body – Planar Motion

- Combination of translation and rotation
- $\sum F_x = m(a_G)_x$ or $\sum F_n = m(a_G)_n$
- $\sum F_y = m(a_G)_y$ or $\sum F_t = m(a_G)_t$
- $\sum \overrightarrow{M}_P = \vec{r} \times m\vec{a}_G + I_G\vec{\alpha}$

This term is zero if point *P* is at center of mass *G*

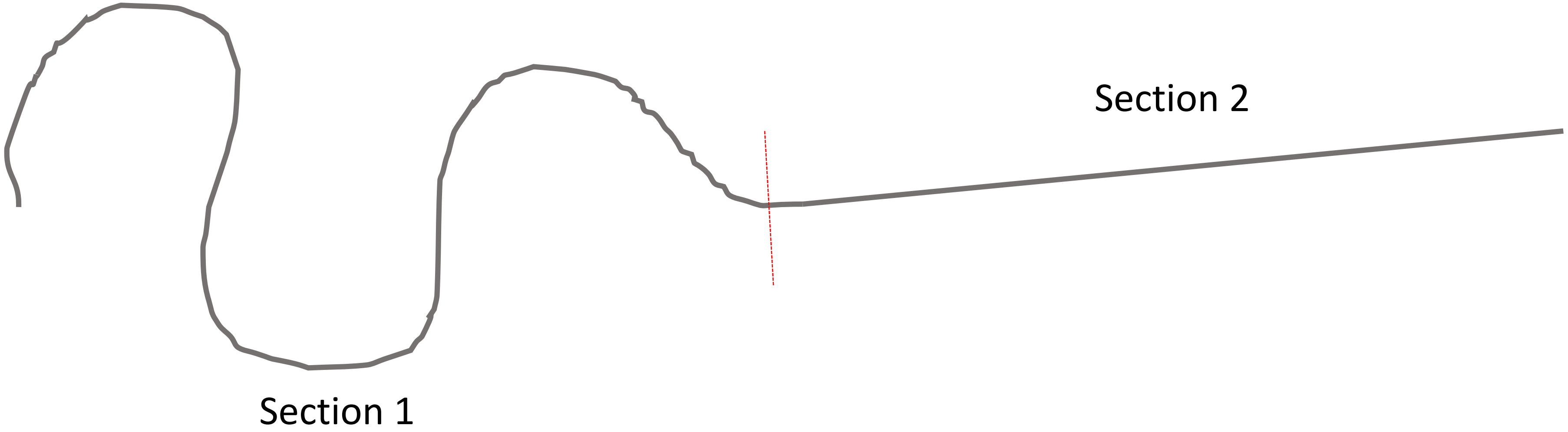
Rigid Body – Energy and Work

- Kinetic energy include both translational and rotational velocity
  - $T = \frac{1}{2}m(v_G)^2 + \frac{1}{2}I_G\omega^2$
- Work is same as before
  - $U_F = \int \vec{F} \cdot d\vec{r}$
- Forces that have zero work
  - External forces at fixed support (displacement is zero at fixed support)
  - Normal and friction with **no slip** for rolling motion (displacement is zero at contact point)
  - All internal forces (equal and opposite pair cancels out)

Theory Questions

Question 1

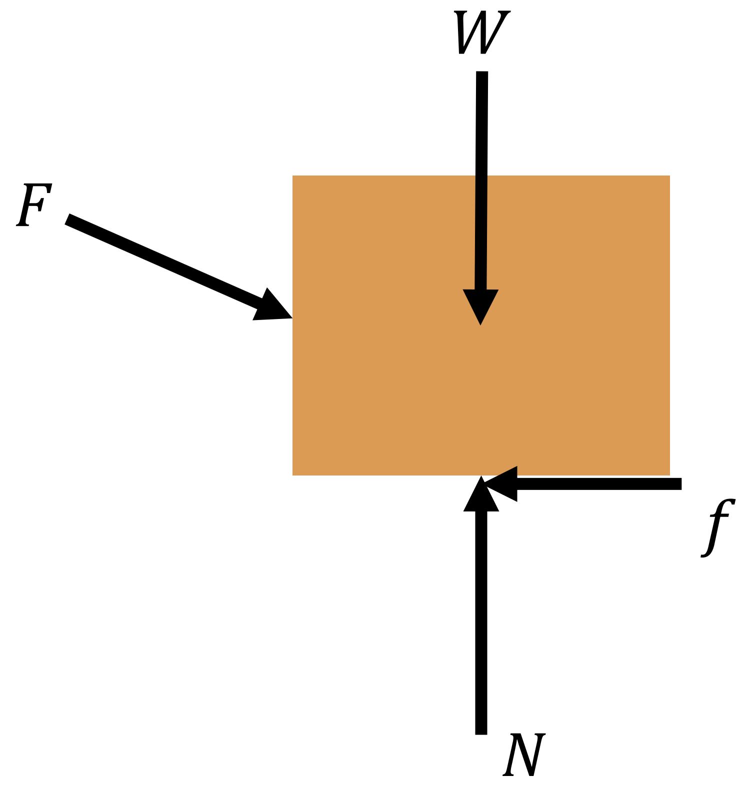
A car follows this route while maintaining a constant velocity of 30 km/h. What are a_n and a_t in section 1 and 2?



$a_t > 0$	$a_n > 0$	$a_t > 0$	$a_n > 0$
$a_t = 0$	$a_n = 0$	$a_t = 0$	$a_n = 0$
$a_t < 0$	$a_n < 0$	$a_t < 0$	$a_n < 0$

Question 2

Consider the following free body diagram for a person pushing a box to the right. What are the signs for the work of each force?



- F is the applied force
- N is the normal force
- f is the friction force
- W is the weight of the box

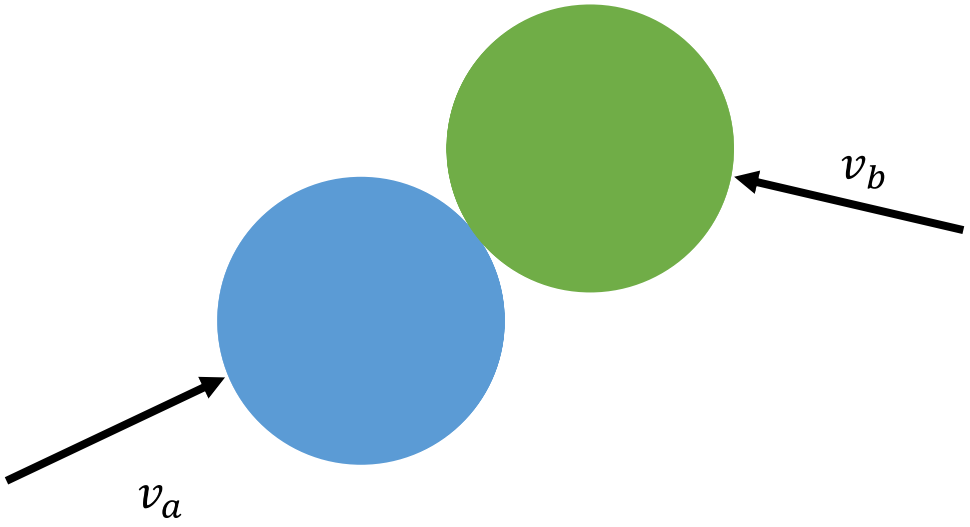
Question 3

The back wheel of a bicycle is driven by the pedal. What is the direction of the friction force for the back wheel and the front wheel?

Image taken from https://www.concepts-of-physics.com/mechanics/direction-of-frictional-force-on-bicycle-wheels.php

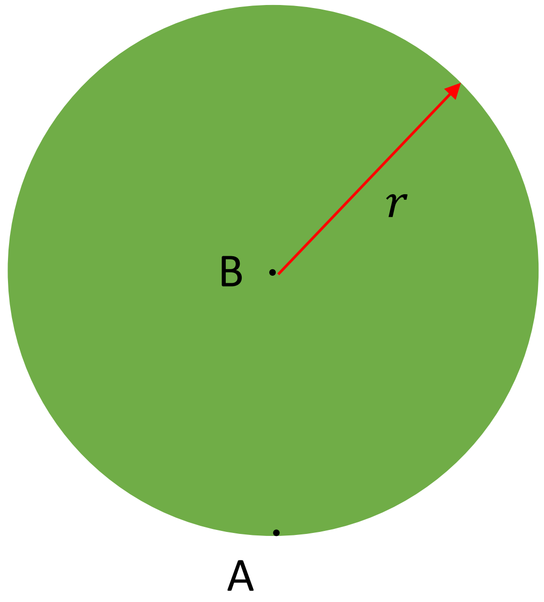
Question 4

Draw the line of impact and plane of collision for the following collision.



Question 5

Find the mass moment of inertia about the axis that leaves the page and intersect point A and B for a disc with mass M and radius r. Assume the disc has a uniform density and point B is located at the center.



Answers

1. $a_{1,t} = 0, a_{1,n} > 0, a_{2,t} = 0, a_{2,n} = 0$
2. $W_F > 0, W_f < 0, W_N = 0, W_W = 0$
3. Friction at rear wheel points to the right. Friction at front wheel point to the left.
4. See figure on right.
5. $I_B = \frac{1}{2}Mr^2$

   $I_A = \frac{3}{2}Mr^2$