Faculty of Engineering and Department of Physics
ENPH 131 Final Examination
Saturday, April 21, 2012; 2:00 pm – 4:30 pm
Universiade Pavilion

Section EB01: Rows 1, 3, 5 (seats 1-16)
Section EB02: Rows 5 (seats 17-50), 7, 9 (seats 1-30)
Section EB03: Rows 9 (seats 31-50), 11, 13 (seats 1-45)
Section EB04: Rows 13 (seats 46-50), 15, 17, 19 (seats 1-10)
Section EB05: Rows 19 (seats 11-50), 21, 23 (seats 1-25)
Section EB06: Rows 23 (seats 26-50), 25, 27 (seats 1-40)

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1. No notes or textbooks allowed.
2. A formula sheet is included (may be removed).
3. The exam has **9 problems** and is out of **60 points**. Attempt all parts of all problems.
4. Show all work in a neat and logical manner. For Questions 1 to 4, only the final answers to them will be marked. For Questions 5 to 9, details and procedures to solve these problems will be marked.
5. Write your solution directly on the pages with the questions. Indicate clearly if you use the backs of pages for material to be marked.
6. Non-programmable calculator allowed. Turn off all cell-phones, laptops, etc.

DO NOT separate the pages of the exam containing the problems.

Last Name: ____

First Name: ____

ID#: ____

Please circle the name of your instructor:

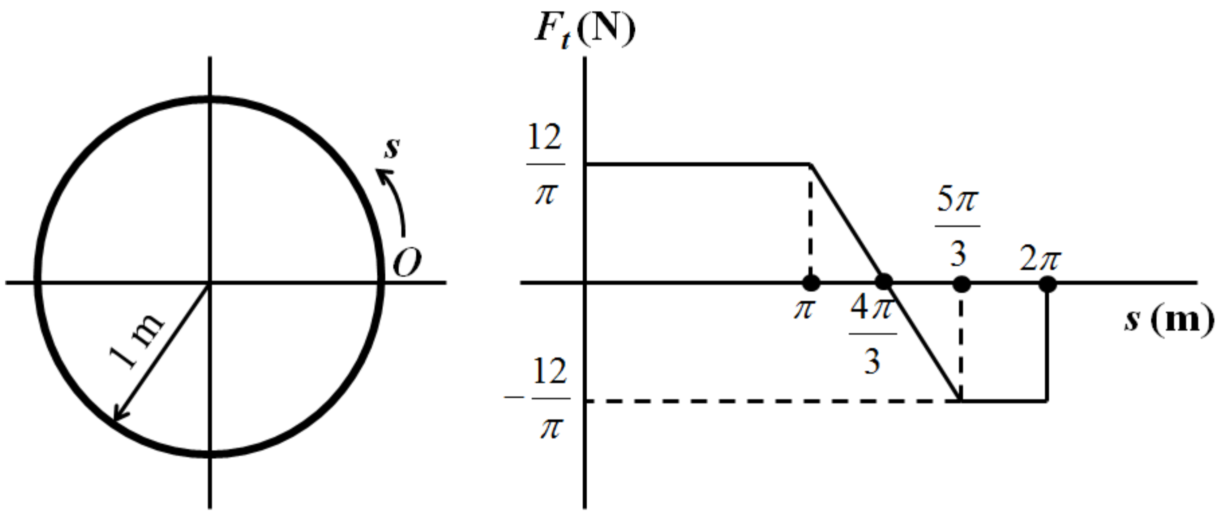
B01: Chow **B02:** Fenrich **B03:** Schiavone

B04: Lavoie **B05:** Wheelock **B06:** Tang

Please do not write in the table below.

Question	**Value (Points)**	**Mark**
1	**6**	
2	**4**	
3	**3**	
4	**3**	
5	**8**	
6	**8**	
7	**10**	
8	**8**	
9	**10**	
Total	**60**	

1. [6 marks] A particle of mass m = 10 kg starts from point O and travels along a circle of radius 1 m in the counterclockwise direction as shown in the left figure below. At s = 0, the speed of the particle is 1 m/s. The total force on the particle **in the tangential direction** (F_t) is plotted as a function of s in the right figure below.



(a) [1] What is the magnitude of the particle's total acceleration when s = 0?

Answer: 1.07 m/s^2

(b) [1] What is the speed of the particle when it again comes to point O ($s = 2\pi$ meters)?

Answer: 1.61 m/s

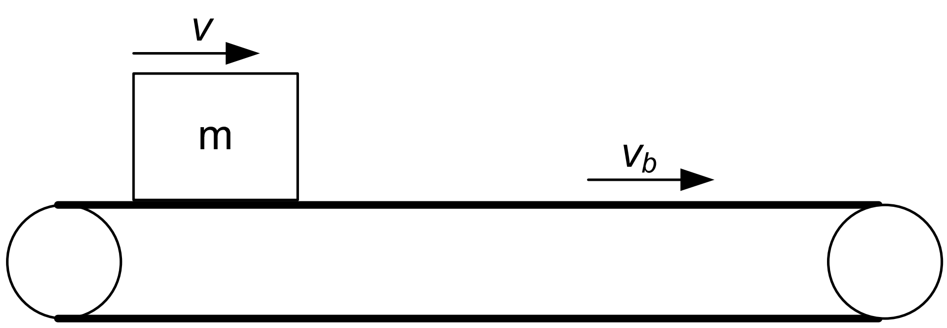
(c) [2] At what s is the total force on the particle **in the normal direction** minimum? What is the minimum net normal force?

Answer: s = 0 10 N

(d) [2] At what s is the total force on the particle **in the normal direction** maximum? What is the maximum net normal force?

Answer: $s = 4\pi/3 = 4.19$ m 38 N

2. [4 marks] A block of mass m lands on a conveyor belt with velocity v as shown below. The conveyor belt moves with constant speed v_b ($<v$). Due to the kinetic friction between the block and the conveyor belt (coefficient of kinetic friction $= \mu_k$), the block eventually stops sliding relative to the conveyor belt. In order to determine the distance d the block slides **relative to the conveyor belt**, the following systems of equations for the block are proposed. In these equations, a is used to denote the acceleration of the block relative to the ground, s is used to denote the displacement of the block relative to the ground, t is used to denote the time during which the block slides relative to the conveyor belt, and g is the magnitude of the gravitational acceleration. Some of these systems of equations are correct while others are not. Circle your choice of TRUE or FALSE after each system of equations. [1 mark for each correct answer]

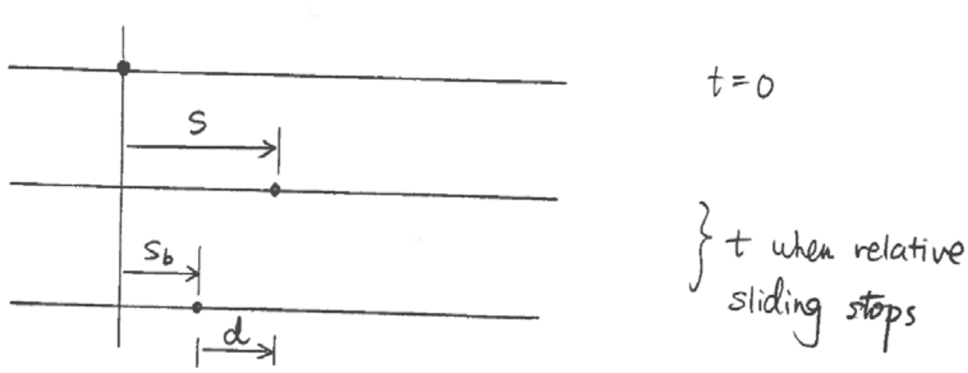


(a) $$\begin{cases} -\mu_k mg = ma \\ v_b^2 = v^2 + 2ad \end{cases}$$ TRUE FALSE

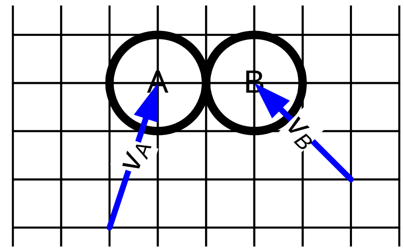
(b) $$\frac{1}{2}mv^2 - \mu_k mgd = \frac{1}{2}mv_b^2$$ TRUE FALSE

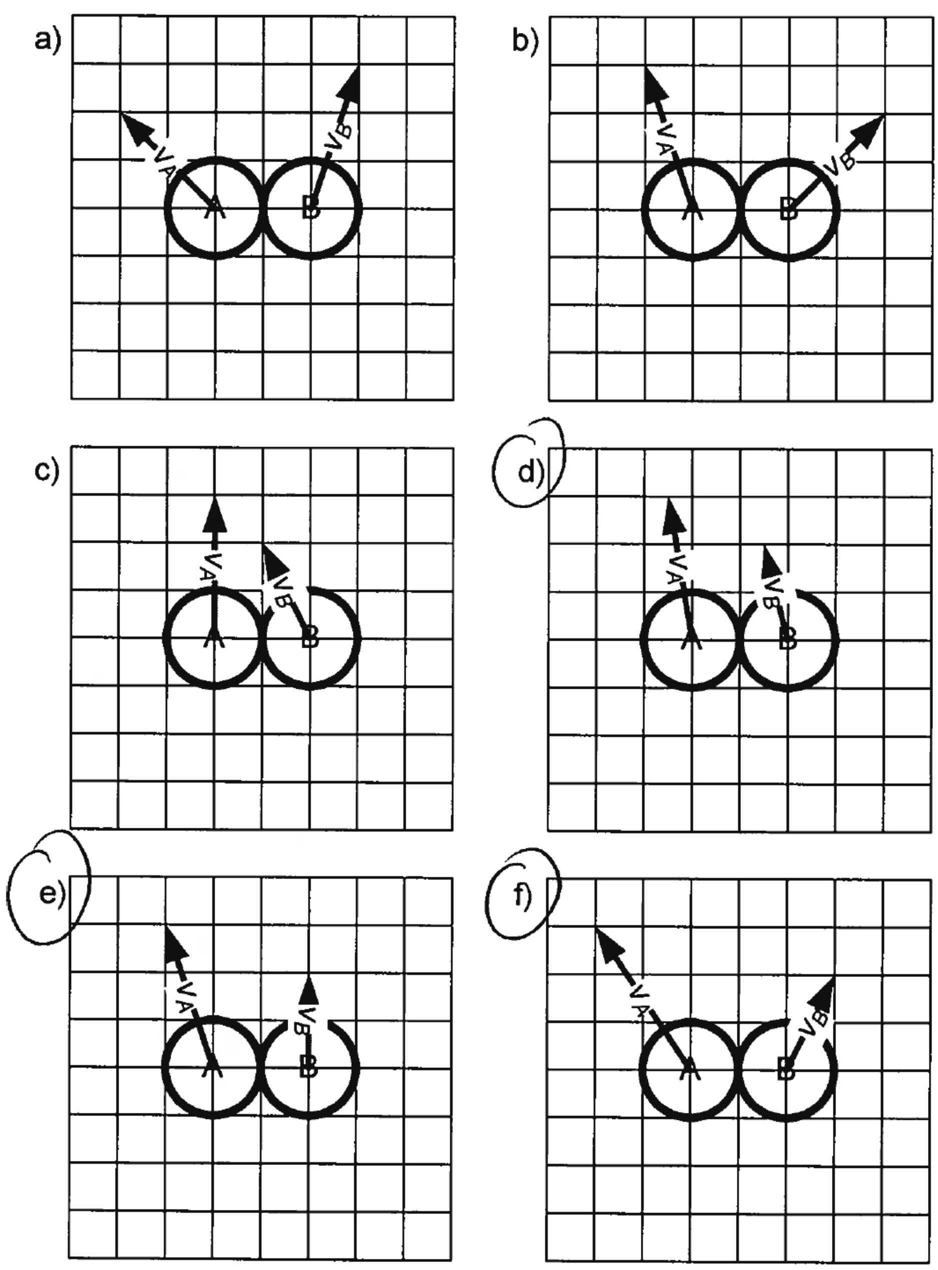
(c) $$\frac{1}{2}m(v - v_b)^2 - \mu_k mgd = 0$$ TRUE FALSE

(d) $$\begin{cases} mv - \mu_k mgt = mv_b \\ s = vt - \frac{\mu_k g}{2}t^2 \\ s = d + v_b t \end{cases}$$ TRUE FALSE



3. [3 marks] Two particles A and B of equal mass have velocities as shown in the figure to the right. If the particles collide, circle the situation(s) shown in the figures below that represent possible velocities for the particles **after impact**. The velocities in all figures are shown to scale. [0.5 mark for each correct answer]





$v_{Ay} = 3$, $v_{By} = 2$

$v_{Ax} + v_{Bx} = -1$

$$e = \frac{v_{Bx} - v_{Ax}}{3}$$

4. [3 marks] A solid uniform ball rolls without slipping up a hill. At the top of the hill, it is moving horizontally when it goes over the edge of the cliff.

   Consider the ball at three locations shown:
   A = just before rolling up the hill;
   B = at the top of the hill moving horizontally; and
   C = the instant before striking the ground.

   Rank the three positions **from highest to lowest** (indicating ties with an equal sign, if appropriate) in terms of:

(a) [1] *Total* kinetic energy

Answer: A = C > B

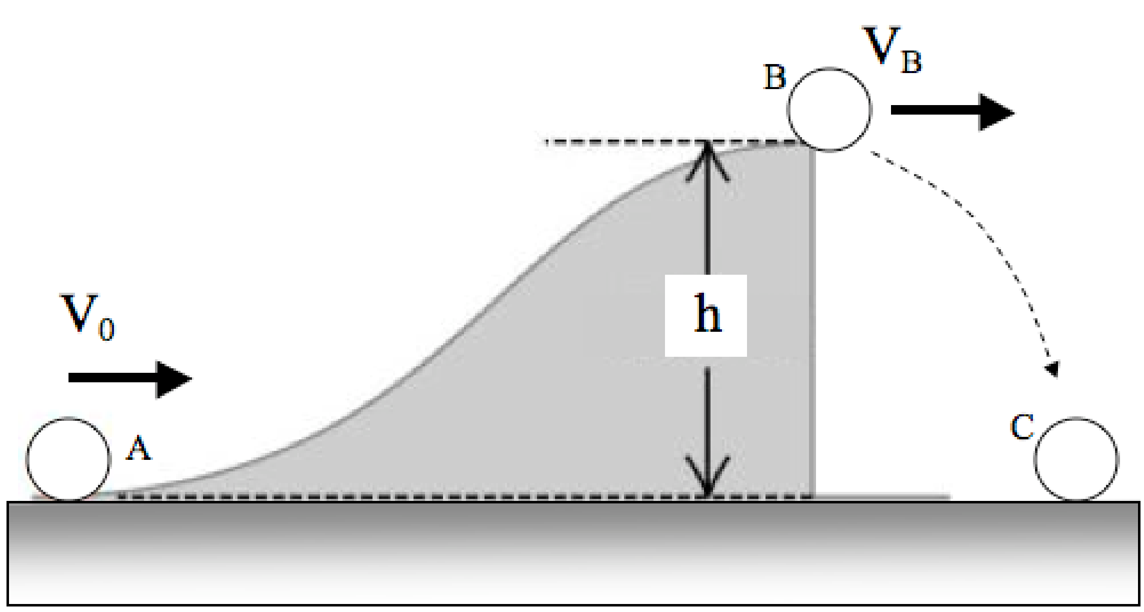
(b) [1] *Rotational* kinetic energy

Answer: A > B = C

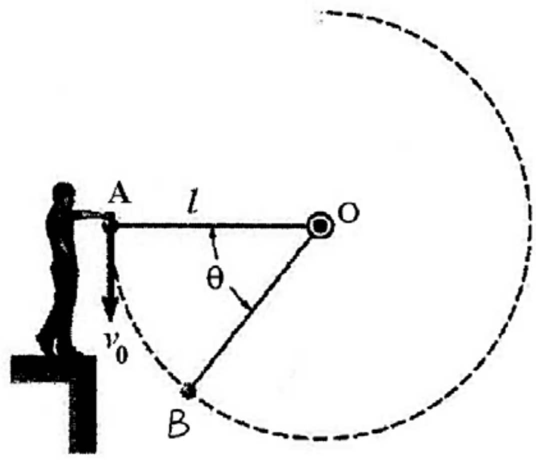
(c) [1] *Translational* kinetic energy

Answer: C > A > B

For a solid uniform ball of mass m and radius R, the moment of inertia about its center of mass is given by $2mR^2/5$.

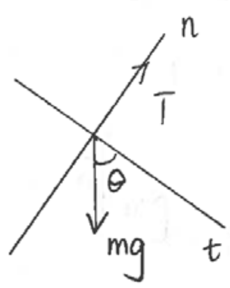


5. [8 marks] The sphere at A is given a downward velocity $\boldsymbol{v}_0$ of magnitude 16.0 ft/s and swings in a vertical plane at the end of a rope of length l = 6.00 ft attached to a support at O. Determine the angle θ at which the rope will break, knowing that it can withstand a maximum tension equal to twice the weight of the sphere.

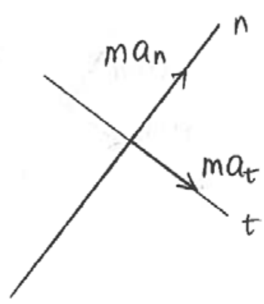


At B:

FBD



KD



$$+\nearrow \quad T - mg\sin\theta = m a_n$$

$$\Rightarrow \quad 2mg - mg\sin\theta = m\frac{V_B^2}{l} \qquad (1)$$

A → B: energy conservation

$$\Rightarrow \quad \frac{1}{2} m V_A^2 + mgl\sin\theta = \frac{1}{2} m V_B^2 \qquad (2)$$

$$(1)\ \&\ (2) \Rightarrow \quad V_A^2 + 2gl\sin\theta = 2gl - gl\sin\theta$$

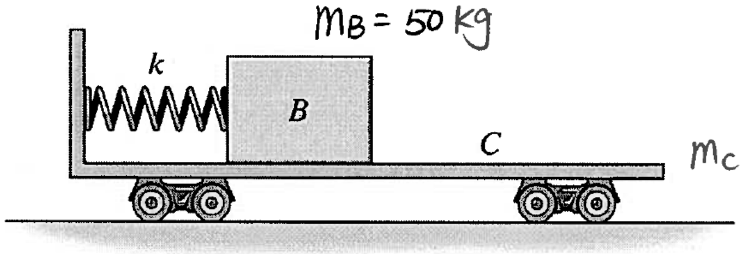
$$\Rightarrow \quad \sin\theta = \frac{2}{3} - \frac{V_A^2}{3gl} = \frac{2}{3} - \frac{16^2}{3(32.2)(6)}$$

$$= 0.225$$

$$\Rightarrow \quad \theta = 13.0^\circ$$

6. [8 marks] The block B has a mass of 50 kg and rests on the smooth surface of the cart C having a mass of 75 kg. A massless spring with a spring constant k = 300 N/m is attached to the cart. The block B is pushed against the spring, compressing it by 0.200 m, as indicated in the figure below. The system is released from rest. Determine the **relative velocity of the block with respect to the cart** when the spring reaches its unstretched position. Neglect the mass of the cart's wheels and also friction between the cart and the ground.

$m_B = 50\ kg$



$m_C = 75\ kg$ $\qquad \longrightarrow \vec{i}$

$\vec{v}_B = v_B \vec{i}$

$\vec{v}_C = v_C \vec{i}$

PROBR1_015-016.jpg

L.M. conservation: $m_B v_B + m_C v_C = 0$ ①

energy conservation: $\frac{1}{2} m_B v_B^2 + \frac{1}{2} m_C v_C^2 = \frac{1}{2} k s^2$ ②

① & ②

$$\Rightarrow \left(\frac{m_C^2}{m_B} + m_C\right) v_C^2 = k s^2$$

$$\Rightarrow v_C = -\sqrt{\frac{300\,(0.2)^2}{\frac{75^2}{50} + 75}} = -\ 0.253\ m/s$$

$$\Rightarrow v_B = -\frac{m_C}{m_B} v_C = 0.380\ m/s$$

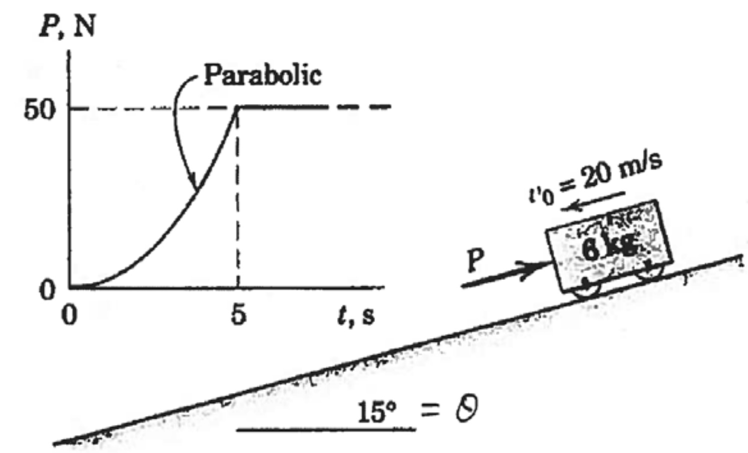
$$\Rightarrow \vec{v}_{B/C} = \vec{v}_B - \vec{v}_C$$

$$= (0.380\ m/s)\ \vec{i} - (-0.253\ m/s)\ \vec{i}$$

$$= (0.633\ m/s)\ \vec{i}$$

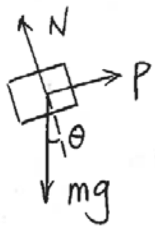
7. [10 marks] The 6-kg cart is moving down the smooth incline with a speed of v_0 = 20 m/s at t = 0, at which time the force P begins to act as shown. The force can be described by $P = 2t^2$ during the first 5 seconds, and is constant afterwards with a value of 50 N. Determine:

(a) [6] The velocity of the cart at time t = 8 s;
(b) [4] The time t at which the cart's velocity is zero.



(a) $P = \begin{cases} 2t^2 & t \le 5s \\ 50 & t > 5s \end{cases}$

FBD:



$\overset{+}{\nearrow}$:

$\int_0^8 P\,dt - \int_0^8 mg\sin\theta\,dt = mv_1 - mv_0$

$\Rightarrow \int_0^5 2t^2 dt + \int_5^8 50\,dt$

$- \int_0^8 (6)(9.81)\sin 15°\,dt = 6[v_1 - (-20)]$

$\Rightarrow v_1 = -1.42$ m/s

$\Rightarrow$ cart is moving down the incline at speed of 1.42 m/s

(b) From (a), it can be seen that $t > 8s$ when the cart's velocity is zero

$\int_8^t 50\,dt - \int_8^t (6)(9.81)\sin 15°\,dt$

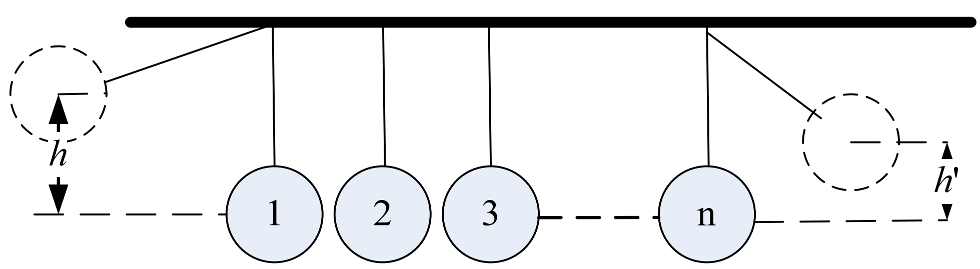
$= 6[0 - (-1.42)]$

$\Rightarrow t = 8.25$ s.

8. [8 marks] n identical spheres are suspended in a line by wires of equal length so that the spheres are almost touching each other. Sphere 1 is released from height *h* and strikes sphere 2, sphere 2 then strikes sphere 3, and sphere 3 strikes sphere 4…; finally sphere (n – 1) strikes sphere n, which rises to a height of *h'* as shown. Assume that the impact between each pair of spheres occurs only once and that the coefficient of restitution between each pair of spheres is *e*.

   (a) [4] Consider the collision between ball (i + 1) and ball i. Determine the ratio of the speed of ball (i + 1) after the impact to that of ball i before the impact (v'_{i+1}/v_i). Express your answer in terms of *e*.

   (b) [4] Determine the ratio of h'/h. Express your answer in terms of *e* and n.



(a) central impact b/w identical particles

$$v_i + \cancel{v_{i+1}}^{\,0} = v_i' + v_{i+1}' \qquad ①$$

$$e = \frac{v_{i+1}' - v_i'}{v_i - \cancel{v_{i+1}}_{\,0}} \Rightarrow e\,v_i = v_{i+1}' - v_i' \qquad ②$$

$$① + ② \Rightarrow 2v_{i+1}' = (1+e)\,v_i \Rightarrow \frac{v_{i+1}'}{v_i} = \frac{1+e}{2}$$

(b) v_{i+1}' in part (a) = speed of ball (i+1) after its impact w/ ball i

= speed of ball (i+1) before its impact w/ ball (i+2)

$$\frac{v_n'}{v_1} = \frac{v_n'}{v_{n-1}'}\cdot\frac{v_{n-1}'}{v_{n-2}'}\cdot \;\cdots\; \frac{v_2'}{v_1} = \left(\frac{1+e}{2}\right)^{n-1}$$

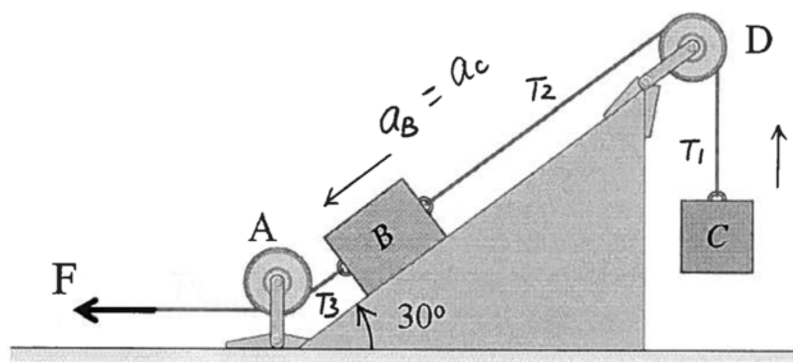
energy conservation: $v_1 = \sqrt{2gh}$, $v_n' = \sqrt{2gh'}$

$$\Rightarrow \frac{h'}{h} = \left(\frac{v_n}{v_1}\right)^2 = \left[\frac{1+e}{2}\right]^{2(n-1)}$$

9. [10 marks] Blocks B (m_B = 20 kg) and C (m_C = 30 kg) shown in the figure are connected by ropes of negligible mass and pulleys A and D of non-negligible mass (m_A = 5 kg and m_D = 10 kg). Both pulleys can be treated as solid uniform disks of radius R = 0.5 m. The ropes are tightly wrapped around the pulleys so that there is no slipping between them. The coefficient of kinetic friction between B and the inclined surface is 0.35. A horizontal force F is applied on the system so that block C has an upward acceleration of 5 m/s^2.

(a) [4] Find the tension in the rope between Pulley D and Block B.
(b) [6] Determine the magnitude of the force F.

For a solid uniform disk of mass *m* and radius *R*, the moment of inertia about its center of mass is given by $mR^2/2$.



C: FBD: T_1, $m_c g$ KD: $m_c a_c$

$\Rightarrow T_1 = m_c(g + a_c)$

$= 30(9.81 + 5) = 444$ N

D: FBD: F_{Dy}, F_{Dx}, T_2, T_1 KD: $I_D \alpha_D$

$(T_2 - T_1)R = \frac{1}{2} m_D R^2 \left(\frac{a_c}{R}\right)$

$\Rightarrow T_2 = T_1 + \frac{1}{2} m_D a_c = 469$ N

B: FBD: N, T_2, $\mu_k N$, T_3, $m_B g$ KD: $m_B a_c$

$T_3 + m_B g \sin 30° - T_2 - \mu_k m_B g \cos 30° = m_B a_c$

$\Rightarrow T_3 = 530$ N

A: FBD: F_{Ay}, F_{Ax}, T_3, F KD: $I_A \alpha_A$

$(F - T_3) R = \frac{1}{2} m_A R^2 \left(\frac{a_c}{R}\right)$

$\Rightarrow F = T_3 + \frac{1}{2} m_A a_c = 543$ N.

Fundamental Equations of Dynamics

KINEMATICS

Particle Rectilinear Motion

Variable a	*Constant $a = a_c$*
$a = \dfrac{dv}{dt}$	$v = v_0 + a_c t$
$v = \dfrac{ds}{dt}$	$s = s_0 + v_0 t + \frac{1}{2}a_c t^2$
$a\,ds = v\,dv$	$v^2 = v_0^2 + 2a_c(s - s_0)$

Particle Curvilinear Motion

x, y, z Coordinates		*r, θ, z Coordinates*	
$v_x = \dot{x}$	$a_x = \ddot{x}$	$v_r = \dot{r}$	$a_r = \ddot{r} - r\dot{\theta}^2$
$v_y = \dot{y}$	$a_y = \ddot{y}$	$v_\theta = r\dot{\theta}$	$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$
$v_z = \dot{z}$	$a_z = \ddot{z}$	$v_z = \dot{z}$	$a_z = \ddot{z}$

n, t, b Coordinates

$v = \dot{s}$	$a_t = \dot{v} = v\dfrac{dv}{ds}$
	$a_n = \dfrac{v^2}{\rho}$ $\quad \rho = \dfrac{[1 + (dy/dx)^2]^{3/2}}{\lvert d^2y/dx^2 \rvert}$

Relative Motion

$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A} \qquad \mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$

Rigid Body Motion About a Fixed Axis

Variable α	*Constant* $\alpha = \alpha_c$
$\alpha = \dfrac{d\omega}{dt}$	$\omega = \omega_0 + \alpha_c t$
$\omega = \dfrac{d\theta}{dt}$	$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha_c t^2$
$\omega\,d\omega = \alpha\,d\theta$	$\omega^2 = \omega_0^2 + 2\alpha_c(\theta - \theta_0)$

For Point P

$s = \theta r \quad v = \omega r \quad a_t = \alpha r \quad a_n = \omega^2 r$

Relative General Plane Motion—Translating Axes

$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A(\text{pin})} \qquad \mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A(\text{pin})}$

Relative General Plane Motion—Trans. and Rot. Axis

$\mathbf{v}_B = \mathbf{v}_A + \mathbf{\Omega} \times \mathbf{r}_{B/A} + (\mathbf{v}_{B/A})_{xyz}$

$\mathbf{a}_B = \mathbf{a}_A + \dot{\mathbf{\Omega}} \times \mathbf{r}_{B/A} + \mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{r}_{B/A}) + 2\mathbf{\Omega} \times (\mathbf{v}_{B/A})_{xyz} + (\mathbf{a}_{B/A})_{xyz}$

KINETICS

Mass Moment of Inertia $\quad I = \int r^2\,dm$

Parallel-Axis Theorem $\quad I = I_G + md^2$

Radius of Gyration $\quad k = \sqrt{\dfrac{I}{m}}$

Equations of Motion

Particle	$\Sigma \mathbf{F} = m\mathbf{a}$
Rigid Body (Plane Motion)	$\Sigma F_x = m(a_G)_x$ $\Sigma F_y = m(a_G)_y$ $\Sigma M_G = I_G\alpha$ or $\Sigma M_P = \Sigma(\mathcal{M}_k)_P$

Principle of Work and Energy

$T_1 + U_{1-2} = T_2$

Kinetic Energy

Particle	$T = \frac{1}{2}mv^2$
Rigid Body (Plane Motion)	$T = \frac{1}{2}mv_G^2 + \frac{1}{2}I_G\omega^2$

Work

Variable force	$U_F = \int F\cos\theta\,ds$
Constant force	$U_F = (F_c\cos\theta)\,\Delta s$
Weight	$U_W = -W\,\Delta y$
Spring	$U_s = -(\frac{1}{2}ks_2^2 - \frac{1}{2}ks_1^2)$
Couple moment	$U_M = M\,\Delta\theta$

Power and Efficiency

$P = \dfrac{dU}{dt} = \mathbf{F}\cdot\mathbf{v} \qquad \epsilon = \dfrac{P_{\text{out}}}{P_{\text{in}}} = \dfrac{U_{\text{out}}}{U_{\text{in}}}$

Conservation of Energy Theorem

$T_1 + V_1 = T_2 + V_2$

Potential Energy

$V = V_g + V_e$, where $V_g = \pm Wy$, $V_e = +\frac{1}{2}ks^2$

Principle of Linear Impulse and Momentum

Particle	$m\mathbf{v}_1 + \Sigma\int \mathbf{F}\,dt = m\mathbf{v}_2$
Rigid Body	$m(\mathbf{v}_G)_1 + \Sigma\int \mathbf{F}\,dt = m(\mathbf{v}_G)_2$

Conservation of Linear Momentum

$\Sigma(\text{syst. } m\mathbf{v})_1 = \Sigma(\text{syst. } m\mathbf{v})_2$

Coefficient of Restitution $\quad e = \dfrac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$

Principle of Angular Impulse and Momentum

Particle	$(\mathbf{H}_O)_1 + \Sigma\int \mathbf{M}_O\,dt = (\mathbf{H}_O)_2$ where $H_O = (d)(mv)$
Rigid Body (Plane motion)	$(\mathbf{H}_G)_1 + \Sigma\int \mathbf{M}_G\,dt = (\mathbf{H}_G)_2$ where $H_G = I_G\omega$ $(\mathbf{H}_O)_1 + \Sigma\int \mathbf{M}_O\,dt = (\mathbf{H}_O)_2$ where $H_O = I_O\omega$

Conservation of Angular Momentum

$\Sigma(\text{syst. } \mathbf{H})_1 = \Sigma(\text{syst. } \mathbf{H})_2$