Faculty of Engineering and Department of Physics

Engineering Physics 131

Midterm Examination

February 26, 2007; 7:00 pm – 8:30 pm

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1. No notes or textbooks allowed.
2. Formula sheets are at the end (may be removed).
3. Non-programmable calculators approved by the Faculty of Engineering permitted.
4. The exam has **7** problems. Attempt all parts of all problems.
5. The value of each problem is indicated on the table below. Budget your time accordingly.
6. Show all work in a neat and logical manner.
7. Indicate clearly if you use the backs of pages for material to be marked.
8. Turn off all cell-phones, laptops, etc.

DO NOT separate the pages of the exam containing the problems.

NAME: ***SOLUTIONS and PARTIAL MARKING SCHEME from INSTRUCTORS***

ID#: ______________________

Please circle the name of your instructor:

B01: Chow

B02: Sigurdson

B03: Raboud

B04: Behzadipour

B05: Ropchan

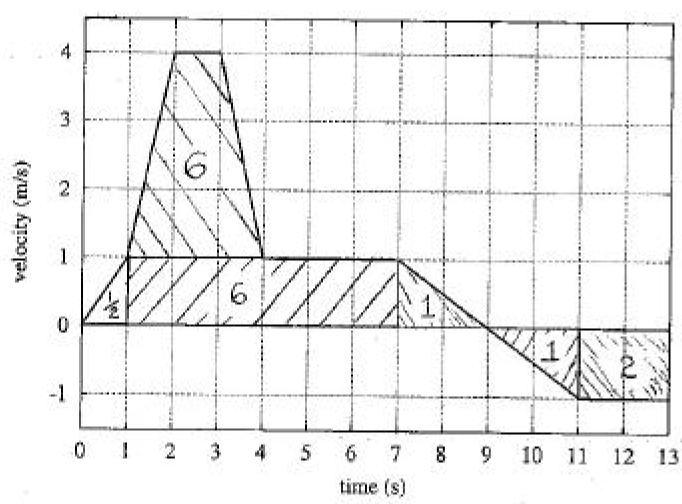
B06: Le

Question	**Value**	**Mark**
1	10 (2 each)	
2	4 (1 each)	
3	6 (1 each)	
4	13 (4 for Pt. 1, 9 for Pt. 2)	
5	25	
6	27	
7	15	
Total	100	

1. The graph below represents the velocity of a fast bee traveling along a *straight line*. At time $t = 0$, the bee is at the hive. (No partial marks for each question, **which is worth 2 marks each. However, you should not be penalized for making a mistake early on and then using that wrong answer later on with the correct method.**)

(a) When is the bee furthest from the hive?
(b) How far is the bee at its furthest point from the hive?
(c) At $t = 13$ s, how far is the bee from the hive?
(d) Over the period from $t = 0$ s to $t = 13$ s, what is the bee's average velocity?
(e) Over the period from $t = 0$ s to $t = 13$ s, what is the bee's average speed?

SOLUTION:



(a) t = 9 seconds

(b) Add up the positive contributions up to 9 seconds: ½+ 6 + 6 +1 = 13.5 m

(c) 13.5 m – 1 m – 2 m = 10.5 m.
You should also get full (two) marks if you put down: the answer in (b) – 3 m.

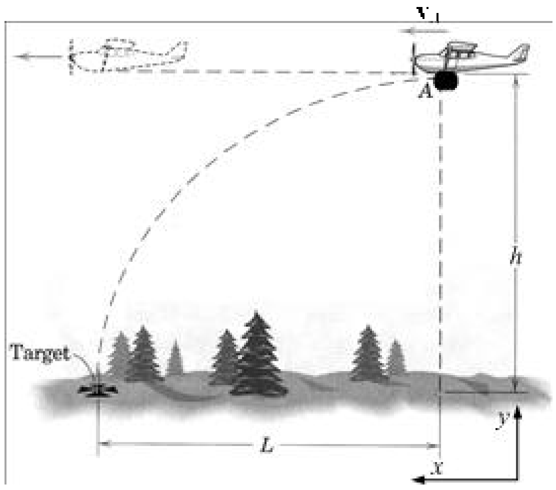
(d) Average velocity = (final displacement – initial displacement)/(change in time)
= 10.5 m / 13 seconds = 0.808 m/s

You should also get full (two) marks if you put down: the answer in (c)/13.0s.

(e) Average speed = (total distance traveled)/(change in time)
= (13.5 m + 1 m + 2 m)/13 seconds = 1.27 m/s

You should also get full (two) marks if you put down: [the answer in (b) + 3 m]/13.0s.

2. A small airplane is flying horizontally with a constant speed of $\mathbf{v}_A$. The pilot releases a medical package at an altitude of h above the ground at time t_r in order for the package to land exactly on the target. Circle the correct answer for the following situations such that the package will still land on the target: **[1 mark each, no part marks]**



(A) If the airplane was flying at a higher altitude with the same velocity, the pilot would have to release the package

(a) Earlier

(b) Later

(c) At the same time

(B) Consider the original condition (airplane flying horizontally at an altitude of h). Suppose the target is on a trailer moving with a speed smaller than the one of the airplane in the positive x direction. The pilot will have to release the package

(a) Earlier

(b) Later

(c) At the same time

(C) Consider the original condition (airplane flying horizontally at an altitude of h). If the release mechanism now applies a pushing force to the package such that it obtains a downward initial velocity when released, the pilot will have to release the package

(a) Earlier

(b) Later

(c) At the same time

(D) Consider the original condition (airplane flying horizontally at an altitude of h). If there is a deceleration in x direction (e.g. due to a wind force), the pilot will have to release the package

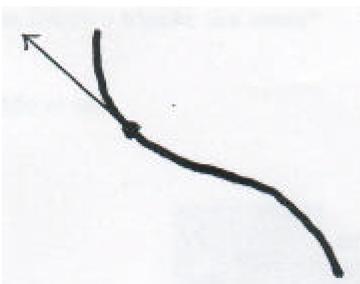
(a) Earlier

(b) Later

(c) At the same time

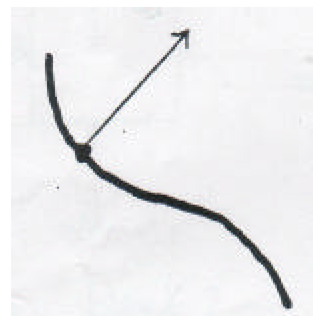
3. Shown below are six identical paths followed by a particle whose position is indicated by a solid dot. In each case, the particle is traveling downwards towards the right. The arrow on each path indicates the resultant acceleration of the particle at the given instant. However, some of the accelerations shown are impossible. **[1 mark each, no part marks]**

In each case, answer the following question:
Is the indicated acceleration possible? Indicate by circling (only) one of the choices: **Yes**, **No**, or, if there is insufficient information, **Can’t Say**.
Note: The position of the particle in e) is at a point of inflection and in a) and d) the vector is tangent to the path.



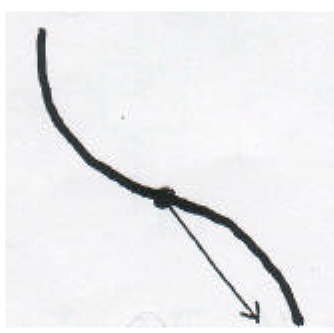
a) Yes No Can’t Say

b) Yes No Can’t Say

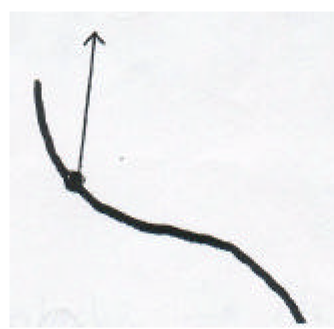


c) Yes No Can’t Say

d) Yes No Can’t Say



e) Yes No Can’t Say

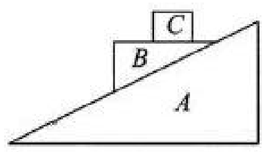


f) Yes No Can’t Say

4. Two triangular blocks *A* and *B* and a rectangular block *C* are placed together as shown. Block *A* is sitting on the ground.

(1) If *A*, *B* and *C* are all static, draw the free body diagrams for *B* and *C*. Indicate the correct direction of all forces. Assume there is friction between all surfaces.

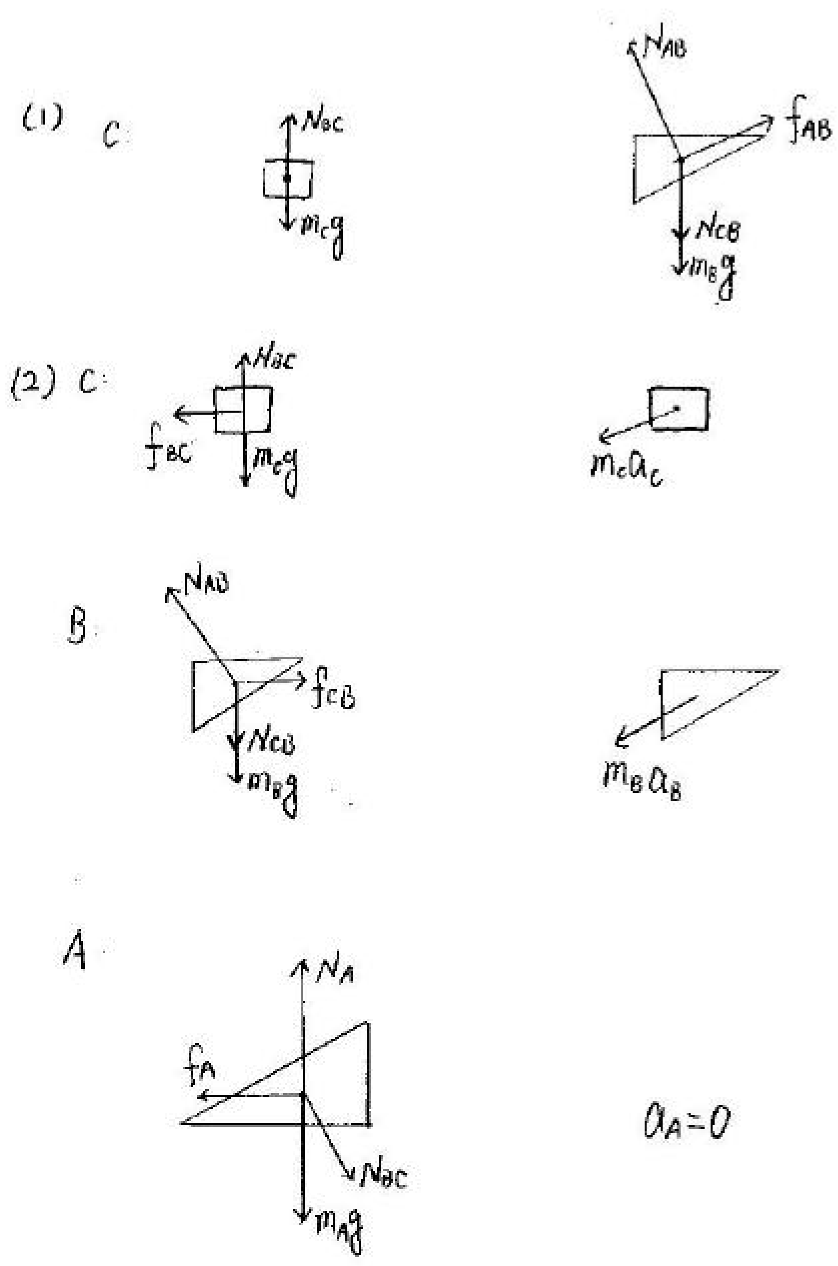
(2) Suppose the surface between *A* and *B* is completely smooth, but the surface between *B* and *C* and the surface between *A* and the ground are not. Blocks *B* and *C* move downward together along the surface, *C* is static relative to *B*, and *A* is at rest on the ground. Draw the free body diagrams and kinetic diagrams for *A*, *B* and *C*. Indicate the correct direction of all forces and accelerations.



Part 1: **2 marks for the correct FBD Block B and 2 marks for the correct FBD Block C. Zero marks if *anything* is wrong (no part marks). e.g. If you added an arrow for friction for Block C, you will get 0 marks.**

Part 2: **2 marks each for the *completely correct* FBD for Block A, B and C. 1 mark each for the *completely correct* kinetic diagram for Block A,B, and C. No part marks for either the FBD or the KD.**

Solutions:



5. A sky diver jumps from a helicopter and is falling straight down at 30 m/s when her parachute opens. From then on, her downward acceleration is approximately $a = g - cv^2$, where g is 9.81 m/s^2, c is a positive constant, and v is the velocity. After an initial "transient" period, she descends at a constant velocity of 5 m/s.

(a) What is the value of c, and what are its SI units?
(b) What maximum deceleration is the sky diver subjected to?
(c) What is her downward velocity when she has fallen 2 m from the point where her parachute opens?

SOLUTION:

1

(a) After initial transient, velocity is a constant, V_f, so acceleration is zero.

4

$$a = g - cv^2$$
$$0 = g - cV_f^2$$
$$g = cV_f^2$$
$$c = \frac{g}{V_f^2} = \frac{9.81\ \text{m/s}^2}{(5\ \text{m/s})^2} = 0.3924\ \text{1/m}$$

$$\boxed{c = 0.392\ \text{m}^{-1}}$$

(b) $a = g - cv^2$ v changes from 30 m/s to 5 m/s

7

$\frac{da}{dv} = -2cv$ Not equal to zero in the range $5\ \text{m/s} \leq v \leq 30\ \text{m/s}$

∴ Maximum value occurs at one of the endpoints.

@ V = 30 m/s,
$$a = 9.81\ \text{m/s}^2 - (0.392\ \text{m}^{-1})(30\ \text{m/s})^2 \qquad \underline{a = -343.4\ \text{m/s}^2}$$

@ V = 5 m/s
$$a = 9.81\ \text{m/s}^2 - (0.3924\ \text{m}^{-1})(5\ \text{m/s})^2 \qquad \underline{a = 0}$$

∴ Maximum deceleration occurs at start when parachute first opens.

$$\boxed{a_{max} = 343\ \text{m/s}^2} \quad \text{deceleration}$$

(c)

s

14

Parachutist   @ s = 0, $V = \frac{ds}{dt} = 30\ \text{m/s}$

Given: $a = g - cv^2$ $a = a(v)$

Want: V @ s = 2 m

∴ use $a = v\frac{dv}{ds}$ relates a, v, s.

$$a = v\frac{dv}{ds}$$
$$g - cv^2 = v\frac{dv}{ds} \qquad \text{separate v and s}$$
$$ds = \frac{v\,dv}{g - cv^2} \qquad \text{integrate}$$
$$\int ds = \int \frac{v\,dv}{g - cv^2}$$

From integral tables provided,
$$\int \frac{x\,dx}{a + bx^2} = \frac{1}{2b}\ln\left|a + bx^2\right| + C_1$$

$\int ds = s$ Here $x = v$, $a = g$, $b = -c$

$$\therefore \int \frac{v\,dv}{g - cv^2} = \frac{-1}{2c}\ln\left|g - cv^2\right| + C_1$$

$$\therefore s = \frac{-1}{2c}\ln\left|g - cv^2\right| + C_1$$

Initial conditions: @ s = 0, V = 30 m/s

$$0 = \frac{-1}{2(0.3924\ \text{m}^{-1})}\ln\left|9.81 - (0.3924)(30)^2\right| + C_1$$
$$\underline{C_1 = 7.4398\ \text{m}}$$

$$\therefore s = \frac{-1}{2c}\ln\left|g - cv^2\right| + 7.4398\ \text{m}$$

At s = 2 m,
$$2\ \text{m} = \frac{-1}{2(0.3924\ \text{m}^{-1})}\ln\left|g - cv^2\right| + 7.4398\ \text{m}$$
$$\ln\left|g - cv^2\right| = -(2 - 7.4398)(2)(0.3924) = 4.2692$$
$$\therefore \left|g - cv^2\right| = 71.464 \qquad g - cv^2 < 0$$
$$\therefore cv^2 - g = 71.464$$
$$v = \sqrt{\frac{71.464 + 9.81}{0.3924}} = 14.39$$

$$\boxed{V = 14.4\ \text{m/s}}$$

Doing the last part by using Limits rather than “Constant of Integration”:

Using limits:

$$\int_0^2 ds = \int_{30}^{V_2} \frac{V\,dV}{g - cV^2}$$

$$s\Big|_0^2 = \left|\frac{-1}{2c}\ln|g - cV^2|\right|_{30}^{V_2}$$

$$2 - 0 = \left(\frac{-1}{2c}\ln|g - cV_2^2|\right) - \left(\frac{-1}{2c}\ln|g - c(30)^2|\right)$$

$$2 - \frac{1}{2c}\ln|g - c\,30^2| = \frac{-1}{2c}\ln|g - cV_2^2|$$

$$-5.4398 = \frac{-1}{2c}\ln|g - cV_2^2|$$

$$(-5.4398)(-2)(0.3924) = \ln|g - cV_2^2|$$

$$4.2692 = \ln|g - cV_2^2|$$

$$71.4609 = |g - cV_2^2| \qquad g - cV_2^2 < 0$$

$$\therefore\ 71.4609 = cV_2^2 - g$$

$$V_2 = \sqrt{\frac{71.4609 + 9.81}{0.3924}} = 14.39$$

$$\boxed{V_2 = 14.4\ \text{m/s}}$$

6. At the instant illustrated, the rocket is experiencing a thrust (and hence acceleration) from its engine and is also being accelerated downward due to the force of its weight. The *net* acceleration of the rocket is:

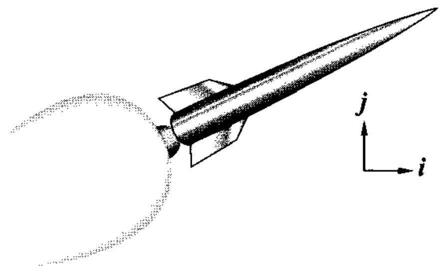
$$\boldsymbol{a} = (5.54\boldsymbol{i} + 3.84\,\boldsymbol{j})\ \text{m/s}^2 .$$

The local acceleration due to gravity at the rocket's position is 9.50 m/s^2 in the $-\boldsymbol{j}$ direction and the rocket's velocity vector is:

$$\boldsymbol{v} = (5000\boldsymbol{i} + 2000\,\boldsymbol{j})\ \text{m/s}.$$

At this instant, determine:

(a) the acceleration due to the rocket's engine only,
(b) the tangential component of the net acceleration,
(c) the normal component of the net acceleration,
(d) the radius of curvature of the rocket's path.



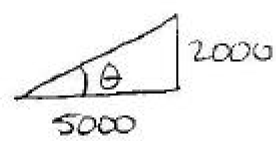
SOLUTION:

$\underline{a}_{THRUST} = \underline{a} - \underline{g}$

$= \underline{a} - (-9.5\,\underline{j})$

$= (5.54\,\underline{i} + 13.34\,\hat{j})\ \frac{m}{s^2}$

b) Angle of tangent $= \tan^{-1}\left(\frac{2}{5}\right) = 21.80°$

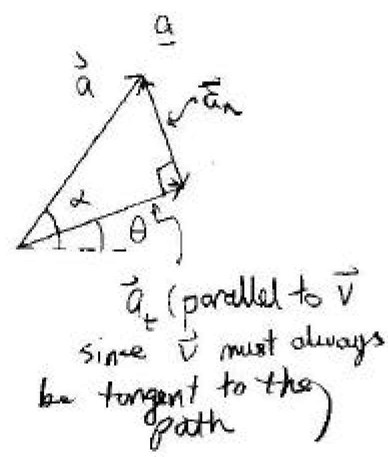


c) $\underline{a} = \sqrt{a_x^2 + a_y^2} = 6.741\ \frac{m}{s^2}$

$\alpha = \tan^{-1}\left(\frac{3.84}{5.54}\right) = 34.73°$

$a_n = a\sin(\alpha - \theta) = 1.508\ m/s^2$

$a_t = a\cos(\alpha - \theta) = 6.570\ m/s^2$



d) $\rho = \frac{v^2}{a_n} = 1.923 \times 10^7\ m$

Additional comments regarding the marking of Problem 6.
(a) 6 marks (d) 5 marks
(b) and (c): 16 marks total

General Marking Scheme for #6 b) and c):

These were marked as a whole and were worth 16 of the 27 marks.
Parts a) and d) were marked quite easily – these two sections were marked quite rigorously.

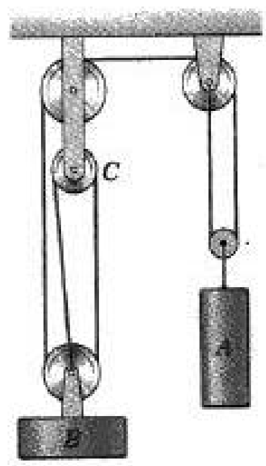
First, if the angle of the velocity was correctly calculated (or a unit vector in its direction) and indicated anywhere in the problem as a whole, but all else was incorrect, this received a total of 4/16.

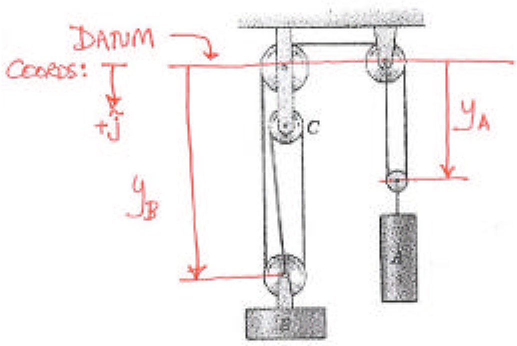
If the angle between the tangent was not calculated at all, and the angle between the acceleration and velocity was not calculated in some way (dot product), there was no hope of solving the problem (0/16).

Roughly, the breakdown for correct answers was: angle (4), a_t (6), a_n (6).

There was quite a bit of variety in the errors people made, but most were due to improper manipulations of vectors or of a dot product.
Error: $\mathbf{a}_t = \|\mathbf{a}\|\mathbf{u}_t$ but used correctly apart from this: Total 8/16
Error: $\mathbf{a}_t = \mathbf{a}\cdot\mathbf{u}_t$ and left as a vector (the equation makes no sense!): Total: 4/16
Error: Added angles of velocity and acceleration instead of doing the appropriate subtraction (fundamental lack of understanding): Total: 4/16
Error: Used $\mathbf{a}_{\text{thrust}}$ instead of $\mathbf{a}$ (fundamental lack of understanding): Total 4/16
Error: Correct a_t but wrong a_n: Total 10/16

7. In the pulley configuration shown, block A has a downward velocity of 0.3 m/s. Determine the relative velocity of block B with respect to block A, i.e. $\mathbf{v}_{B/A}$.





(3) Datum, diagram, coordinate system

Length of cable $L = 3y_B + 2y_A + \text{constant}$ (3)

take $\frac{d}{dt} L = 0 = 3v_B + 2v_A$

$\therefore \boxed{v_B = -\frac{2}{3} v_A}$ (3)

also: $\bar{v}_B = \bar{v}_A + \bar{v}_{B/A} \Rightarrow \boxed{\bar{v}_{B/A} = \bar{v}_B - \bar{v}_A}$ (3)

$\therefore \bar{v}_{B/A} = -\frac{2}{3}\bar{v}_A - \bar{v}_A = -\frac{5}{3}\bar{v}_A = -\frac{5}{3}(0.3)\tilde{j}$

taking $+\downarrow$ → $0.3\tilde{j}$

$= -0.5 \frac{m}{s}$

$\tilde{j}\downarrow$ $\boxed{\bar{v}_{B/A} = -0.5 \frac{m}{s} \tilde{j}}$

(3)
−1 if no true coordinate system indicated
−1 if no DIRECTION ($\bar{V}_{B/A}$ is a VECTOR!)

OR $\bar{V}_{B/A} = 0.5 \frac{m}{s}$ directly upwards

OR $\bar{V}_{B/A} = 0.5 \frac{m}{s} \tilde{j}$ $\uparrow\tilde{j}$